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Connections Between the High and Low Energy Violation of Lepton and Flavor Numbers in the Minimal Left-Right Symmetric Model

Thesis submitted for the degree of

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Introduction

A theory behind neutrino masses has not yet been discovered. Soon after the existence of a neutrino was postulated in 1930 by Pauli, the young scientist Ettore Majorana in 1937 offered an alternative way to describe mathematically those particles which are absolutely neutral, i.e., those which are identical to their antiparticles [1]. An immediate phenomenological consequence that could distinguish them from the standard Dirac formulation was found soon after. If neutrinos are Majorana particles then double beta decay may occur without them having to be emitted [2]. This process known as neutrinoless double beta decay ($0\nu\beta\beta$) has not yet been observed. Later on, Bruno Pontecorvo introduced the concept of neutrino oscillations [3] in analogy with the oscillations of neutral kaons and opened a new branch of physics. He understood that if neutrinos are massive they can oscillate among different flavors. It is thanks to neutrino oscillation experiments that we are able obtain information on neutrino masses and mixing, they have established that neutrino masses are small but not zero. For a historical review see [4]. The Standard Model (SM) [5] which describes massless neutrinos thus need to be updated. One natural extension of the SM, in which neutrinos are necessarily massive is given by the Left-Right models [6]. In these models a connection between the large scale of parity restoration and the smallness of neutrino masses can be made [7]. This is nowadays known as the seesaw mechanism [7, 8]. The left-right theories offer plenty of new phenomenology [9] but more important, they offer the possibility to access to the high energy analogous of $0\nu\beta\beta$ [10]. The connection between the low energy and high energy manifestation of the same lepton number violation was recently addressed in [11] and is the main subject developed in this thesis.

Before starting with the details of the Left-Right model we comment on the general features of the seesaw mechanism. One way to parametrize the masses of neutrinos is to add to the SM Lagrangian the non-renormalizable Weinberg operator [12]:

$$\mathcal{L}_5 = \frac{c_{\alpha\beta}}{2\Lambda}\ell_L^\alpha \ell_L^\beta \phi\,\phi \tag{1}$$

where ℓ_L is the lepton doublet and ϕ is the Higgs doublet, $c_{\alpha\beta}$ is a constant matrix in the flavor space and Λ/c is the scale at which the neutrino masses are generated. The neutrino masses will be generated spontaneously through the Higgs mechanism [13] once the Higgs field takes a vacuum expectation value. The neutrino mass matrix will then be:

$$(M_\nu)_{\alpha\beta} = \frac{c_{\alpha\beta}v^2}{\Lambda} \tag{2}$$

where we have denoted $v = \langle\phi\rangle$. The coupling to the Higgs boson can also be obtained from the effective operator. In the mass basis of neutrinos it reads:

$$\mathcal{L}_{Y_\nu} = \frac{h}{\sqrt{2}v} m_{\nu_i}\nu_{iL}\nu_{iL} \tag{3}$$

where $\phi^0 = v + h/\sqrt{2}$. But due to the smallness of neutrino masses the direct test of these couplings does not seem to be realistic at present. The only hope to reveal the theory behind neutrino masses is to probe the scale $\Lambda \sim c \times 10^{14}\,\text{GeV}$. This can be done essentially in two different ways. Either by low energy high precision experiments or directly by searching new particle states with the help of high energy colliders. At the present time, we can hope to probe c as large as $c \lesssim 10^{-10}$.

The effective Weinberg operator can be realized at tree level in three different ways when one assumes only one kind of new particles. The new degrees of freedom can directly be associated with the scale of new physics. The three ways to do this are:

- Type I seesaw [7, 8]: It assumes the existence of right-handed neutrinos ν_R with zero hypercharge and singlets of $SU(2)_L$. These neutrinos are thus completely sterile under the SM gauge group. They are allowed then to have a non-zero mass term M_R at tree level, and a Dirac Yukawa coupling with the Higgs boson $y_D \bar{\nu}_L \nu_R H$. Then the scale of neutrino mass generation will be simply $\Lambda/c = M_R/y_D^2$ and this means that $M_R \sim y_D^2 \times 10^{14}\,\mathrm{GeV}$.

- Type II seesaw[14]: It assumes the existence of a scalar triplet $\Delta_L = (\Delta_L^{++}, \Delta_L^{+}, \Delta_L^{0})$ of $SU(2)_L$ with hypercharge 2. They are allowed then to have a non-zero mass term M_Δ at tree level, Yukawa coupling with the lepton doublet $y_\Delta \ell_L \ell_L \Delta_L$, and also a coupling with the Higgs doublet $\mu\, \phi\phi\Delta_L^*$ The scale of neutrino mass generation will then be $\Lambda/c = M_\Delta^2/(\mu y_\Delta)$ and therefore $M_\Delta \sim \sqrt{\mu y_\Delta} \times 10^7\,\mathrm{GeV}$.

- Type III seesaw[15]: It assumes the existence of a fermion triplet $\Sigma = (\Sigma^+, \Sigma^0, \Sigma^-)$ of $SU(2)_L$ with zero hypercharge. In analogy to Type I one has $M_\Sigma \sim y_\Sigma^2 \times 10^{14}\,\mathrm{GeV}$.

Each of the seesaw types then provides us with a minimum number of fields needed to describe the masses of neutrinos. They have been studied extensively in the literature, for a detailed review see [16]. The type I seesaw has limited phenomenology because if one wishes to produce a ν_R with the help of the Yukawa interaction, a large Yukawa coupling is needed, but this will make M_R out of the reach of any nearby experiment. Some cancellations in the flavor space or approximate lepton number symmetries [17] are also possible which can artificially allow large Yukawas and therefore increase the phenomenological domain of type I seesaw. The type II seesaw, on the contrary, is very attractive. A small enough μ allows for a low scale of M_Δ with large Yukawa couplings at the same time. From decay of the doubly charged scalar Δ_L^{++} into dileptons, these y_Δ couplings can be determined and can then be confronted with the mass matrix of light neutrinos [19]. The type III seesaw can also have interesting low scale collider phenomenology because of their gauge interactions [20].

We also address in this thesis a study of the structure of the seesaw mechanism and its connection with the Higgs mechanism. In the Left-Right model the neutrino masses will receive contributions from type I and type II seesaw. The nice feature of the model is that the right-handed neutrinos cease to be sterile and become fully interacting particles, just like their left-handed counterpart. This leads, as we have said, to the remarkable lepton number violation signature in colliders [10]. Moreover the Yukawa coupling y_D to the Higgs boson, responsible for type I seesaw contribution turns out to depend only on few observables and is therefore deeply connected to the previous signal of lepton number violation [21].

In short, this work addresses the interconnection between the high energy collider phenomenology and the low energy processes such as $0\nu\beta\beta$, lepton flavor violation and the electromagnetic dipole moments of leptons.

Chapter 1

The Left-Right Model

The most attractive characteristic of the Left-Right symmetric models comes from the relation between the high and low mass scales of the theory. In the Standard Model parity is maximally violated being essentially the gauging of the V-A theory [22, 23]. Neutrinos are massless because there are only left-handed components charged under the SM gauge group. At first sight there is no relation between the absence of V+A charged currents and the absence of massive neutrinos. Today we know, thanks to neutrino oscillations, that neutrinos have small but not zero masses, and we have no reason to believe that V+A currents do not exist. So the brilliant idea behind the Left-Right models is the connection between the large scale of the V+A currents and the smallness of neutrino masses [7].

We should also keep in mind that the Left-Right model originally described Dirac neutrinos [6], and it was only after the observation made above that the interest in the Majorana nature came back to life once more. Somehow, we believe, the model went back on the right track to the historical developments that date back to 1937, when Majorana introduced the concept of a truly neutral particle in an attempt to describe the neutron and the neutrino.

The Left-Right model is based on the gauge group:

$$G_{LR} = SU(3)_c \times SU(2)_L \times SU(2)_R \times U(1)_{B-L} \tag{1.1}$$

At low energies, it is required that G_{LR} breaks spontaneously to the Standard Model gauge group:

$$G_{SM} = SU(3)_c \times SU(2)_L \times U(1)_Y \tag{1.2}$$

We expect three extra gauge bosons, two charged $W_R^{\pm}$ analogous to the $W^{\pm}$ boson (which will be denoted hereafter by $W_L^{\pm}$ in order to keep the historical symmetric notation), and one neutral Z' analogous to the Z boson. The values of new the gauge couplings g_R and g_{B-L} are related to the known coupling g_Y by:

$$\frac{1}{g_R^2} + \frac{1}{g_{B-L}^2} = \frac{1}{g_Y^2} \tag{1.3}$$

thus their values are neither too small nor too large and it is wrong to believe that one may recover the SM by making $g_R = 0$. The electric charge generator is given by:

$$Q = T_{3L} + T_{3R} + \frac{B-L}{2} \tag{1.4}$$

where T_{3L}, T_{3R} are the diagonal generators of $SU(2)_L$ and $SU(2)_R$, and $B - L$ is the baryon minus lepton number charge operator. The fermionic particle content is chosen symmetrically as doublet representations of $SU(2)_L$ and $SU(2)_R$:

$$q_L = \begin{pmatrix} u_L \\ d_L \end{pmatrix}, q_R = \begin{pmatrix} u_R \\ d_R \end{pmatrix}, \ell_L = \begin{pmatrix} \nu_L \\ e_L \end{pmatrix}, \ell_R = \begin{pmatrix} N_R \\ e_R \end{pmatrix}, \tag{1.5}$$

with $B - L = -1$ for leptons and $B - L = \frac{1}{3}$ for quarks. We are assuming implicitly the existence of three generations of leptons and quarks. The neutrino field N has not yet been observed. We believe that its mass is considerably larger than that of light neutrinos ν, and for this reason they will be called heavy neutrinos.

1.1 The Left-Right symmetry.

The presence of right-handed currents allows us to reconsider the meaning of parity violation in the SM. With the enlarged gauge group and a symmetric particle content one can consider the possibility of having spontaneous violation of parity. In general this extra symmetry has to relate only the left-handed sector with the right handed one. There are two ways of introducing the symmetry: as generalized parity $\mathcal{P}$ or as generalized charged conjugation $\mathcal{C}$. For fermions they coincide with the usual parity and charge conjugation. For gauge boson, they are chosen in such a way as to keep the gauge interactions invariant:

$$\mathcal{P}: \{W_L, q_L, \ell_L\} \leftrightarrow \{W_R, q_R, \ell_R\} \tag{1.6a}$$

$$\mathcal{C}: \{W_L, q_L, \ell_L\} \leftrightarrow \{-W_R^\dagger, (q_R)^c, (\ell_R)^c\} \tag{1.6b}$$

A consequence of these symmetries is the equality of gauge couplings:

$$g_L = g_R = g. \tag{1.7}$$

This can of course be affected by radiative correction. The couplings can be slightly different and formally (1.7) should be taken only as a good approximation.

