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PARTIAL-SYMMETRIES OF WEAK INTERACTIONS

SHELDON L. GLASHOW †

Institute for Theoretical Physics, University of Copenhagen, Copenhagen, Denmark

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Abstract: Weak and electromagnetic interactions of the leptons are examined under the hypothesis that the weak interactions are mediated by vector bosons. With only an isotopic triplet of leptons coupled to a triplet of vector bosons (two charged decay-intermediaries and the photon) the theory possesses no partial-symmetries. Such symmetries may be established if additional vector bosons or additional leptons are introduced. Since the latter possibility yields a theory disagreeing with experiment, the simplest partially-symmetric model reproducing the observed electromagnetic and weak interactions of leptons requires the existence of at least four vector-boson fields (including the photon). Corresponding partially-conserved quantities suggest leptonic analogues to the conserved quantities associated with strong interactions: strangeness and isobaric spin.

1. Introduction

At first sight there may be little or no similarity between electromagnetic effects and the phenomena associated with weak interactions. Yet certain remarkable parallels emerge with the supposition that the weak interactions are mediated by unstable bosons. Both interactions are universal, for only a single coupling constant suffices to describe a wide class of phenomena: both interactions are generated by vectorial Yukawa couplings of spin-one fields ††. Schwinger first suggested the existence of an "isotopic" triplet of vector fields whose universal couplings would generate both the weak interactions and electromagnetism — the two oppositely charged fields mediate weak interactions and the neutral field is light [2]). A certain ambiguity beclouds the self-interactions among the three vector bosons; these can equivalently be interpreted as weak or electromagnetic couplings. The more recent accumulation of experimental evidence supporting the $\Delta I = \frac{1}{2}$ rule characterizing the non-leptonic decay modes of strange particles indicates a need for at least one additional neutral intermediary [3]).

The mass of the charged intermediaries must be greater than the K-meson mass, but the photon mass is zero — surely this is the principal stumbling block in any pursuit of the analogy between hypothetical vector mesons and photons. It is a stumbling block we must overlook. To say that the decay intermediaries

† National Science Foundation Post-Doctoral Fellow. Present Address: Physics Department, California Institute of Technology, Pasadena.

†† A scalar intermediary is also conceivable. See ref. [1]).

together with the photon comprise a multiplet leads to no more than an excessively obscure notation unless a principle of symmetry is discovered which can relate the forms of weak and electromagnetic couplings. Because of the large mass splittings among the vector mesons, only a very limited symmetry among them may be anticipated. The purpose of this note is to seek such symmetries among the interactions of leptons in order to make less fanciful the unification of electromagnetism and weak interactions.

2. Partially-Symmetric Interactions

In the conventional Lagrangian formulation of quantum field theory the relation between symmetries of the Lagrange function and conservation laws is well known. We recently introduced the notion of "partial-symmetry" — invariance of only part of the Lagrange function under a group of infinitesimal transformations [4]). The part of the Lagrange function bilinear in the field variables which produces masses of the elementary particles need not be invariant under a partial-symmetry. Corresponding "partial-conservation laws" become conservation laws only with the neglect of appropriate masses or mass differences. This is the only sort of symmetry which could relate the massive decay intermediaries to the massless photon.

The most familiar example of a partial-symmetry is produced by an infinitesimal change of scale. If we change the coordinates, $x_\mu \to (1+\lambda)x_\mu$, and at the same time replace each field variable χ by $(1-\lambda)\chi$, that part of the integrated Lagrange function not involving dimensional parameters will be left unchanged. As long as all the interactions involve only dimensionless coupling constants the scale transformation will be a partial-symmetry. To require this partial-symmetry excludes such interactions as ps—pv meson theory and direct four-Fermion couplings.

