Ludwig-Maximilians-Universität München

QCD and Standard Model

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Guidelines :

- The exam consists of 7 problems.
- The duration of the exam is 4 hours.
- Please write your name or matriculation number on every sheet that you hand in.
- Your answers should be comprehensible and readable.

GOOD LUCK !

Exercise 1	9 P
Exercise 2	23 P
Exercise 3	13 P
Exercise 4	9 P
Exercise 5	20 P
Exercise 6	16 P
Exercise 7	10 P

Total	100 P

Problem 1 (9 points)

Consider the following Lagrangian of a real scalar field Φ in 4 spacetime dimensions

$$\mathcal{L} = \frac{1}{2}\partial_\mu \Phi \partial^\mu \Phi + \frac{m^2}{2}\Phi^2 - \frac{\lambda}{4}\Phi^4 \ ,$$

with $m^2, \lambda > 0$.

a) What is the symmetry of the above Lagrangian ?

b) Minimize the potential and determine the ground state of the system. Is the symmetry broken spontaneously ?

c) How many Goldstone bosons are in the spectrum ? Justify your answer.

Problem 2 (23 points)

Consider a theory invariant under a local SU(2) symmetry with a scalar field in the adjoint representation of the group, i.e., $\phi = \sum_{i=1}^{3} \phi^i T^i$, where the ϕ^i's are real and T^i's the Hermitian generators of SU(2).

a) Write down the most general renormalizable gauge SU(2)- and Lorentz-invariant Lagrangian in four spacetime dimensions.

b) Arrange the potential such that the vacuum expectation value (vev) of the scalar field is non-zero. Find the vev by explicitly minimizing the potential.

c) What is the unbroken symmetry group ? How many gauge bosons acquire a mass and how many remain massless ? Justify your answer.

d) Determine the masses of the gauge bosons.

e) Can this model alone describe the gauge electroweak interactions ? Justify your answer.

Problem 3 (13 points)

a) Add a right-handed neutrino to the Lagrangian of the SM, which only has a Majorana mass term. Write down the corresponding Lagrangian. Does this particle conserve lepton number ? Verify your statement by an explicit calculation.

b) Is a Majorana mass term invariant under parity transformations? Verify your statement by an explicit calculation.
Hint : Do not forget the phase.

c) Draw a Feynman diagram for the process $n + n \to p + p + e^- + e^-$. Can this process happen within the SM (including the above modification)?

Problem 4 (9 points)

a) Draw two tree-level Feynman diagrams that describe the decay of the Higgs particle to the W and Z gauge bosons (one each).

b) Are these processes kinematically allowed?

c) Write down the interaction vertex governing the Higgs-W^+W^- coupling.

Problem 5 (20 points)

Let us focus on the fermionic sector of the SM (without right-handed neutrinos).

a) Set all the Yukawa couplings to zero. What is the global symmetry group in this case (apart from the gauged $SU(2) \times U(1)$)?

b) Assume that the Yukawa couplings are non-zero diagonal matrices. What is the global symmetry group in this case?

Problem 6 (16 points)

Consider the following Lagrangian of a complex scalar field ϕ in 4 spacetime dimensions

$$\mathcal{L} = \partial_\mu \phi^* \partial^\mu \phi + m^2 \phi^* \phi - \lambda (\phi^* \phi)^2 + \mu^2 (\phi^* + \phi)^2 \ .$$

Here $m^2, \lambda, \mu^2 > 0$ and $\mu \ll m$ are real parameters.

a) Determine the ground state of the theory.

b) Find the masses of the particles when the Lagrangian is expanded around the vacuum.

c) What happens in the limit $\mu \to 0$? Why? Explain.

Problem 7 (10 points)

Let us assume that in the Standard Model there are two Higgs doublets H^1 and H^2 with the same hypercharge as the conventional Higgs. Let the fields take the following vacuum expectation values

$$H_0^1 = \begin{pmatrix} 0 \\ v_1 \end{pmatrix} \;, \quad H_0^2 = \begin{pmatrix} v_2 \\ 0 \end{pmatrix} \;.$$

a) Write down the unbroken generators, if there are any.

b) What is the unbroken group?

c) How many gauge bosons acquire mass and how many remain massless?