1.1.1 Symmetry breaking.

The Left-Right symmetry forces us to have a symmetric particle content. The minimum set of scalar fields needed to accomplish the spontaneous symmetry breaking and to provide fermion masses at tree level are:

$$\begin{aligned} &\text{Bidoublet: } \Phi(1,2,2,0), \\ &\text{Triplets Left : } \Delta_L(1,3,1,+2), \\ &\text{Triplet Right : } \Delta_R(1,1,2,+2). \end{aligned} \tag{1.8}$$

The bidoublet Φ will be responsible for breaking the SM symmetry and giving mass to charged fermions. It is made out of two doublets ϕ_1 and ϕ_2 of $SU(2)_L$ and at the same time the rows form two doublets of $SU(2)_R$. It is given by:

$$\Phi = \begin{pmatrix} \phi_1 & \phi_2^c \end{pmatrix} = \begin{pmatrix} \phi_1^0 & \phi_2^+ \\ \phi_1^- & -\phi_2^{0^*} \end{pmatrix}. \tag{1.9}$$

The triplets will break the Left-Right symmetry as well as the $B-L$ thereby giving Majorana mass to neutrinos. They belong to the adjoint representation of $SU(2)_L$ and $SU(2)_R$ and are given by:

$$\Delta_L = \begin{pmatrix} \frac{\Delta_L^+}{\sqrt{2}} & \Delta_L^{++} \\ \Delta_L^0 & -\frac{\Delta_L^+}{\sqrt{2}} \end{pmatrix}, \qquad \Delta_R = \begin{pmatrix} \frac{\Delta_R^+}{\sqrt{2}} & \Delta_R^{++} \\ \Delta_R^0 & -\frac{\Delta_R^+}{\sqrt{2}} \end{pmatrix} \tag{1.10}$$

The scalar potential is the most general renormalizable potential made out of Φ, Δ_L and Δ_R, invariant under $\mathcal{G}_{LR}$ and the Left-Right symmetry. Under this last symmetry the transformation of the scalar fields follows from the invariance of the Yukawa interactions with leptons and quarks:

$$\mathcal{P} : \left\{\Phi, \Delta_L, \Delta_R\right\} \leftrightarrow \left\{\Phi^\dagger, \Delta_R, \Delta_L\right\} \tag{1.11a}$$

$$\mathcal{C} : \left\{\Phi, \Delta_L, \Delta_R\right\} \leftrightarrow \left\{\Phi^T, \Delta_R^*, \Delta_L^*\right\} \tag{1.11b}$$

This imposes additional constraints on the Yukawa couplings and on the parameters of the potential. The most general potential is given in Appendix 1. The minimum of the potential is achieved with:

$$\langle\Phi\rangle = \mathrm{diag}(v_1, -v_2 e^{-ia}), \quad \langle\Delta_L^0\rangle = v_L = |v_L| e^{i\theta_L}, \quad \langle\Delta_R^0\rangle = v_R. \tag{1.12}$$

It is possible to keep only two vevs real by using the broken gauge symmetries. In our convention we take $\langle\phi_1^0\rangle$ and $\langle\Delta_R^0\rangle$ real and instead of v_1 and v_2 we will work with v and

β defined as follows:

$$v_1 = v\cos\beta, \qquad v_2 = v\sin\beta \tag{1.13}$$

The vev v_L will contribute to the masses of neutrinos and v_R to the scale associated with V+A currents. Therefore, the only physically acceptable vacuum must satisfy the following condition:

$$v_L \ll v \ll v_R. \tag{1.14}$$

The symmetry breaking can be understood in simple terms by imagining that the scale of Left-Right symmetry is much larger than the weak scale. This will allow us to leave aside the bidoublet and concentrate only on the scalar potential of the triplets:

$$\langle V_\Delta \rangle = -\mu^2(v_L^2 + v_R^2) + \lambda(v_L^4 + v_R^4) + \rho\, v_L^2 v_R^2 \tag{1.15}$$

or alternatively, up to a constant:

$$\langle V_\Delta \rangle = \lambda\left(v_L^2 + v_R^2 - \frac{\mu^2}{2\lambda}\right)^2 + (\rho - 2\lambda)\, v_L^2 v_R^2 \tag{1.16}$$

It is clear that when $\rho - 2\lambda < 0$ the vacuum that minimize the potential is Left-Right symmetric: $v_L = v_R$. On the contrary, when $\rho - 2\lambda > 0$ one of the vev's has to be necessarily zero, and thus the vacuum is asymmetric with $v_L = 0,\ v_R = \sqrt{\frac{\mu^2}{2\lambda}}$. However, once the SM symmetry is broken a small value of v_L will be generated. In fact, adding the quartic coupling $\Delta V \propto \mathrm{tr}\Delta_L \Phi \Delta_R \Phi^\dagger = v_L v_R v^2$ to the potential (1.16) one finds $v_L \propto \frac{v^2}{v_R}$. This proves that the Left-Right symmetry can be spontaneously broken in fully agreement with (1.14) .

Once the symmetry is broken, the original neutral gauge fields will mix and produce

Z', Z, A:

$$\begin{pmatrix} Z' \\ Z \\ A \end{pmatrix} = \begin{pmatrix} 1 & 0 & 0 \\ 0 & c_{\theta_W} & -s_{\theta_W} \\ 0 & s_{\theta_W} & c_{\theta_W} \end{pmatrix} \begin{pmatrix} c_{\theta_R} & 0 & -s_{\theta_R} \\ 0 & 1 & 0 \\ s_{\theta_R} & 0 & c_{\theta_R} \end{pmatrix} \begin{pmatrix} W_L^3 \\ W_R^3 \\ B_{B-L} \end{pmatrix} \tag{1.17}$$

The first rotation is necessary to identify Z' and B_Y and happens at the first stage when $\mathcal{G}_{LR}$ breaks to $\mathcal{G}_{SM}$. The second rotation through the weak mixing angle θ_W takes places when the $\mathcal{G}_{SM}$ breaks to $U(1)_{em}$ and serves to identify the photon A and the Z boson. The Left-Right symmetry implies that:

$$\sin\theta_R = \frac{\sin\theta_W}{\sqrt{\cos\theta_W}}. \tag{1.18}$$

The resulting gauge bosons will not be mass eigenstates and an additional rotation will be necessary. To reduce the clutter we skip writing down the mass matrices and present only well known results. For the charged gauge bosons we will need the following rotation:

$$\begin{pmatrix} W^- \\ W_R^- \end{pmatrix} \to \begin{pmatrix} 1 & \xi^* \\ -\xi & 1 \end{pmatrix} \begin{pmatrix} W^- \\ W_R^- \end{pmatrix} \tag{1.19}$$

where the complex angle $\xi = |\xi| e^{ia}$ contains the same phase of the bidoblet. For the neutral gauge bosons we will need:

$$\begin{pmatrix} Z \\ Z' \end{pmatrix} \to \begin{pmatrix} 1 & \zeta \\ -\zeta & 1 \end{pmatrix} \begin{pmatrix} Z \\ Z' \end{pmatrix} \tag{1.20}$$

with ζ real. The gauge boson masses are given by:

$$M_W^2 \simeq \frac{1}{2} g^2 v^2, \qquad M_Z^2 \simeq \frac{M_W^2}{c_W^2}, \tag{1.21a}$$

$$M_{W_R}^2 \simeq g^2 v_R^2, \qquad M_{Z'}^2 \simeq \frac{2c_W^2}{c_W} M_{W_R}^2, \tag{1.21b}$$

and the gauge mixing angles by:

$$\sin\xi \simeq \frac{M_W^2}{M_{W_R}^2}\sin 2\beta, \qquad \sin\zeta \simeq \frac{M_W^2}{M_{W_R}^2}\frac{c_{2W}^{\frac{3}{2}}}{2c_W^4}. \tag{1.22}$$

1.1.2 Gauge interactions of fermions

Using (1.17) one can write the gauge interaction of fermions in a familiar form:

$$\begin{aligned}\sum i\overline{\Psi}\gamma^\mu D_\mu \Psi = \sum \overline{\Psi} i\partial\!\!\!/ \Psi - eA_\mu J^\mu_{em} - \frac{g}{c_{\theta_W}}(Z_\mu J^\mu_Z + \frac{c^2_{\theta_W}}{\sqrt{c_{2\theta_W}}}Z'_\mu J^\mu_{Z'}) \\ - \frac{g}{\sqrt{2}}(W^+_{L\mu}J^\mu_{W_L} + W^+_{R\mu}J^\mu_{W_R}) - \frac{g}{\sqrt{2}}(W^-_{L\mu}J^{\mu\dagger}_{W_L} + W^-_{R\mu}J^{\mu\dagger}_{W_R})\end{aligned} \tag{1.23}$$

where the fermionic currents are given by:

$$J^\mu_{em} = -\bar{e}\gamma^\mu e + \frac{2}{3}\bar{u}\gamma^\mu u - \frac{1}{3}\bar{d}\gamma^\mu d \tag{1.24a}$$

$$J^\mu_Z = \sum \overline{\Psi}\gamma^\mu (T^3_L - s^2_{\theta_W} Q)\Psi \tag{1.24b}$$

$$J^\mu_{Z'} = \sum \overline{\Psi}\gamma^\mu \Big(T^3_R - \frac{s^2_{\theta_W}}{c^2_{\theta_W}}(Q - T_{3L})\Big)\Psi \tag{1.24c}$$

$$J^\mu_{W_L} = \overline{\nu_L}\gamma^\mu e_L + \overline{u_L}\gamma^\mu d_L \tag{1.24d}$$

$$J^\mu_{W_R} = \overline{N_R}\gamma^\mu e_R + \overline{u_R}\gamma^\mu d_R \tag{1.24e}$$

These currents will get mixed after rotating the gauge bosons to their mass eigenstates by means of (1.19) and (1.20). This will result in mixed interactions of the form $W_{L\mu}J^\mu_{W_R}$, $Z_\mu J^\mu_{Z'}$ and so on.

1.1.3 Yukawa interactions of leptons

The interactions between leptons and scalars are governed by the following Lagrangian:

$$L_\Phi = \overline{\ell_L}\big(y_1\Phi + y_2\Phi^c\big)\ell_R + \text{h.c.} \tag{1.25a}$$

$$L_\Delta = -\frac{1}{2}y_{\Delta_L}\overline{\ell^c_L}\epsilon\Delta_L\ell_L - \frac{1}{2}y_{\Delta_R}\overline{\ell^c_R}\epsilon\Delta_R\ell_R + \text{h.c.} \tag{1.25b}$$

Here $\Phi^c = i\sigma_2 \Phi^* i\sigma_2$. The Yukawa matrices are given in term of the vevs (1.12) and (1.13) and the mass matrices as follows:

$$y_1 = -\frac{1}{vc_{2\beta}}(c_\beta M_D^\dagger + e^{ia} s_\beta M_\ell), \qquad y_{\Delta_L} = \frac{M_L}{v_L} \tag{1.26a}$$

$$y_2 = \frac{1}{vc_{2\beta}}(e^{-ia} s_\beta M_D^\dagger + c_\beta M_\ell), \qquad y_{\Delta_R} = \frac{M_R^*}{v_R} \tag{1.26b}$$

We can insert these relations back in (1.25a) to get a more physical insight. For this, we first notice that the bidoublet is made out of two $SU(2)_L$ doublets $\Phi = (\phi_1, \phi_2^c)$. These fields are mixed and so it is desirable to work in the physical basis. For this let us call φ_1 the field that takes the vev $\langle \varphi_1 \rangle = v$ and φ_2 the field which does not and so $\langle \varphi_2 \rangle = 0$. We can then write:

$$\varphi_1 = c_\beta \phi_1 + e^{-ia} s_\beta \phi_2, \quad \varphi_2 = -e^{ia} s_\beta \phi_1 + c_\beta \phi_2 \tag{1.27}$$

where we have introduced the shorthand notation $c_\beta = \cos\beta$, etc. Inserting (1.27) and (1.26) in (1.25a) we obtain:

$$\begin{aligned} L_\Phi = &- \overline{\ell_L}\left[\frac{M_D^\dagger}{v}\varphi_1 - \frac{M_\ell + e^{-ia} s_{2\beta} M_D^\dagger}{vc_{2\beta}}\varphi_2\right] N_R \\ &+ \overline{\ell_L}\left[\frac{M_\ell}{v}\varphi_1^c - \frac{M_D^\dagger + e^{ia} s_{2\beta} M_\ell}{vc_{2\beta}}\varphi_2^c\right] e_R + \text{h.c.} \end{aligned} \tag{1.28}$$