Another kind of partial-symmetry has been recently examined by Gell-Mann and his collaborators [5]). They are led to a proportionality between the divergence of the axial-vector weak interaction current and the pion field. This they recognize as a partial-conservation law. (They must neglect both weak and electromagnetic interactions. We shall demand partial-symmetry of these interactions themselves.) It can result from the invariance of strong interactions under the infinitesimal unitary transformation,

$$\psi_N \to (1+ia\gamma_5\tau^{\pm})\psi_N,$$

where ψ_N are the nucleon fields, together with appropriate accompanying transformations of meson and hyperon fields. Requiring this partial-symmetry in order to generate a partially-conserved axial-vector current provides them a powerful restriction on the acceptable form of the strong interactions. If we may invert historical sequence for the sake of pedagogy, in the same fashion the assertion of Feynman and Gell-Mann [6]) that the vector weak-interaction current is conserved could have led to the discovery that the strong interactions are charge-independent.

A last prerequisite to our discussion is the possibility of constructing partially-symmetric interactions among a triplet of vector mesons, $Z_\mu{}^1$, $Z_\mu{}^2$ and $Z_\mu{}^3$. We shall assume that under the CP transformation, $Z_0{}^1 \rightarrow -Z_0{}^1$, $Z_0{}^2 \rightarrow +Z_0{}^2$, and $Z_0{}^3 \rightarrow +Z_0{}^3$. The most general self-interaction tri-linear in the fields and consistent with CP invariance † is

$$g_1 Z^1_{\mu\nu} Z_\mu{}^2 Z_\nu{}^3 + g_2 Z^2_{\mu\nu} Z_\mu{}^3 Z_\nu{}^1 + g_3 Z^3_{\mu\nu} Z_\mu{}^1 Z_\nu{}^2 + f_1 Z^1_{\mu\nu} Z^2_{\nu\lambda} Z^3_{\lambda\mu} + f_2 Z^2_{\mu\nu} Z^3_{\nu\lambda} Z^1_{\lambda\mu} + f_3 Z^3_{\mu\nu} Z^1_{\nu\lambda} Z^2_{\lambda\mu}.$$

To obtain a three-parameter group of partial-symmetries, we must choose $g_1 = g_2 = g_3$ and $f_1 = f_2 = f_3$. The resulting partial-conservation laws are

$$\partial_\mu (Z^1_{\mu\nu} Z_\nu{}^2 - Z^2_{\mu\nu} Z_\nu{}^1) = (M_1{}^2 - M_2{}^2) Z_\mu{}^1 Z_\mu{}^2,$$
$$\partial_\mu (Z^2_{\mu\nu} Z_\nu{}^3 - Z^3_{\mu\nu} Z_\nu{}^2) = (M_2{}^2 - M_3{}^2) Z_\mu{}^2 Z_\mu{}^3,$$
$$\partial_\mu (Z^3_{\mu\nu} Z_\nu{}^1 - Z^1_{\mu\nu} Z_\nu{}^3) = (M_3{}^2 - M_1{}^2) Z_\mu{}^3 Z_\mu{}^1,$$

corresponding to the infinitesimal transformations

$$\mathbf{Z} \rightarrow (1 + i\mathbf{a} \cdot \mathbf{t})\mathbf{Z},$$

where the $\mathbf{t}$ are the conventional anti-symmetric imaginary 3×3 matrices.

Without the quadrupole couplings (they involve implicit cubes of momenta), and with $M_1 = M_2$ and $M_3 = 0$, this partially-symmetric interaction describes the electrodynamics (with $Z_\mu{}^3$ the vector potential) of a charged pair of vector bosons, $Z^\pm = (Z^1 + iZ^2)/\sqrt{2}$, with gyromagnetic ratio of two and with no electric quadrupole coupling. Some properties of this version of spin-one electrodynamics we have discussed elsewhere [4]).