In the same way, we can expand (1.25b) and write:

$$\begin{aligned} L_\Delta = &- \frac{1}{2}\frac{M_L}{v_L}(\Delta_L^0 \overline{\nu_L}\nu_L - \sqrt{2}\Delta_L^+ \overline{\nu_L} e_L - \Delta_L^{++}\overline{e_L^c} e_L) \\ &- \frac{1}{2}\frac{M_R^*}{v_R}(\Delta_R^0 \overline{N_R} N_R - \sqrt{2}\Delta_R^+ \overline{N_R} e_R - \Delta_R^{++}\overline{e_R^c} e_R) + \text{h.c.} \end{aligned} \tag{1.29}$$

1.1.4 Yukawa interactions of Quarks

In the same way as before we rewrite the quarks bidoublet interactions analogously to (1.28):

$$
\begin{aligned}
L_\Phi =& \overline{q_L}\big(y_1^q\Phi + y_2^q\Phi^c\big)q_R + \text{h.c.} && (1.30a)\\
=& -\overline{q_L}\left[\frac{M_u}{v}\varphi_1 - \frac{M_d + e^{-ia}s_{2\beta}M_u}{vc_{2\beta}}\varphi_2\right]u_R && (1.30b)\\
&+\overline{q_L}\left[\frac{M_d}{v}\varphi_1^c - \frac{M_u + e^{ia}s_{2\beta}M_d}{vc_{2\beta}}\varphi_2^c\right]d_R + \text{h.c.} && (1.30c)
\end{aligned}
$$

1.1.5 The Left-Right symmetry and the mass matrices

The Yukawa couplings are not arbitrary complex matrices. The Left-Right symmetry imposes the following relations among them:

$$
\begin{aligned}
\mathcal{P}:&\quad y_{1,2} = y_{1,2}^\dagger, \quad y_{\Delta_L} = y_{\Delta_R}, \qquad y_{1,2}^q = y_{1,2}^{q\dagger} && (1.31a)\\
\mathcal{C}:&\quad y_{1,2} = y_{1,2}^T, \quad y_{\Delta_L} = y_{\Delta_R}^*, \qquad y_{1,2}^q = y_{1,2}^{qT} && (1.31b)
\end{aligned}
$$

In the $\mathcal{C}$ case we have symmetric quark and charged lepton matrices and also the Dirac neutrino mass matrix. In the $\mathcal{P}$ case these matrices are almost hermitian. Hermiticity is lost because of the complex phase e^{ia} in the bidoublet vev.

We are interested in the relations resulting on the mixing matrices. In the quark sector, when diagonalizing the mass matrices M_u and M_d by an appropriate rotations of the quark fields we will encounter a mixing matrix in the charged current. We write the quark mass matrices in term of the mass eigenvalues:

$$
M_u = U_L m_u U_R^\dagger, \quad M_d = D_L m_d D_R^\dagger \tag{1.32}
$$

where m_u, m_d are the diagonal quark masses and U_L, U_R, D_L, D_R are the rotation matrices of the fields u_L, u_R, d_L, d_R necessary to achieve the diagonalization. The mixing matrices that will appear in the charged V-A and V+A currents are:

$$V-A \;\; \text{quark mixing}: \quad V_L^q = U_L^\dagger D_L \tag{1.33a}$$

$$V+A \;\; \text{quark mixing}: \quad V_R^q = U_R^\dagger D_R \tag{1.33b}$$

Now when M_u, M_d are almost hermitian then $U_R = U_L S_u$ and $D_R = D_L S_d$, with S_u, S_d diagonal sign matrices[1]. This implies that:

$$\mathcal{P}: \quad V_R^q \simeq S_u V_L^q S_d \tag{1.34}$$

When M_u and M_d are symmetric we have $U_R = U_L^* K_u^*$, $D_R = D_L^* K_d$, with K_u, K_d diagonal matrices of phases[2] and therefore:

$$\mathcal{C}: \quad V_R^q = K_u (V_L^q)^* K_d \tag{1.35}$$

Analogously, in the lepton sector we will encounter leptonic mixing matrices when diagonalizing the mass matrices of charged leptons, light neutrinos and heavy neutrinos. The light neutrino mass matrix M_ν is diagonalized by V_L, the heavy neutrino mass M_N by V_R, and we can write:

$$M_\nu = V_L^* m_\nu V_L^\dagger, \qquad M_N = V_R m_N V_R^T \tag{1.36}$$

In the basis in which the charged leptons matrix is diagonal, the rotation matrices coincide with the leptonic mixing matrices:

$$V-A \;\; \text{leptonic mixing}: \quad V_L \tag{1.37a}$$

$$V+A \;\; \text{leptonic mixing}: \quad V_R \tag{1.37b}$$

Due to the nature of the seesaw formula which contains an extra matrix M_D, no connection

[1]For example, $m_u = X m_u X$ has $X = U_L^\dagger U_R = S_u$ as solution.
[2]The condition reduces to $m_u = X^* m_u X$ and therefore $X = U_L^T U_R = K_u^*$.

between the left and right-handed leptonic mixing matrices follows directly from $\mathcal{C}$ or $\mathcal{P}$.

1.2 Seesaw mechanism

The mass matrix of neutrinos is given in the (ν_L, N_L) basis by the following 6×6 matrix:

$$\begin{pmatrix} M_L & M_D^T \\ M_D & M_R \end{pmatrix} \tag{1.38}$$

By an appropriate rotation of the neutrino fields it can be brought to a block diagonal form. To do this, let us imagine that the mass scale of M_R is much larger than that of M_L and M_D. More precisely, we will only consider the first terms in the expansion in M_R^{-1} when performing the diagonalization. We thus expect the heavy neutrino mass at the large scale to be approximately unperturbed:

$$M_N = M_R \tag{1.39}$$

The rotation is then made through a "small" mixing matrix Θ as follows:

$$\begin{pmatrix} \nu_L \\ N_L \end{pmatrix} \to \begin{pmatrix} 1 & \Theta^\dagger \\ -\Theta & 1 \end{pmatrix} \begin{pmatrix} \nu_L \\ N_L \end{pmatrix} \tag{1.40}$$

The matrix Θ is given by:

$$\Theta = M_N^{-1} M_D \tag{1.41}$$

and mass matrix of light neutrinos is given by the canonical seesaw formula:

$$M_\nu = M_L - M_D^T \frac{1}{M_N} M_D \tag{1.42}$$

Recalling that $v_L \propto v^2/v_R$, we observe that both terms in the neutrino mass matrix are in general of the same order. Therefore the largeness of v_R ensures small neutrino masses.

This is known as the seesaw mechanism [7, 8]. But we remark that the scale of N may be well below the scale of W_R. The same is also true for M_D, which can be many orders of magnitude below M_W. Just like it happens in the standard model if we compare the mass of W with the mass of the electron. Meaning that Yukawas can be arbitrary small, and yet we do not understand how the mass hierarchies are generated. But this by no means lowers the stature of the seesaw mechanism. There is a large portion of the parameter space with low scale of heavy neutrinos, susceptible of experimental verification, that must be taken seriously.

1.2.1 Light-Heavy Neutrino mixing - Inverting the seesaw formula

The light neutrino mass matrix has almost been reconstructed by oscillation experiments. Only the mass scale of the lightest neutrino, their hierarchy and the CP violating phases need to be determined. The LHC offers the possibility to measure the heavy neutrino masses and right handed mixing matrix in the near future. We then may consider inverting the seesaw formula to hopefully obtain some knowledge of the Dirac mass matrix itself, and in turn predict the coupling of neutrinos to the Higgs boson.

In this section we show that, thanks to the Left-Right symmetry responsible for making M_D symmetric, the Dirac matrix can indeed be determined [21]. The left-right symmetry $\mathcal{C}$ provides us with the following relations:

$$M_D = M_D^T \tag{1.43a}$$

$$M_L = \epsilon\, M_N \tag{1.43b}$$

where ϵ has been defined as the ratio of the vevs of the triplets:

$$\epsilon = \frac{v_L}{v_R}. \tag{1.44}$$

Inserting (1.43) in the seesaw formula (1.42) and writing it in terms of Θ we obtain:

$$M_\nu = M_N(\epsilon - \Theta^2) \tag{1.45}$$

Then we can invert this equation and solve for Θ [21]:

$$\Theta = \sqrt{\epsilon - M_N^{-1} M_\nu} \tag{1.46}$$

The mixing matrix Θ, which in general is undetermined [24], turns out to depend only on the parameter ϵ and on the mass matrices of light and heavy neutrinos. We recall that the square root of a matrix has a certain number of discrete solutions and, only in some degenerate cases, the root may contain arbitrary coefficients. Nevertheless these arbitrariness should be regarded as unphysical. To understand what happened, we shall count the number of parameters before and after the diagonalization. We started with two complex symmetric matrices M_N and M_D, and with one complex parameter ϵ. We ended with two complex symmetric matrices M_ν and M_N and with one complex matrix Θ. This last matrix Θ can have only one free parameter ϵ and not n^2 if the degrees of freedom are to be conserved. The matrix Θ found in (1.46) have this remarkable property. The Dirac mass matrix is given simply by:

$$M_D = M_N \sqrt{\epsilon - M_N^{-1} M_\nu} \tag{1.47}$$

The physical matrix that couples the mass eigenstates of light and heavy neutrinos to the Higgs boson is given by:

$$M_D^{\text{phys}} = V_R^\dagger M_D V_L \tag{1.48}$$

The remarkable fact is that once M_ν, M_N and ϵ are measured, the coupling M_D^{phys} is predicted. This allows a verification of the seesaw mechanism.

1.2.2 Examples of M_D

The general procedure of extracting the root of a matrix is outlined in Appendix 2. Here we present two cases in which the Dirac mass matrix takes a simple form.

Case 2×2

In the 2×2 case there exists an exact formula for the square root of a matrix[3]. In this case (1.47) reduces to:

$$M_D = \pm \frac{1}{\sqrt{\tau + 2\delta}} \left[(\epsilon + \delta) M_N - M_\nu \right], \tag{1.49}$$

with

$$\tau = \mathrm{tr}(\epsilon - M_N^{-1} M_\nu), \qquad \delta = \pm\sqrt{\det(\epsilon - M_N^{-1} M_\nu)}. \tag{1.50}$$

There are four solutions in total.

Case $V_R = V_L^*$

This choice of the leptonic mixing angles is analogous to what happens in the quark sector where, up to phases, the $\mathcal{C}$ symmetry implies that $V_R^q = V_L^{q*}$. In the lepton sector this is not mandatory but is a clean example which we believe worth mentioning. Using (1.36) in (1.47) together with $V_R = V_L^*$ it is easy to find[4]:

$$M_D = V_L^* m_N \sqrt{\epsilon - \frac{m_\nu}{m_N}} V_L^\dagger. \tag{1.51}$$

Each eigenvalue comes with a $\pm$ sign in the front:

$$\pm \left\{ m_{N_1} \sqrt{\epsilon - \frac{m_{\nu_1}}{m_{N_1}}}, \pm m_{N_2} \sqrt{\epsilon - \frac{m_{\nu_2}}{m_{N_2}}}, \pm m_{N_3} \sqrt{\epsilon - \frac{m_{\nu_3}}{m_{N_3}}} \right\}, \tag{1.52}$$

---

[3]The square root of a 2×2 matrix is given by $\sqrt{A} = \pm_1 \frac{A \pm_2 \sqrt{\det(A)}\, I_{2\times 2}}{\sqrt{\mathrm{tr}(A) \pm_2 2\sqrt{\det(A)}}}$.

[4]The mixing matrix is $\Theta = V_L \sqrt{\epsilon - \frac{m_\nu}{m_N}} V_L^\dagger$

In our convention m_{N_2} and m_{N_3} carry complex Majorana phases, and for this reason the $\pm$ sings are not very important. Moreover, in this case the number of families is also irrelevant and we are dealing with a trivial generalization of the one dimensional case.

The physical coupling to the Higgs is diagonal:

$$(M_D^{\text{phys}})_{ij} = \delta_{ij} m_{N_j} s_j \sqrt{\epsilon - \frac{m_{\nu_j}}{m_{N_j}}} \tag{1.53}$$

with $s_i = \pm$. There are 8 solutions in total for three generations.

1.2.3 General Parametrization of M_D.

In this section we comment about an attempt that was made to invert the seesaw formula in favor of the Dirac mass matrix. This was done for the cases in which M_D possesses no symmetries. Consider for simplicity the case in which M_L is set to zero: [5]

$$M_\nu = -M_D^T \frac{1}{M_N} M_D \tag{1.54}$$

The so called Casas-Ibarra parametrization of the Dirac mass matrix reads [24]:

$$M_D = i V_R \sqrt{m_N} O \sqrt{m_\nu} V_L^\dagger \tag{1.55}$$

with O an orthogonal complex matrix:

$$O O^T = 1. \tag{1.56}$$

Therefore (1.55) provides in general no unique solution but a family of M_D parametrized by three complex parameters in the orthogonal complex matrix O.

The point is that when M_D is symmetric, and we believe this is a well motivated consequence of the Left-Right symmetry, we have 3 additional complex relations which fix

[5] The case with nonzero M_L was discussed in [25].

the orthogonal complex matrix completely. The seesaw formula leads us directly to:

$$M_D = iM_N\sqrt{M_N^{-1}M_\nu} \tag{1.57}$$

with only a finite number of discrete solutions.

1.3 Summary

In Table 1.1 we collect the interactions of leptons with gauge bosons and with physical scalar fields. In Table 1.2 we do the same for quarks. In Table 1.3 we write all the physical observables of the theory appearing in Table 1.1 and 1.2. In the next chapter part of the phenomenological consequences of these interactions, in particular the leptonic ones, will be discussed in detail.

leptons &	interaction Lagrangian
W_L	$-\frac{g}{\sqrt{2}}\Big[\bar{e}_L \not{W}_L V_L \nu_L + \bar{e}\,\not{W}_L(\Theta^\dagger V_R^*)\gamma_L N\Big] + \text{h.c.}$
W_R	$-\frac{g}{\sqrt{2}}\bar{e}_R \not{W}_R V_R N_R + \text{h.c.}$
Z	$-\frac{g}{c_W}\Big\{\bar{e}\not{Z}\big[(-\frac{1}{2}+s_W^2)\gamma_L + s_W^2\gamma_R\big]e + \frac{1}{2}\bar{\nu}_L\not{Z}\nu_L\Big\}$ $-\frac{g}{c_W}\frac{1}{2}\bar{\nu}\not{Z}(V_L^\dagger\Theta^\dagger V_R^*)\gamma_L N + \text{h.c.}$
Z'	$-\frac{g}{c_W\sqrt{c_{2W}}}\Big\{\bar{e}\not{Z}'\Big[\frac{1}{2}s_W^2\gamma_L + (-\frac{1}{2}+\frac{3}{2}s_W^2)\gamma_R\Big]e + \frac{s_W^2}{2}\bar{\nu}_L\not{Z}'\nu_L$ $+\frac{c_W^2}{2}\bar{N}_R\not{Z}'N_R - \frac{c_{2W}}{2}\Big[\bar{\nu}\not{Z}'(V_L^\dagger\Theta^\dagger V_R^*)\gamma_L N + \text{h.c.}\Big]\Big\}$
Higgs	$-\frac{H}{v}\big(\overline{e_L}m_\ell e_R + \overline{N_R}M_D^{\text{phys}}\nu_L\big) + \text{h.c.}$ $-\frac{H}{v}\Big[\overline{\nu}(V_L^T M_I V_L)\gamma_L\nu + \overline{N}(V_R^\dagger \delta M_N V_R^*)\gamma_L N\Big] + \text{h.c.}$
neutral Δ_L, Δ_R	$-\frac{1}{2}\frac{1}{v_L}\Big[H_L^0\overline{\nu}\big(V_L^T M_{II}V_L\big)\gamma_L\nu + iA_L^0\overline{\nu}\big(V_L^T M_{II}V_L\big)\gamma_L\nu\Big] + \text{h.c.}$ $-\frac{1}{2}\frac{m_N}{v_R}H_R^0\overline{N}N$
$\Delta_L^+, \Delta_L^{++}, \Delta_R^{++}$	$\frac{1}{2}\frac{M_N}{v_R}\Big[\sqrt{2}\Delta_L^+\overline{\nu}\gamma_L e + \Delta_L^{++}\overline{e^c}\gamma_L e\Big] + \frac{1}{2}\frac{M_N^*}{v_R}\Delta_R^{++}\overline{e^c}\gamma_R e + \text{h.c.}$
netural φ_2	$\frac{V_L^\dagger m_\ell V_R + e^{-ia}s_{2\beta}(M_D^{\text{phys}})^\dagger}{vc_{2\beta}}H_2^0\overline{\nu_L}N_R + \text{h.c}$ $\frac{V_L(M_D^{\text{phys}})^\dagger V_R^\dagger + e^{ia}s_{2\beta}m_\ell}{vc_{2\beta}}H_2^{0*}\overline{e_L}e_R + \text{h.c}$
charged φ_2	$\frac{m_\ell V_R + e^{-ia}s_{2\beta}V_L(M_D^{\text{phys}})^\dagger}{vc_{2\beta}}H_2^-\overline{e_L}N_R + \text{h.c}$ $-\frac{(M_D^{\text{phys}})^\dagger V_R^\dagger + e^{ia}s_{2\beta}V_L^\dagger m_\ell}{vc_{2\beta}}H_2^+\overline{\nu_L}e_R + \text{h.c}$

Table 1.1: Summary of the leptonic gauge and scalar interactions in the mass basis. Here M_I and M_{II} are the seesaw contributions to neutrino masses $M_\nu = M_I + M_{II}$. The matrix δM_N is the correction to the heavy right handed neutrino mass matrix $\delta M_N = \frac{1}{2}(M_D\Theta^\dagger + \Theta^* M_D)$

quarks &	interaction Lagrangian
W_L	$-\frac{g}{\sqrt{2}}\bar{d}_L \not{W}_L V_L^q u_L + \text{h.c.}$
W_R	$-\frac{g}{\sqrt{2}}\bar{d}_R \not{W}_R V_R^q u_R + \text{h.c.}$
Z	$-\frac{g}{c_W}\bar{u}\not{Z}\big[(\frac{1}{2}-\frac{2}{3}s_W^2)\gamma_L - \frac{2}{3}s_W^2\gamma_R\big]u$ $-\frac{g}{c_W}\bar{d}\not{Z}\big[(-\frac{1}{2}+\frac{1}{3}s_W^2)\gamma_L + \frac{1}{3}s_W^2\gamma_R\big]d$
Z'	$-\frac{g}{c_W\sqrt{c_{2W}}}\bar{u}\not{Z}'\Big[-\frac{1}{6}s_W^2\gamma_L + (\frac{1}{2}-\frac{7}{6}s_W^2)\gamma_R\Big]u$ $-\frac{g}{c_W\sqrt{c_{2W}}}\bar{d}\not{Z}'\Big[-\frac{1}{6}s_W^2\gamma_L + (-\frac{1}{2}+\frac{5}{6}s_W^2)\gamma_R\Big]d$
Higgs	$-\frac{H}{v}\left(\overline{u_L}m_u u_R + \overline{d_L}m_d d_R\right) + \text{h.c.}$
netural φ_2	$\frac{V_L^{q\dagger}m_d V_R^q + e^{-ia}s_{2\beta}m_u}{v c_{2\beta}} H_2^0 \overline{u_L}u_R + \text{h.c}$ $\frac{V_L^q m_u V_R^{q\dagger} + e^{ia}s_{2\beta}m_d}{v c_{2\beta}} H_2^{0*} \overline{d_L}d_R + \text{h.c}$
charged φ_2	$\frac{m_d V_R^q + e^{-ia}s_{2\beta}V_L^q m_u}{v c_{2\beta}} H_2^- \overline{d_L}u_R + \text{h.c}$ $-\frac{m_u V_R^{q\dagger} + e^{ia}s_{2\beta}V_L^{q\dagger}m_d}{v c_{2\beta}} H_2^+ \overline{u_L}d_R + \text{h.c}$

Table 1.2: Summary of the quark gauge and scalar interactions in the mass basis.

g	gauge coupling
M_W, M_{W_R}	mass of W and W_R
$v = \dfrac{\sqrt{2}M_W}{g}$	physical vev breaking the SM
$v_R = \dfrac{M_{W_R}}{g}$	physical vev breaking $SU(2)_R \times U(1)_{B-L}$
V_L^q, V_R^q	quark mixing matrices
V_L, V_R	leptonic mixing matrices
m_u, m_d, m_ℓ	diagonal mass matrices of quarks and charged leptons
$M_\nu = V_L^* m_\nu V_L^\dagger$	light left-handed neutrino mass matrix
$M_N = V_R m_N V_R^T$	heavy right-handed neutrino mass matrix
$\Theta = \sqrt{\epsilon - M_N^{-1} M_\nu}$	light-heavy neutrino mixing matrix
$M_D^{\text{phys}} = V_R^\dagger M_N \Theta V_L$	higgs coupling to light and heavy neutrinos
$v_L = \langle \Delta_L^0 \rangle$	complex vev of Δ_L^0
$\beta = \arctan \dfrac{\langle \phi_1 \rangle}{\langle \phi_2^c \rangle}$	β angle of the doublets inside the bidoublet
e^{ia}	complex phase of $\langle \phi_2 \rangle$

Table 1.3: Summary of the physical observables appearing in Table 1.1 and Table 1.2

Chapter 2

Phenomenology

In this chapter we perform a study of the main phenomenological implications of the Left-Right model. We aim here to establish a connection between the high energy physics of colliders and the low energy physics of neutrinoless double beta decay, electromagnetic dipole moments and lepton flavour violating processes.

2.1 Measuring the scale of parity restoration

Colliders offer the best tool to access the new physics. The LHC can hopefully sit on the resonance of W_R as shown in Figure 2.1. Once produced, W_R will decay $\sim 75\%$

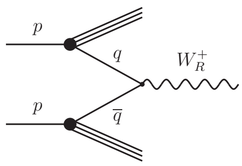

Figure 2.1: The production of W_R at the LHC.

into quarks and $\sim 25\%$ into charge leptons and heavy neutrinos. The decay channel into heavy neutrinos is open only for those $m_N < M_{W_R}$. These Majorana fermions will in turn decay 50% into leptons and 50% into anti-leptons plus two jets. The same-sign lepton decay of W_R will signal the violation of lepton number by two units [10]. The corresponding Feynman diagram is shows in Figure 2.2. The KS and also the lepton

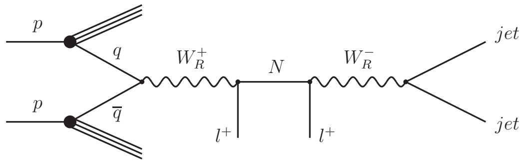

Figure 2.2: The Keung and Senjanovic production of same-sign charged lepton pairs.

flavour violating channel will allow the determination of the right handed leptonic mixing matrix V_R. Therefore a direct reconstruction of the heavy neutrino mass matrix M_N is promising. A full Monte Carlo simulation was done in [26]. The LHC at 14 TeV will be able to reach up to ~ 6 TeV with an integrated luminosity of ~ 300 fb^{-1}.