3. Interactions of Leptons

We consider the interactions of a multiplet of vector bosons with a triplet of real Majorana fields, $\psi_1 = (\psi^+ + \psi^-)/\sqrt{2}$, $\psi_2 = i(\psi^+ - \psi^-)/\sqrt{2}$ and ψ_3. The mass-producing term has the form $m\psi\beta t_3{}^2\psi$, so that we may regard $\psi^\pm$ as the positon and negaton and ψ_3 as that variety of neutrino produced in association with negatons and positons. To escape the $\mu \rightarrow e + \gamma$ difficulty [7]), we assume (with Schwinger [2])) that a quite distinct triplet of fields describes muons and those neutrinos produced with muons. No interaction shall be introduced that couples the one triplet to the other. The interactions between the Fermions and

† Our choice of the CP behaviour of the Z-meson triplet is such as to exclude the interactions $\varepsilon^{\mu\nu\lambda\sigma} Z^1_{\mu\nu} Z^2_\lambda Z^3_\sigma$ and $\varepsilon^{\mu\nu\lambda\sigma} Z^1_{\mu\nu} Z^2_{\lambda\beta} Z^3_{\beta\sigma}$.

the vector mesons should include both electromagnetism and such weak interactions as are necessary to produce observed decay phenomena.

The interaction Lagrangian,

$$g\mathbf{Z}_\mu \cdot \mathbf{J}_\mu = g\mathbf{Z}_\mu \cdot [(\mathbf{Z}_{\mu\nu} \times \mathbf{Z}_\nu) + i\psi\beta\gamma_\mu \mathbf{O}\psi], \tag{3.1}$$

includes both a symmetrical self-interaction of the Z-triplet and the Yukawa interaction of the bosons to the Fermions. The common coupling strength is in accord with the universality both of the electric charge and of the weak interaction coupling constant. For partial-symmetry, the three imaginary anti-symmetric matrices $\mathbf{O}$ must satisfy the commutation relations of an angular momentum,

$$\mathbf{O} \times \mathbf{O} = i\mathbf{O}. \tag{3.2}$$

In that case three partially-conserved currents

$$\mathbf{J}_\mu = \mathbf{Z}_{\mu\nu} \times \mathbf{Z}_\nu + \psi\beta\gamma_\mu \mathbf{O}\psi \tag{3.3}$$

result from the three infinitesimal transformations (i.e., partial-symmetries)

$$\psi \to (1 + i\mathbf{a} \cdot \mathbf{O})\psi, \quad \mathbf{Z} \to (1 + i\mathbf{a} \cdot \mathbf{t})\mathbf{Z}. \tag{3.4}$$

The $\mathbf{O}$ must commute with $\beta\gamma_\mu$ in order to leave invariant the kinematic terms of the Lagrange function. They must have the form $\mathbf{O} = \mathbf{A} + i\gamma_5 \mathbf{S}$, where the $\mathbf{A}$ are anti-symmetric, the $\mathbf{S}$ are symmetric, and both are Hermitean 3×3 matrices acting between the ψ_i. Further restrictions upon the $\mathbf{O}$ arise if we demand CP invariance and limit the currents to neutral and singly charged. The simplest choice is evidently $\mathbf{O} = \mathbf{t}$. Certainly $J_\mu{}^3$ is the total electrical current, so that $Z_\mu{}^3$ may be interpreted as the electromagnetic vector potential. But the charged currents do not display symmetric parity violation, hence they cannot reproduce the observed weak interactions.

Other matrices satisfying (3.2) but violating parity conservation are obtained from the $\mathbf{t}$ by a unitary transformation,

$$U\mathbf{t}U^{-1} = \mathrm{e}^{-(\gamma_5 t_3{}^2\theta)}\, \mathbf{t}\, \mathrm{e}^{+(\gamma_5 t_3{}^2\theta)}.$$

Clearly U commutes with t_3 so that the leptons' electrical current $j_\mu{}^3$ remains unchanged. To obtain symmetric parity violation in the charged currents we take $\theta = \frac{1}{4}\pi$, whereupon

$$'O_1 = (t_1 + i\gamma_5\{t_2, t_3\})/\sqrt{2}, \qquad 'O_2 = (t_2 - i\gamma_5\{t_1, t_3\})/\sqrt{2}, \tag{3.5}$$

where curly brackets signify anti-commutators. Unfortunately the charged currents $j_\mu{}^{1,2} = \psi\beta\gamma_\mu\, 'O_{1,2}\psi$ are not acceptable weak interaction currents. The parity-violating unitary transformation U is not invariant under the CP transformation so that, under CP,

$$'O_1 \to \tfrac{1}{2}(t_1 - i\gamma_5\{t_2, t_3\}).$$

In terms of charge eigenstates these currents involve the interaction of both negaton and positon to the same handed neutrino. The theory generated with $'\mathbf{O}$, partially-symmetric and parity violating though it may be, is not a faithful model of the weak and electromagnetic interactions of leptons.

Recent experiments have determined the form of the charged leptonic currents. We shall choose O_1 and O_2 in accordance with these experiments. Negatons are produced in elementary-particle decays only in association with left-handed neutrinos whereas positons are accompanied by right-handed ones. The correct charged currents $j_\mu^{1,2} = \psi\beta\gamma_\mu O_{1,2}\psi$ are produced with [2])

$$O_1 = (t_1 + i\gamma_5\{t_1, t_3\})/\sqrt{8}, \qquad O_2 = (t_2 + i\gamma_5\{t_2, t_3\})/\sqrt{8}. \tag{3.6}$$

The decay interaction is partially-symmetric only when all three vector bosons participate, $g\mathbf{Z}_\mu \cdot \mathbf{j}_\mu$, where the neutral current is given by $j_\mu^3 = \psi\beta\gamma_\mu O_3\psi$ and

$$O_3 = -i[O_1, O_2] = \tfrac{1}{4}(t_3 + i\gamma_5(3t_3^2 - 2)). \tag{3.7}$$

It is readily seen that these three O_i satisfy (3.2). Since j_μ^3 is not the lepton electric current, Z_3 cannot be interpreted as the electromagnetic field. Thus the theory containing only the necessary weak interactions of two oppositely charged decay intermediaries together with the electromagnetic interactions of both the leptons and the bosons is not partially-symmetric.

4. Partially-Symmetric Synthesis

In order to achieve a partially-symmetric theory of weak and electromagnetic interactions, we must go beyond the hypothesis of only a triplet of vector bosons and introduce an additional neutral vector boson Z_S. It will have the same behaviour under CP as Z_3 and it is coupled to its own neutral lepton current $J_\mu^S = \psi\beta\gamma_\mu S\psi$. The three partial symmetries of section 3 are undisturbed by this new interaction provided that

$$[\mathbf{O}, S] = 0. \tag{4.1}$$

We use the $\mathbf{O}$ that yield correct weak interactions of charged currents, (3.6) and (3.7), and we define

$$S = \tfrac{3}{4}(t_3 - i\gamma_5(t_3^2 - \tfrac{2}{3})). \tag{4.2}$$

Note that (4.1) is satisfied, and moreover,

$$O_1^2 + O_2^2 + O_3^2 + S^2 = 1 \tag{4.3}$$

and

$$Q = t_3 = O_3 + S. \tag{4.4}$$

The last relation suggests the analogous expression relating strangeness and the third component of isobaric spin to the electrical charge of strongly interacting particles. Far more transparent expressions for $\mathbf{O}$ and S and for the relations among them emerge in a notation wherein the handedness of leptons is diagonal †.