2.2 Testing the Higgs mechanism.

The scalar interactions allow us to test the multiple Higgs mechanism present in the theory. On one side we have the Majorana nature of heavy neutrinos with Δ_R being the Higgs responsible for its mass generation and on the other hand we have the Dirac nature of charged leptons and quarks in which the masses are governed by the Standard Model Higgs boson. The light neutrino is particularly interesting because it is receiving, through the seesaw mechanism, contributions from both sides. In the following, we outline how to probe experimentally this interrelated Higgs and seesaw mechanism.

The Yukawa couplings of leptons to the left and right doubly-charged triplets are tied to the masses of the heavy neutrinos:

$$Y_L = Y_R^* = \frac{g}{M_{W_R}} V_R m_N V_R^T \tag{2.1}$$

The decay signatures of Δ_L^{++} and Δ_R^{++} into pairs of same-sign leptons will therefore provide information about the heavy neutrino mass matrix. If one could in principle measure all the channels into dilepton one would get 6 parameters $(Y_\Delta)_{ij}$. Combining this with the m_N and V_R from the same-sign charge lepton signal we could in the best scenario get information on the Majorana phases. In any case the confirmation of (2.1) will be a direct test of the Majorana Higgs mechanism.

In the Dirac Higgs mechanism, it is a well know fact that the couplings of charged fermions to the Higgs boson are predicted to be proportional to the masses of the same fermions. The same would be true if we were dealing only with Dirac neutrinos. For Majorana neutrinos the coupling to the Higgs boson is essentially different. Nevertheless, in the left-right model this coupling is predicted, meaning that the seesaw mechanism by no means overshadows the Higgs mechanism [21]. This coupling M_D^{phys} was given in (1.48). Here we write it explicitly in term of neutrino masses and mixing:

$$M_D^{\text{phys}} = m_N V_R^T V_L \sqrt{\frac{g v_L}{M_{W_R}} - (V_R^T V_L)^\dagger \frac{1}{m_N} (V_R^T V_L)^* m_\nu} \tag{2.2}$$

It depends only on V_R, m_N and ϵ. We have already seen that the first two can be obtained in principle from the KS signal and from the decay of the doubly charged scalars. The vev $v_L = \langle \Delta_L \rangle$ can also be hunted in the colliders provided it is not too small. It parametrizes the strength of the interaction between Δ_L^{++} and the W_L boson. With the predicted coupling of neutrinos to the Higgs boson we need only to wait for the experimental confirmation.

2.3 Experimental limits on particle masses

In order to have a picture on the mass scale of the particles that we will be dealing in the following sections, we present in Table 2.1 the experimental limits on particle content of the theory masses that arise from direct searches in the collider. The theoretical limit on the heavy doublet arises from the precision measurements on the mass difference of K_L-K_S mesons.

2.4 Lepton flavour violation

If neutrinos were massless, we could have absorbed the leptonic mixing matrix by a redefinition of the neutrino fields. This could have been done by rotating the neutrino fields

Particle	Lower limit	ref.
W_R	2.9 TeV	[27]
Z'	3 TeV	[28]
A^0_ϕ	~10 TeV (Theory)	[29]
H^0_ϕ	~10 TeV (Theory)	[29]
H^+_ϕ	~10 TeV (Theory)	[29]
A^0_L	45 GeV	[28]
H^0_L	45 GeV	[28]
Δ^+_L	70-90 GeV	[30]
Δ^{++}_L	100-355 GeV	[31]
H^0_R	-	-
Δ^{++}_R	113-251 GeV	[31, 32]

Table 2.1: Experimental limits on the particle masses adapted from [33].

in the same way charged leptons were rotated. In that case the individual flavor lepton number would be conserved. We thus see that flavour conservation depends only on whether neutrinos are massive particles or not. Today we know that neutrinos of different flavour oscillate into each other. This is interpreted as neutrinos being created as flavour eigenstates, instead of mass eigenstates. Neutrino oscillation experiments provide us with useful information about the left-handed leptonic mixing matrix V_L and the mass squared difference of light neutrinos Δm^2_ν [16]. In Table 2.2 we collect the oscillation parameters obtained from recent experiments.

However, there are many other processes that violate the lepton flavor number. Table 2.3 shows the experimental limits on rare leptonic decay modes of the muon and tau leptons. Also Table 2.4 shows limits on LFV decay modes for light mesons and the Z boson. The most sensitive limits are given by $\mu \to e\gamma$, $\mu \to e^+ee$, $K^0_L \to e^\pm\mu^\mp$. This last one can be mediated by the heavy doublet at tree level, but we know that it has to have large mas from the limits on K_L-K_S mass difference. Thus the most relevant processes are provided by the leptonic rare decays of the muon.

	Central value	99% CL Range
$\sin^2\theta_{12}$	0.320	$31.3° < \theta_{12} < 37.5°$
$\sin^2\theta_{13}$	$\mathrm{N}: 0.0246,\ \mathrm{I}: 0.0245$	$7.5° < \theta_{13} < 10.5°$
$\sin^2\theta_{23}$	$\mathrm{N}: 0.613,\ \mathrm{I}: 0.600$	$36.8° < \theta_{23} < 55.5°$
Δm^2_{21} in eV^2	7.62×10^{-5}	$(7.12 - 8.20) \times 10^{-5}$
Δm^2_{31} in eV^2	$\mathrm{N}: 2.55 \times 10^{-3},\ \mathrm{I}: 2.43 \times 10^{-3}$	$(2.21 - 2.74) \times 10^{-3}$

Table 2.2: Oscilation parameters. Table adapted from [34]. There is a small difference between the normal (N) and the inverted (I) hierachy.

The contribution of light neutrino masses to $\mu \to e\gamma$ was found to be [49]:

$$\mathrm{BR}(\mu \to e\gamma) = \frac{3\alpha}{32} \left| (V_L)_{ei} (V_L)^*_{\mu i} \frac{m^2_{\nu_i}}{M^2_W} \right|^2 \tag{2.3}$$

And the contribution to $\mu \to e^+ee$ contains an additional factor of α. Plugging numbers we see that both branching ratios are predicted to be smaller than 10^{-50}. This is a general result, any contribution of light neutrinos in LFV other than oscillations can, most of the time, be safely be neglected.

The effective operator involved in $\mu \to e\gamma$ is of dimension 5, and involves a generic coupling $em_\mu G_F^2 M_W^2/\Lambda^2$ with Λ the scale of new physics. The branching ratio is effectively given by:

$$B(\mu \to e\gamma)_{NP} \propto \alpha \frac{M_W^4}{\Lambda^4} \tag{2.4}$$

Meaning that the experiments on $\mu \to e\gamma$ are probing the scale $\Lambda \sim 20$TeV. But of course if there is a loop involved the scale will be an order of magnitude smaller. In the same way, the effective operator of $\mu \to e^+ee$ is of dimension 6 and we can write:

$$B(\mu \to e^+ee)_{NP} \propto \frac{M_W^4}{\Lambda^4} \tag{2.5}$$

And the experiments on $\mu \to e^+ee$ are in principle probing the scale $\Lambda \sim 80$TeV. From

LFV process	Upper Limits on BR	Reference
$\mu \to e\gamma$	2.4×10^{-12}	[39]
$\tau \to e\gamma$	3.3×10^{-8}	[40]
$\tau \to \mu\gamma$	4.4×10^{-8}	
$\mu \to e^+ee$	1.0×10^{-12}	[38]
$\tau \to e^+ee$	2.7×10^{-8}	[41]
$\tau \to e^+\mu\mu$	1.7×10^{-8}	
$\tau \to e^+e\mu$	1.8×10^{-8}	
$\tau \to \mu^+ee$	1.5×10^{-8}	
$\tau \to \mu^+\mu\mu$	2.1×10^{-8}	
$\tau \to \mu^+e\mu$	2.7×10^{-8}	

Table 2.3: Leptonic LFV decay modes

this point of view, all the physics that generates the masses of light neutrinos will therefore be somehow restricted by these processes. In the next section we show how the interplay of the different scales of the model can produce small rates of LFV while allowing, at the same time, a low scale of W_R of no more than few TeV. It turns out that the Λ used above is in fact $\Lambda \simeq \frac{m_S}{m_N} M_{W_R}$ where S is one of the scalar fields and N is a heavy neutrino. By having m_N below m_S we can generate an artificially large Λ and can therefore comply with the experimental constraints.

2.4.1 LFV decay modes of Leptons

Tree level decay $\mu \to e^+ee$

We begin by writing the effective Hamiltonian for the tree level exchange of Δ_L^{++} and Δ_R^{++}. From (1.29) one obtains the following effective four-fermions interaction:

$$H = c^L_{ijkl}\overline{\ell^c_i}\gamma_L\ell_j\overline{\ell_k}\gamma_L\ell^c_l + c^R_{ijkl}\overline{\ell^c_i}\gamma_R\ell_j\overline{\ell_k}\gamma_R\ell^c_l \tag{2.6}$$

LFV process	Upper Limits on BR	Reference
$\pi^0 \to e^+\mu^-$	3.4×10^{-9}	[42]
$\pi^0 \to e^-\mu^+$	3.8×10^{-10}	[43]
$K^+ \to \pi^+\mu^+e^-$	1.3×10^{-11}	[44]
$K^+ \to \pi^+\mu^-e^+$	5.2×10^{-10}	[42]
$K^0_L \to e^\pm\mu^\mp$	4.7×10^{-12}	[45]
$K^0_L \to \pi^0 e^\pm\mu^\mp$	7.6×10^{-11}	[46]
$Z \to e^\pm\mu^\mp$	1.7×10^{-6}	[47]
$Z \to e^\pm\tau^\mp$	9.8×10^{-6}	[47]
$Z \to \mu^\pm\tau^\mp$	1.2×10^{-5}	[48]

Table 2.4: LFV decay modes of light mesons and the Z boson.

where the coefficients $c^{L,R}_{ijkl}$ are found to be:

$$c^L_{ijkl} = \sqrt{2}G_F \frac{M_W^2}{M_{W_R}^2} \frac{(M_N^*)_{ij}(M_N)_{kl}}{m^2_{\Delta_L^{++}}} \tag{2.7}$$

$$c^R_{ijkl} = \sqrt{2}G_F \frac{M_W^2}{M_{W_R}^2} \frac{(M_N)_{ij}(M_N^*)_{kl}}{m^2_{\Delta_R^{++}}} \tag{2.8}$$

Assuming only that the mass of the decaying particle m_{ℓ_1} is much larger than the masses of the decay products we obtain the following three-body decay rate:

$$\begin{aligned} \Gamma(\ell_j \to \ell_i^c \ell_k \ell_l) &= \frac{m^5_{\ell_j}}{768\pi^3} \left(|c^L_{ijkl}|^2 + |c^R_{ijkl}|^2\right) \\ &= \frac{G_F^2 m^5_{\ell_j}}{384\pi^3} \frac{M_W^4}{M_{W_R}^4} \left| \frac{(M_N)_{ij}(M_N)_{kl}}{m^2_{\Delta^{++}}} \right|^2 \end{aligned} \tag{2.9}$$

where $m^{-4}_{\Delta^{++}} = m^{-4}_{\Delta_L^{++}} + m^{-4}_{\Delta_R^{++}}$. For the muon only the decay $\mu \to e^+ee$ is kinematically allowed. For the tau there are six channels allowed $\tau \to \{e^+ee, e^+e\mu, e^+\mu\mu\}$ and those with e^+ replaced by μ^+. The strongest constraint will arise from $\mu \to e^+ee$. The branching

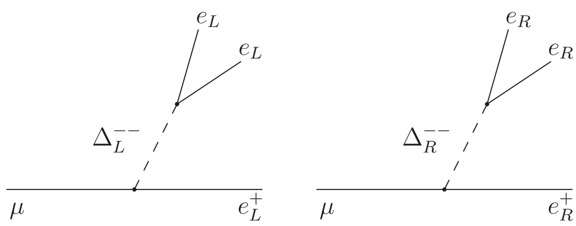

Figure 2.3: Feynman diagrams of $\mu \to e^+ee$

ratio normalized to the standard muon decay is given by:

$$\mathrm{BR}(\mu \to e^c ee) \simeq 10^{-7} \left(\frac{3.5\mathrm{TeV}}{M_{W_R}}\right)^4 \left| (V_R)_{\mu i}(V_R)_{ei}(V_R)_{ej}(V_R)_{ej} \frac{m_{N_i} m_{N_j}}{m^2_{\Delta^{++}}} \right|^2 \tag{2.10}$$

When M_{W_R} is not more than few TeV, there are only three ways that can make this branching ratio small enough to comply with the experimental constraint given in Table2.3:

$$\mathrm{BR}^{\exp}(\mu \to e^+ ee) < 10^{-12} \tag{2.11}$$

For example, an almost diagonal right-handed mixing matrix $V_R \simeq 1$, or an almost degenerate heavy neutrino spectra $m_{N_1} \simeq m_{N_2} \simeq m_{N_3}$ are enough to make (2.10) vanish. But far more interesting is when the ratio $m_{N_i}/m_{\Delta^{++}}$ is small itself. Indeed, for M_{W_R} at the reach of LHC, the following ratio:

$$\frac{m_{N_i}}{m_{\Delta^{++}}} \lesssim \frac{1}{10} \tag{2.12}$$

will ensure that for any V_R and any heavy neutrino hierarchy the branching ratio agrees with the experimental limit given in Table 2.3.