The interaction Lagrange function including the couplings of four vector bosons is

$$e \sec\theta \; \mathbf{Z}_\mu \cdot [(\mathbf{Z}_{\mu\nu} \times \mathbf{Z}_\nu) + \psi\beta\gamma_\mu \mathbf{O}\psi] + e \csc\theta \; Z_\mu{}^S \psi\beta\gamma_\mu S\psi. \qquad (4.5)$$

The parameter θ appears in order to permit an arbitrary choice of the strengths of the triplet and singlet interactions. The three partial-symmetries (3.4) have been preserved and an additional partial-symmetry yielding a partial-conservation law for $J_\mu{}^S$ is obtained,

$$\psi \to (1+ibS)\psi, \qquad \mathbf{Z} \to \mathbf{Z}, \qquad Z^S \to Z^S. \qquad (4.6)$$

The reader may wonder what has been gained by the introduction of another neutral vector meson. Neither Z_3 nor Z_S interacts with the electrical current so that neither interaction may be identified with electromagnetism. (To have chosen $J_\mu{}^S$ to be the electrical current would have violated (4.1) and lost partial-symmetry.) However, both the neutral fields have the same CP property so that linear combinations of the two fields may correspond to "particles." The most general form for the boson mass producing part of the Lagrange function is a positive-definite bilinear expression in $Z_\mu{}^3$ and $Z_\mu{}^S$ whose diagonalization identifies those linear combinations of the fields which display unique masses (i.e., the "particles"). Most generally we may have

$$L_M = \tfrac{1}{2} M_A{}^2 (Z_\mu{}^3 \cos\theta' + Z_\mu{}^S \sin\theta')^2 + \tfrac{1}{2} M_B{}^2 (Z_\mu{}^S \cos\theta' - Z_\mu{}^3 \sin\theta')^2,$$

in which the fields

$$A_\mu = Z_\mu{}^3 \cos\theta' + Z_\mu{}^S \sin\theta', \qquad B_\mu = Z_\mu{}^S \cos\theta' - Z_\mu{}^3 \sin\theta' \qquad (4.7)$$

describe spin-one particles with masses M_A and M_B. In terms of these fields the interaction Lagrange function (4.5) becomes

$$eA_\mu J_\mu{}^Q + eF_{\mu\nu}\mathbf{Z}_\mu t_3 \mathbf{Z}_\nu + e \sec\theta (Z_\eta{}^1 j_\mu{}^1 + Z_\mu{}^2 j_\mu{}^2)$$
$$- e\tan\theta (B_{\mu\nu}\mathbf{Z}_\mu t_3 \mathbf{Z}_\nu + B_\mu \mathbf{Z}_{\mu\nu} t_3 \mathbf{Z}_\nu) + eB_\mu \psi\beta\gamma_\mu (S\cot\theta - O_3 \tan\theta)\psi, \qquad (4.8)$$

when θ' is put equal to θ. The total electrical current of leptons and bosons is denoted by $J_\mu{}^Q$,

$$J_\mu{}^Q = \mathbf{Z}_{\mu\nu} t_3 \mathbf{Z}_\nu + \psi\beta\gamma_\mu (S+O_3)\psi.$$

The interaction of A_μ is precisly the electrodynamic interaction of the charged

† We are indebted to Professor M. Gell-Mann for this observation, and for presenting part of our work, in his notation, at the 1960 Conference on High Energy Physics in Rochester, New York.

Fermions and of the charged vector mesons (with gyromagnetic ratio of two). We may identify A_μ with the electromagnetic field if we put $M_A = 0$. This isolation of the electromagnetic interaction has been possible only because of (4.4) and our apparently arbitrary choice of S is now justified. The interactions of Z_1 and Z_2 generate the correct parity-violating decay interactions with coupling constant $g_W = e \sec \theta$. Remaining terms in (4.8) comprise the interaction of B with a neutral partially-conserved current. They are the price we must pay for partial-symmetry. The symmetries of (4.5) under the four-parameter group of transformations (3.4 and 4.6) are unaffected by the re-definition of fields required by the mass-producing part of the Lagrange function. Expressed in terms of A and B one of these symmetries yields the conservation of electrical charge; the other three shuffle the photon with decay-intermediaries and lead only to partial-conservation laws for the three weak interaction currents. For no choice of θ is the interaction of the neutral current small compared with weak interactions involving charged currents. The masses of the charged intermediaries M_Z and of the neutral M_B are as yet arbitrary.

5. Discussion

It seems remarkable that both the requirement of partial-symmetry and quite independent experimental considerations indicate the existence of neutral weakly interacting currents. It would be gratifying if the introduction of only a single neutral vector-meson field B could secure both partial-symmetry and the $\Delta I = \frac{1}{2}$ rule. Whether this is possible depends upon the extension of our work to the interactions of the vector-meson multiplet with strongly interacting particles. But the roles of B and of $Z^{\pm}$ are far from symmetrical in the leptonic decays of strange particles. Indeed, the modes $K \to \pi + \nu_R + \nu_L$ and $K \to \pi + e^+ + e^-$ have never been seen. Since the coupling strengths of Z to charged leptonic currents and of B to its neutral current are limited by the requirement of partial-symmetry, the absence of neutral leptonic decay modes must be attributed to a mass splitting between $Z^{\pm}$ and B. The unobserved modes are then suppressed by the factor $(M_Z/M_B)^4$. Does not this mass splitting prevent the symmetrical participation of the three decay intermediaries prerequisite to a $\Delta I = \frac{1}{2}$ rule? While in a leptonic decay mode, the momentum of the decay intermediary is just the sum of the momenta of the emerging leptons, in non-leptonic modes one must integrate over all momenta of the intermediary. Thus the matrix element for non-leptonic modes is expected to be less sensitive a function of the vector-meson mass than the matrix element for non-leptonic modes. Vector meson theory is less well-behaved than quantum-electrodynamics or pseudoscalar meson theory, so that it is not unreasonable to suppose that the non-leptonic modes are dominated by contributions at virtual momenta far beyond M_B. If this is so the mass difference between

$Z^{\pm}$ and B would be without significant effect and a $\Delta I = \frac{1}{2}$ rule might be secured.

To assure the experimentally observed selection rules of strangeness as well as of isotopic spin, two neutral decay intermediaries are needed [3]). One of these has CP behaviour opposite to that of A and B. It is not discouraging that this particle has not yet appeared in the interactions of the vector-meson multiplet with leptons, since no such interaction exists consistent with CP invariance.

We have argued that any underlying symmetries relating weak interactions and electromagnetism are obscured by the masses of elementary particles. Without a theory of the origins of these masses, any study based upon the analogy between decay-intermediaries and photons may make use only of partial-symmetries. The simplest partially-symmetric system exhibiting all known interactions of the leptons, the weak and the electromagnetic, has been determined. Although we cannot say *why* the weak interactions violate parity conservation while electromagnetism does not, we have shown *how* this property can be embedded in a unified model of both interactions. Unfortunately our considerations seem without decisive experimental consequence. For this approach to be more than academic, a partially-symmetric system correctly describing all decay modes of all elementary particles should be sought.

Appendix

A.1. ANOTHER ALTERNATIVE

In section 4 we said it is necessary to go beyond the framework of a triplet of vector mesons and a triplet of leptons in order to obtain a partially-symmetric system including the known interactions. This is true, but it is not entirely obvious that the introduction of additional lepton fields rather than an additional boson would not also do the trick. Two triplets of real Fermion fields were needed to describe all the leptons — one triplet including the electrons and one the muons, each with its own neutrino. Let us keep our original triplet of vector mesons while introducing two new (i.e., unobserved) Fermion fields. They will correspond to a massive neutral muon and to its distinct antiparticle. Altogether there are now eight kinds of Fermions. Half of them are defined to be leptons: μ^+, μ^0, e^-, and ν; and their antiparticles are anti-leptons. Conservation of lepton number will assure the absence of unwanted modes of muon decay. Four by four matrices which describe isobaric spin $\frac{1}{2}$ are the most convenient to use †. The doublet (μ^+, μ^0) is characterized by $\zeta_3 = +1$, while (e^-, ν) has $\zeta_3 = -1$. Similarly, the doublet (μ^+, ν) has $\tau_3 = +1$, while (e^-, μ^0) has $\tau_3 = -1$. The electrical charge has the form $Q = \frac{1}{2}(\tau_3 + \zeta_3)$. We assume that the μ^0 is sufficiently massive so that the possibility of $\pi \to \mu^0 + e$ is avoided. The problem is again to find a set of **O** satisfying (3.2) which generates charged