Radiative decay $\mu \to e\gamma$

We now calculate the radiative decay rate of $\ell_1 \to \ell_2\gamma$ with the help of [50] and update the formulas presented in [51] by including the mixing of light and heavy neutrinos. The relevant Feynman diagrams are shown in Figure 2.4. The effective Hamiltonian reads as follows:

$$H_{\ell_1 \to \ell_2\gamma} = \overline{\ell_2}\,\sigma_{\mu\nu}\big[(\sigma_{LL} + \sigma_{LR})_{21}\gamma_R + (\sigma_{RR} + \sigma_{RL})_{21}\gamma_L\big]\ell_1 F^{\mu\nu} \tag{2.13}$$

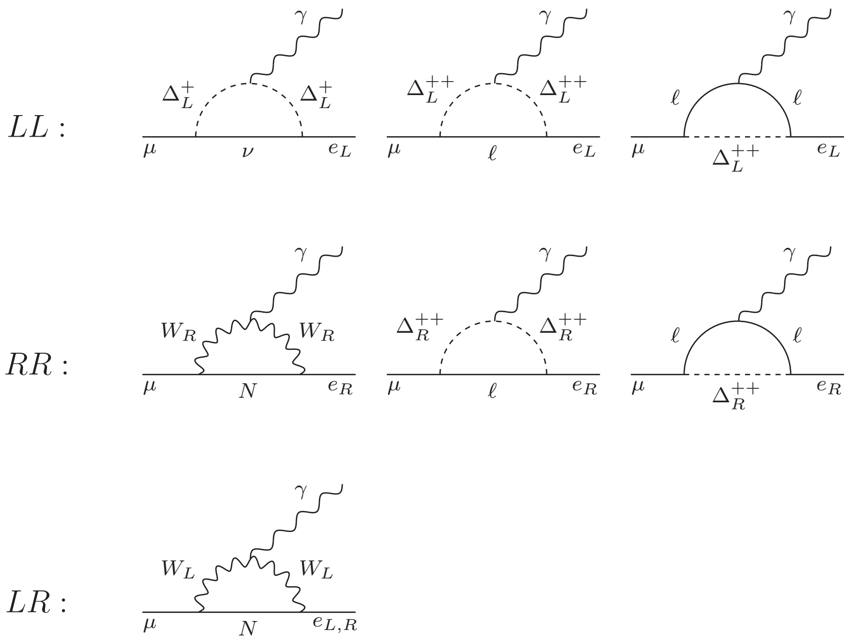

Figure 2.4: Feynman diagrams of $\mu \to e\gamma$.

and total decay rate is given by the following generic formula:

$$\Gamma(\ell_1 \to \ell_2 \gamma) = \frac{m_{\ell_1}^3}{16\pi}\Big(|(\sigma_{LL} + \sigma_{LR})_{21}|^2 + |(\sigma_{RR} + \sigma_{RL})_{21}|^2\Big) \tag{2.14}$$

The matrices $\sigma_{LL}, \sigma_{LR}, \sigma_{LR}$ and σ_{RL} are found to be:

$$\sigma_{LL} = m_{\ell_1}\frac{eG_F}{4\sqrt{2}\pi^2}\frac{M_W^2}{M_{W_R}^2}\left(\frac{1}{24}\frac{M_N^* M_N}{m_{\Delta_L^+}^2} + \frac{1}{3}\frac{M_N^* M_N}{m_{\Delta_L^{++}}^2}\right) \tag{2.15a}$$

$$\sigma_{RR} = m_{\ell_1}\frac{eG_F}{4\sqrt{2}\pi^2}\frac{M_W^2}{M_{W_R}^2}\left(\frac{1}{8}\frac{M_N M_N^*}{M_{W_R}^2} + \frac{1}{3}\frac{M_N M_N^*}{m_{\Delta_R^{++}}^2}\right) \tag{2.15b}$$

$$\sigma_{LR} = m_{\ell_1}\frac{eG_F}{4\sqrt{2}\pi^2}\left(\Theta^\dagger V_R^* F_1(t) V_R^T \Theta - \xi^* \Theta^\dagger V_R^* F_2(t) V_R^T \frac{M_N^*}{m_{\ell_1}}\right) \tag{2.15c}$$

$$\sigma_{RL} = m_{\ell_1}\frac{eG_F}{4\sqrt{2}\pi^2}\left(|\xi|^2 V_R^* F_1(t) V_R^T - \frac{M_N}{m_{\ell_1}} V_R^* F_2(t) V_R^T \xi \Theta\right) \tag{2.15d}$$

where the loop fuctions entering in σ_{LR} and σ_{RL} are:

$$F_1(t) = \frac{-4t^3 + 45t^2 - 33t + 10}{12(t-1)^3} - \frac{3t^3 \log t}{2(t-1)^4} \tag{2.16}$$

$$F_2(t) = \frac{t^2 - 11t + 4}{2(t-1)^2} + \frac{3t^2 \log t}{(t-1)^3} \tag{2.17}$$

where $t = m_N^2/M_W^2$. Moreover, as we are interested in a low scale of heavy neutrinos, we have taken the limit $m_N \ll M_{W_R}$ and simplified an extra loop function present in the gauge component of σ_{RR}. The branching ratio $\mu \to e\gamma$ normalized to the standard muon decay can be expressed ignoring square of ξ and Θ and after some simplifications as:

$$\begin{aligned} \mathrm{BR}(\mu \to e\gamma) \simeq 10^{-9} \left(\frac{3.5\mathrm{TeV}}{M_{W_R}}\right)^4 \Bigg(&\left|\frac{1}{3}\frac{M_N^* M_N}{m_L^2} - e^{-ia} s_{2\beta} \frac{M_D^\dagger}{m_\mu} V_R F_2(t) V_R^\dagger\right|^2_{e\mu} \\ &+ \left|\frac{1}{3}\frac{M_N M_N^*}{m_R^2} - e^{ia} s_{2\beta} V_R F_2(t) V_R^\dagger \frac{M_D}{m_\mu}\right|^2_{e\mu} \Bigg) \end{aligned} \tag{2.18}$$

where we have shortened the expression defining:

$$\frac{1}{m_L^2} = \frac{1}{24}\frac{1}{m^2_{\Delta_L^+}} + \frac{1}{3}\frac{1}{m^2_{\Delta_L^{++}}}, \qquad \frac{1}{m_R^2} \simeq \frac{1}{8}\frac{1}{M^2_{W_R}} + \frac{1}{3}\frac{1}{m^2_{\Delta_R^{++}}} \tag{2.19}$$

The mass of Δ_L^+ can not be much lower than the mass of Δ_L^{++}. To see this we refer to the Appendix A. From Table A.1 the following mass difference can be obtained:

$$M^{++}_{\Delta_L} - M^{+}_{\Delta_L} = \frac{\alpha_3}{2} v^2 \cos 2\beta \tag{2.20}$$

where α_3 is a dimensionless parameter of the scalar potential. This means that during the symmetry breaking of $\mathcal{G}_{SM}$ the masses of the scalar triplet will get split. Coming back to (2.19), we can safely ignore the singly charged scalar contribution. Moreover the scale of the doubly charged scalar Δ_R^{++} is phenomenologically less constrained and can therefore be much lighter than that of the right-handed gauge boson W_R. We thus follow

X	Au	Al	Ti	Pb	Cu	S
$B(\mu X \to eX) <$	7×10^{-13}	-	4.3×10^{-12}	4.6×10^{-11}	1.6×10^{-8}	7×10^{-11}

Table 2.5: Limits on muon conversion.

the scenario with light Δ_R^{++}. From now on, we will always use approximately:

$$\frac{1}{m_L^2} \simeq \frac{1}{3}\frac{1}{m^2_{\Delta_L^{++}}}, \qquad \frac{1}{m_R^2} \simeq \frac{1}{3}\frac{1}{m^2_{\Delta_R^{++}}}, \tag{2.21}$$

in the branching ratio (2.18).

By comparing this branching ratio (2.18) with that of $\mu \to e^+ee$ found in the previous section in (2.10) we see that roughly the branching ratio of $\mu \to e\gamma$ is expected to be smaller than that of $\mu \to e^+ee$ by a factor of 10^{-2} because of the loop suppression. The rate of both processes is controlled mainly by the mass of W_R, and secondly by the lightest of the doubly charged particles $\Delta_R^{++}, \Delta_L^{++}$. Both branching ratios have a strong flavor dependence on the right-handed mixing matrix, with the right handed neutrino mass matrix controlling the flavor structure. The process $\mu \to e^+ee$ is controlled by $(M_N)_{e\mu}(M_N)_{ee}$ whereas $\mu \to e\gamma$ is controlled by $(M_N M_N^\dagger)_{e\mu}$ which has the extra feature of being independent of the Majorana phases. Interestingly the Dirac mass matrix contribution is relevant when the following generic relation is satisfied:

$$\frac{m_D}{m_\mu} \sim \frac{m_N^2}{m^2_{\Delta^{++}}} \tag{2.22}$$

Thus, for the parameter spaces accessible to the LHC (for example $m_N \sim 100\,\mathrm{GeV}$ and $m_{\Delta^{++}} \sim 1\,\mathrm{TeV}$), a Dirac mass m_D of the order few MeV would be on the border limit.

2.4.2 Muon-electron conversion

The lepton flavor can also be violated in the process $\mu + X \to e + X$ [53], where X is a given muonic atom. The muon instead of decaying naturally into an electron, neutrino and antineutrino, gets transformed into an electron. The observation of an electron with fixed energy $m_\mu - B$, with B the binding energy of the muonic atom, will signal the muon conversion and the violation of lepton flavor number. Table 2.5 shows the experimental upper limits on the branching ratio normalized to the standard muon capture in the same nuclei.