† The notation of J. Schwinger is employed. See ref. [2]).

currents embodying symmetric parity violation yet conserving CP. Since we have introduced only a triplet of vector mesons, the neutral must be the photon and $O_3 = \frac{1}{2}(\tau_3+\zeta_3)$. Such a set of **O** is

$$O_1 = [\tau_1(1+i\gamma_5\zeta_3)+\zeta_1(1-i\gamma_5\tau_3)]/\sqrt{8},$$
$$O_2 = [\tau_2(1+i\gamma_5\zeta_3)+\zeta_2(1-i\gamma_5\tau_3)]/\sqrt{8}.$$

With these **O** we may construct a partially-symmetric theory including both weak and electromagnetic interactions of the leptons. Unfortunately it is the wrong theory. If the electron is associated with a left-handed neutrino, the μ^- will be associated with a right-handed one. (A change of sign produces a converse assignment.) But it cannot be arranged for both electron and μ^- to be coupled with the same handed neutrino. Experiments indicate that the handedness of the neutrino is determined solely by the lepton's charge. With this theory muons would decay according to the schemes: $\mu^+ \rightarrow e^+ + 2\nu_R$ and $\mu^- \rightarrow e^- + 2\nu_L$. The electron spectrum from muon decay would necessarily be of the $\rho = 0$ shape and is now probably excluded by experiment. A more acceptable set of just three **O** has not been found.

A.2. OTHER INTERACTIONS

One might conclude from existing experimental determinations of the electrical charge and of the weak interaction strength g_W^2/M_Z^2 that $M_Z > \approx 137$ nucleon masses. But whatever mechanism is responsible for the large mass of the intermediaries should also produce a large wave-function renormalization of the massive vector fields not shared by the photon. This results in a reduction of the weak coupling strength compared with the electric charge and consequently relaxes this lower limit to the Z-meson mass.

Both its mass and its charge renormalization would arise if there were strong interactions quadratic in the Z-field. Should these couplings involve strongly interacting particles they would generate effective six-Fermion interactions of comparable strength to the existing four-Fermion couplings responsible for decay phenomena. A possible experimental consequence of these interactions is the stimulated decay of a muon:

$$\mu^- + N = e^- + \nu_R + \nu_L + N,$$

in the presence of nuclear matter Observable effects upon the decay rate of negative muons bound to heavy nuclei might result, but at this time we cannot exclude the possible existence of strong interactions of the decay intermediaries. Of course these interactions could also be detected in a search for the real production of decay intermediaries.

A.3. ANOTHER SYMMETRY

If only for completeness, we exhibit one remaining symmetry of (4 5)

Defining $W = 1 - t_3^2 + i\gamma_5 t_3$, we discover $W\mathbf{O} = \mathbf{O}W = \mathbf{O}$ and $WS = SW$. The infinitesimal transformation $\psi \to (1 + ia\gamma_5 W)\psi$ is not merely a partial symmetry, but it is a complete symmetry providing the neutrino mass is zero. The corresponding conserved current,

$$J_\mu^{\mathrm{W}} = \psi\beta\gamma_\mu(t_3 - i\gamma_5(1 - t_3^2))\psi,$$

describes the "neutrinic" quantum number of Schwinger [2]). It is the leptons' electrical charge plus the number of left-handed neutrinos less the number of right-handed ones.

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