In the left-right model the process can be mediated by the photon or by any of the two heavy neutral gauge bosons Z, Z'. The photonic interaction involves on-shell photons which are contained in the electric field of the atom [54]. Their contribution to amplitude will be the same as we used before for $\mu \to e\gamma$ and the rate will contain an extra factor α. Therefore these photonic on-shell interactions will not give any new constraint. The interesting contribution then comes from off-shell photonic interactions. It turns out that the contribution of the doubly charged particles $\Delta^{++}_{L,R}$ is always logarithmically enhanced [55]:

$$\text{enhancement factor} \sim \log \frac{m_\mu^2}{m_{\Delta^{++}}^2} \tag{2.23}$$

Figure 2.5 shows the corresponding Feynman diagrams. This enhancement allows one to ignore the contribution of the Z boson[1] and focus only on the doubly charged particles. The decay rate for a general effective Hamiltonian can be found in [52]. In our case we have for the branching ratio normalized with respect to the capture rate the following expression [51]:

$$B(\mu N \to eN) = \frac{512\pi\alpha^2}{3} \frac{\Gamma_\mu}{\Gamma_{\text{cap}}} \frac{(V^{(p)})^2}{m_\mu^5} \frac{M_W^4}{M_{W_R}^4} \left(\log \frac{m_\mu^2}{m_{\Delta^{++}}^2} \right)^2 \left| \frac{M_N M_N^*}{m_\Delta^{++}} \right|^2_{e\mu} \tag{2.24}$$

where $V^{(p)}$ (of dimension $[m]^{5/2}$) takes into account the effect of the electric charge density of the proton on the outgoing electron, and Γ_{cap} is the capture rate. These values can be found tabulated for every element in [52]. For example, for Au we have $V^{(p)}_{\text{Au}} = 0.0974 m_\mu^{5/2}$ and $\Gamma^{\text{Au}}_{\text{cap}} \simeq 6\ \Gamma_\mu$. Taking the enhancement in the region of phenomenologically interest

[1]However they were calculated in [51]

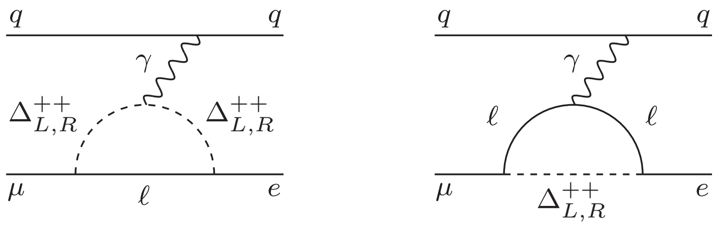

Figure 2.5: Feynman diagrams of muon conversion.

$\log^2 \sim 2 \times 10^2$ one finds:

$$B(\mu Au \to eAu) \simeq 5 \times 10^{-10} \left(\frac{3.5\text{TeV}}{M_{W_R}}\right)^4 \left|\frac{M_N M_N^*}{m_\Delta^{++}}\right|^2_{e\mu} \tag{2.25}$$

The muon conversion mediated by the doubly charged scalars is therefore slightly more constrained than the decay $\mu \to e\gamma$. Both branching ratios are theoretically of the same order because the additional factor of α in the muon conversion gets balanced by the large logarithm coming from the loop, but at the same time the limits on the conversion are roughly one order of magnitude stronger.

2.5 Interplay between different LFV processes

Lepton flavor violation could vanish if the right-handed mixing matrix is nearly diagonal. So a definite prediction can not be made and the different rates depend crucially on V_R. For a generic V_R one could also have a particular mass spectrum of right-handed heavy neutrinos which could make the rate of one or more processes vanish as well. The interesting processes with high sensitivity are $\mu \to e^+ee$, $\mu \to e\gamma$, and μ-e in the nuclei. The common flavor dependence of $\mu \to e\gamma$ and μ-e conversion is given by:

$$(M_N M_N^*)_{e\mu} = \Delta m_{12}^2 c_{13} s_{12}(c_{12}c_{23} - e^{-id} s_{12}s_{13}s_{23}) + e^{-id}\Delta m_{13}^2 s_{13}c_{13}s_{23} \tag{2.26}$$

with Δm_{12}^2, Δm_{13}^2 the mass squared difference of the heavy neutrinos and with $s_{12} = \sin\theta_{12}^R$, etc. This factor can vanish for some Δm_{13}^2 and Δm_{13}^2. For $\mu \to e^+ee$ one have

LFV process	Future sensitivity	ref.
$\mu \to e^+ee$	10^{-15}	Mu3e
$\mu \to e\gamma$	10^{-15}	Meg
$\mu Al \to eAl$	10^{-17}	Meco

Table 2.6: Upcoming LFV experiments.

additionally the freedom of the Majorana phases and they can easily make the rate vanish as well. So without the masses m_N we can not constrain our parameter space either. The problem is that we have only three competitive processes. We could have done more if we would have the three radiative decay of $\ell_1 \to \ell_2\gamma$ and the seven three body decay $\ell_i \to \ell_i\ell_j\ell_k$ with similar experimental sensitivity. They would make 10 constraints on 3 masses m_{N_i}, 3 angles and 4 phases in V_R. But this scenario is far from being realistic. Table 2.6 shows some future experiments and their experimental reach. When combining this future prospects from LHC searches on W_R and the heavy neutrino mass matrix we could then translate the LFV limits into quantitative constraints on the scalar masses.

The prospects on processes $\mu \to e\gamma$ and μ-e in the nuclei are particularly encouraging. The ratio of both processes can be used to get useful information on the masses of the doubly charged scalar particle. One has for doubly charged contributions the following ratio [51]:

$$R_X = \frac{B(\mu X \to eX)}{B(\mu \to e\gamma)} = c_X \times \alpha \times \frac{\frac{1}{m^4_{\Delta_L^{++}}}\left(\log\frac{m^2_\mu}{m^2_{\Delta_L^{++}}}\right)^2 + \frac{1}{m^4_{\Delta_R^{++}}}\left(\log\frac{m^2_\mu}{m^2_{\Delta_R^{++}}}\right)^2}{\frac{1}{m^4_{\Delta_L^{++}}} + \frac{1}{m^4_{\Delta_R^{++}}}} \tag{2.27}$$

where c_X depends only on the particular muonic atom:

$$c_X = \frac{\Gamma_\mu}{\Gamma_{\text{cap}}}\frac{(32\pi V^{(p)})^2}{m^2_\mu} \tag{2.28}$$

The value of c_X is shown for some interesting atoms in Table 2.7. The ratio R_X is therefore

X	Au	Al	Ti	Pb	Cu	S
c_X	3.3	1.7	2.8	2.4	2.7	1.9

Table 2.7: c_X defined in the text basically governs the ratio $B(\mu X \to eX)/B(\mu \to e\gamma)$.

predicted by the theory to be close to one. Its precise value will then permit to probe the scalar sector of the theory and can be used to complement collider searches in the hunt for the doubly charged particles.

2.6 Neutrinoless double beta decay

Apparently Majorana himself was thinking about an experiment that could distinguish "truly" neutral particles, which are identical to its antiparticles, from "normal" neutral particles which have distinct antiparticles [4]. At that time the only known electrically neutral particles were the photon, the neutron and the hypothetical neutrino. It was clear the photon having only real components was a truly neutral particle in that sense. Now we know that the neutron is made out of three electrically charged quarks, and therefore is not equal to the antineutron. The question whether or not the neutrino is a "truly" neutral particle is still open. The experimental idea came shortly after, if neutrinos are Majorana particles then double beta decay may occur without the emission of neutrinos [2].

The standard double beta decay, can only be seen in those nuclei in which single beta decay is kinematically forbidden. It has been experimentally observed for many nuclei: ^{76}Ge, ^{82}Se, ^{100}Mo, ^{130}Te , ^{136}Xe, ^{150}Nd, etc. The outgoing electrons have a continuum energy spectrum due to the presence of two antineutrinos. The sum of their energies are less than the available energy Q which is of the order of few MeV. Table 2.8 shows the double beta decay half-life and the current experimental limits on $0\nu\beta\beta$ for some interesting nuclei.

The contribution of light neutrino masses to the $0\nu\beta\beta$ decay rate is proportional to

	^{76}Ge	^{82}Se	^{100}Mo	^{130}Te	^{136}Xe	^{150}Nd
$\frac{T^{1/2}_{2\nu\beta\beta}}{10^{21}\text{yr}}$	1.74 [56]	0.096 [57]	0.007[58]	0.7 [59]	2.11 [60]	0.009 [61]
$\frac{T^{1/2}_{0\nu\beta\beta}}{10^{25}\text{yr}}$	2.2 [62]	0.036 [63]	0.11 [63]	0.013 [59]	0.006 [64]	0.002 [61]

Table 2.8: Double beta decay half-life and limits on neutrinoless double beta decay

the 1-1 element of the neutrino mass matrix $(M_\nu)_{ee}$ and can be written as follows:

$$\mathcal{A}_\nu \propto G_F^2 \frac{(M_\nu)_{ee}}{k^2} \tag{2.29}$$

where $k \sim 100\,\text{MeV}$ is of the order of the Fermi momentum of the nucleus and may be considered a measure of the neutrino virtuality. The absolute value of $(M_\nu)_{ee}$ is plotted in Figure 2.6 as a function of the lightest neutrino mass [65]. The indeterminacy of the Majorana phases is causing the slight dispersion of the points. Current experimental constraints on the decay rate translate generically into [16]:

$$(M_\nu)_{ee} \lesssim \text{eV} \tag{2.30}$$

Recently a strong limit from searches on ^{136}Xe has been reported in [66]:

$$(M_\nu)_{ee} \lesssim (0.140 - 0.380)\text{eV} \tag{2.31}$$

up to the uncertainty of the nuclear matrix elements.

One may attempt to draw some premature conclusions from Figure 2.6 without considering possible new physics effects. For example, one may conclude that the hierarchy of light neutrino masses can be probed by $0\nu\beta\beta$. Future experiments on $0\nu\beta\beta$ with sensitivity less than 0.1 eV will then be probing the inverted hierarchy. The normal hierarchy still needs to wait for better sensitivity and in some cases the rate may even vanish completely because of freedom on the Majorana phases. So the hope for a future observation relies

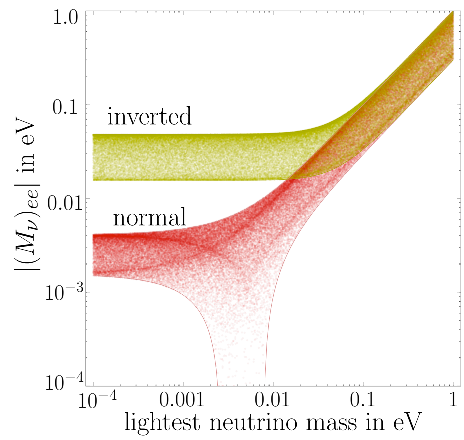

Figure 2.6: The canonical contribution form light neutrino masses. The mixing angles are fixed at $\{\theta_{12}, \theta_{23}, \theta_{13}\} = \{35°, 45°, 9°\}$, while the Dirac and Majorana phases vary in the interval $[0, 2\pi]$.

mainly on the inverted hierarchy mass spectrum.

Nevertheless, cosmology is pushing from above the sum of all neutrino masses [68], which can be translated into the lightest neutrino mass less than $\sim 0.1\,\text{eV}$. This is in apparent contradiction with the recent limit (2.31). There is also another claim that $0\nu\beta\beta$ decay have been seen for ^{76}Ge [67] and that it corresponds to $(M_\nu)_{ee} \simeq 0.4\text{eV}$. Together these three facts can be interpreted as the manifestation of new physics.

In any case, in a complete theory of neutrino masses all the conclusions one can draw from Figure 2.6 are not necessary valid. The simplest addition of heavy right handed neutrinos, for example, through the heavy Majorana mass matrix can also produce sizeable rates of $0\nu\beta\beta$ as was noticed in [69], some years before the seesaw mechanism was discovered. In short, Figure 2.6 represents only one part of the story.

In general all the physics behind the generation of light Majorana neutrino masses, provided its scale is not so high, can cause $0\nu\beta\beta$. We can estimate the size of this scale. The operator contributing to the decay rate is of dimension 9, and therefore the new physics will produce the following amplitude:

$$\mathcal{A}_{\text{NP}} \propto G_F^2 \frac{M_W^4}{\Lambda^5} \tag{2.32}$$

where Λ is the scale of new physics. When the scale Λ is close to the TeV scale, the new physics starts competing with the contribution of light neutrinos. In the following section we analyse the rate produced by the Left-Right model. Specially attractive is the right-handed neutrino contribution, analogous to the left-handed neutrinos, which goes through the W_R boson instead of the W_L boson. In this case the scale Λ will split into the scale M_{W_R} and m_N and the theory produces for m_N of few GeV considerable rates of $0\nu\beta\beta$.

2.6.1 Decay Rate

One can write the effective Hamiltonian of $0\nu\beta\beta$ in the following form[2]:

$$H_{\text{eff}} = -\frac{G_F^2}{k^2}\left(m_{LL}^{ee}\overline{e_L}e_L^c J_L^2 - m_{RR}^{ee}\overline{e_R}\, e_R^c J_R^2 - m_{LR}^{ee}\overline{e_L}\gamma^\mu\gamma^\alpha\gamma^\nu e_R^c J_L^\mu \hat{k}^\alpha J_R^\nu\right) \tag{2.33}$$

Here J_L, J_R denote the left- and right-handed quark currents respectively and k is a constant vector needed to get the right dimension. It is expected to be of the order $\sim 100\,\text{MeV}$ as it is describing the virtuality of light neutrinos. The constant k has been factorized for convenience in order to have all the coefficients normalized to the canonical contribution. The diagrams contributing to the effective Hamiltonian are shown in Figure 2.7.

We have omitted few diagrams which are irrelevant. For example, the mixing ξ between W_L-W_R does not play any significant role for the substitution of a heavy W_R by a light ξW_L will only constraint $\sin 2\beta$ to be smaller than one. This is because the mixing is already predicted to be small $\xi \simeq M_W^2/M_{W_R}^2 \sin 2\beta$. We have neglected the contributions coming from the heavy doublet [70] because the limits on its mass have to be larger than 10 TeV [71]. The heavy neutrino masses were assumed to lie above the Fermi momentum of the nucleus which is around 100 MeV, and so the momentum dependence of their propagator can be neglected . Nevertheless, it can be restored at the end if needed [72].

[2]Strictly speaking one should not write effective interactions for light neutrinos which are propagating. But this step will be justified in a moment.

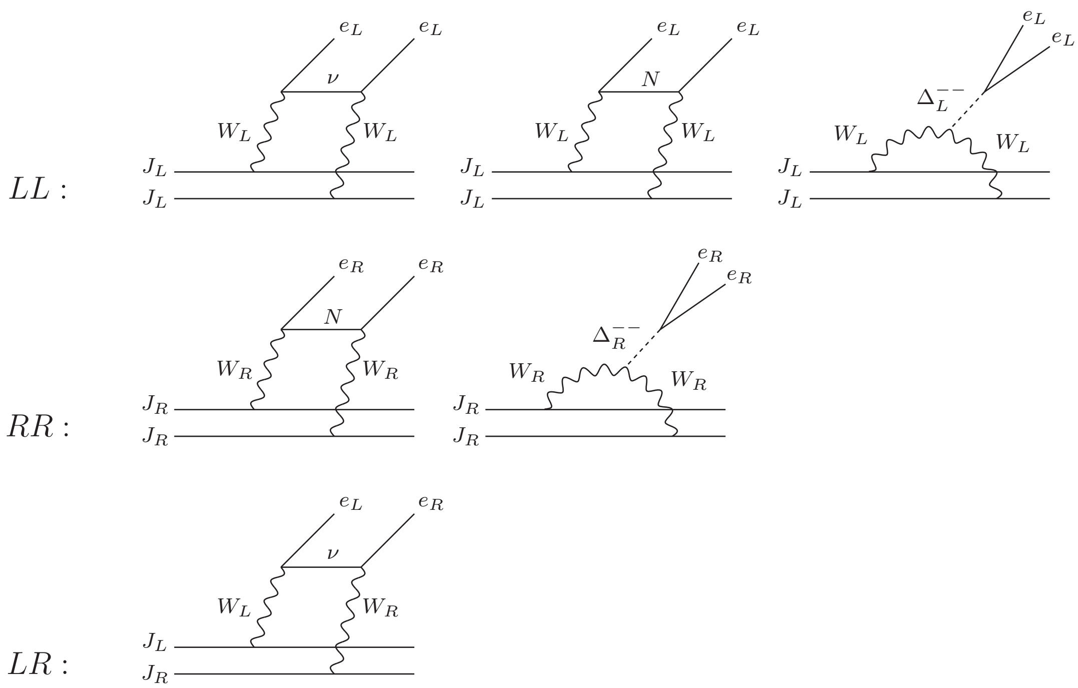

Figure 2.7: Feynman diagrams contributing to the amplitude of $0\nu\beta\beta$. As shown in the text the m_{LL} amplitude is dominated by light neutrinos. The RR and the LR amplitude can separately dominate the decay rate.

A straightforward calculation gives the following m_{LL}, m_{RR} and m_{LR} matrices:

$$m_{LL} = M_\nu^* - \Theta^\dagger \frac{k^2}{M_N} \left(1 + 2\frac{M_N M_N^*}{m^2_{\Delta_L^{++}}} \right) \Theta^* \tag{2.34a}$$

$$m_{RR} = \frac{M_W^4}{M_{W_R}^4} \frac{k^2}{M_N^*} \left(1 + 2\frac{M_N^* M_N}{m^2_{\Delta_R^{++}}} \right) \tag{2.34b}$$

$$m_{LR} = 2\frac{M_W^2}{M_{W_R}^2} k\Theta^\dagger \tag{2.34c}$$

We remind here that M_ν, M_N and Θ are the light neutrino mass matrix, the heavy neutrino mass matrix and the light-heavy neutrino mixing matrix respectively. The effective Hamiltonian (2.33) contains only the ee element of the matrices (2.34). The m_{LL} amplitude is composed of three contributions. The first one is the canonical contribution due to the Majorana masses of light neutrinos. The second happens when each light neutrino is replaced by a heavy neutrino. The third contribution is due to the doubly charged left triplet. The m_{RR} amplitude has heavy neutrinos coupled to the right-handed gauge boson in complete analogy to the canonical contribution. It also contains the contribution of the doubly charged right triplet. The m_{LR} amplitude occurs when a heavy neutrino is converted into a light neutrino.

In deriving (2.34) we did not take into account that the light neutrino masses are generated by the seesaw mechanism. The seesaw formula (1.42) with $M_D = M_N \Theta$ reads:

$$M_\nu = \epsilon M_N - \Theta^T M_N \Theta \tag{2.35}$$

Inserting this equation in the amplitude m_{LL} we get:

$$m_{LL} = \epsilon^* M_N^* - \Theta^\dagger \left(1 + \frac{k^2}{M_N^* M_N} + 2\frac{k^2}{m^2_{\Delta_L^{++}}} \right) M_N^* \Theta^* \simeq M_\nu^* \tag{2.36a}$$

The last equality is valid when the masses m_{N_i} are not exactly of order k.[3]

In order to proceed with the decay rate, we need to have the value of the following nuclear matrix elements between the initial and the final nucleus:

$$\left\langle \frac{J_L^2}{p^2} \right\rangle, \langle J_R^2 \rangle, \left\langle J_L^\mu J_R^\nu \frac{p^\alpha}{p^2} \right\rangle \tag{2.37}$$

where the momentum p is implicitly being integrated. We define formally the value of k as the following ratio of matrix elements [11]:

$$k^2 = \langle J_R^2 \rangle \div \left\langle \frac{J_L^2}{p^2} \right\rangle \tag{2.38}$$

This k is the same in (2.33), physically it is expected to be of the order of the Fermi momentum of the nucleus. In Table 2.9 we took the matrix elements available in the literature and checked that the ratio (2.38) is indeed giving the right momentum. It turns

[3] The case in which the heavy sterile neutrinos contribute substantially to the m_{LL} amplitude through the light-sterile neutrino mixing [73] requires independent choices of the mixing matrix Θ (or equivalently of M_D) and the heavy neutrino mass matrix M_N in order to make $\Theta^T M_N \Theta$ smaller than $k^2 \Theta^T M_N^{*-1} \Theta$. In our case, the left-right symmetry takes over such arbitrariness with Θ given by (1.46).

	ref.	^{76}Ge	^{82}Se	^{100}Mo	^{130}Te	^{136}Xe	^{150}Nd
k in MeV	[75]	99	94	207	102	132	97
	[76]	190	186	189	180	280	210
	[77]	184	-	193	198	-	-
$G\lvert\mathcal{M}_\nu\rvert^2$ in $\dfrac{1}{10^{25}\,\text{yr}\times\text{eV}^2}$	[75]	4.5	17.3	8.2	21.4	4.7	310
	[76]	4.2	16.5	7.7	20.4	4.6	290
	[77]	10.8	-	60.8	48.8	-	-
Q in MeV	[16]	2.039	2.995	3.034	2.529	2.476	3.367

Table 2.9: Nuclear factors relevant for $0\nu\beta\beta$.

out that the nuclear matrix element involving left and right-handed currents is suppressed, see for example [78]. Meaning that the naive estimation is off by one order of magnitude.

The decay rate can then be written as follows:

$$\Gamma = G \cdot \left|\mathcal{M}_\nu\right| \left(|m_\nu^{ee}|^2 + |m_N^{ee} + m_\Delta^{ee}|^2 + |m_\Theta^{ee}|^2\right) + \text{interference terms} \tag{2.39}$$

where G is a phase factor, $|\mathcal{M}_\nu|$ is the nuclear matrix element relevant for the light neutrino exchange, and with the amplitudes given by:

$$m_\nu^{ee} = (V_L^2)_{ie} m_{\nu_i} \tag{2.40a}$$

$$m_N^{ee} = \frac{M_W^4}{M_{W_R}^4}(V_R^2)_{ie}\frac{k^2 m_{N_i}}{k^2 + m_{N_i}^2} \tag{2.40b}$$

$$m_\Delta^{ee} = 2\frac{M_W^4}{M_{W_R}^4}(V_R^2)_{ie}\frac{k^2 m_{N_i}}{m^2_{\Delta_R^{++}}} \tag{2.40c}$$

$$m_\Theta^{ee} = 2\frac{M_W^2}{M_{W_R}^2} k \Theta_{ee}^* \qquad (\textit{suppressed by} \sim 10^{-1}) \tag{2.40d}$$

The values $G|\mathcal{M}_\nu|$ for different nuclei are also shown in Table (2.38) together with the available kinetic energy Q. The interference terms [79] are small due to the difference in chirality of the outgoing electrons. They are suppressed by a factor between m_e/Q and m_e^2/Q^2. The generic bounds arising from the limits on the total decay rate can then be translated from (2.31):

$$|(M_\nu)_{ee}| \lesssim 0.3\,\mathrm{eV} \tag{2.41}$$

$$|(M_N)_{ee}| \gtrsim 10\,\mathrm{GeV} \times \left(\frac{3.5\ \mathrm{TeV}}{M_{W_R}}\right)^4 \tag{2.42}$$

$$|\Theta_{ee}| \lesssim 4 \times 10^{-5} \times \left(\frac{M_{W_R}}{3.5\ \mathrm{TeV}}\right)^2 \tag{2.43}$$

From this last limit on the element Θ_{ee} one can get a rough bound on the elements of the Dirac mass matrix:

$$m_D < \mathrm{MeV} \times \frac{m_N}{100\,\mathrm{GeV}} \times \left(\frac{M_{W_R}}{3.5\ \mathrm{TeV}}\right)^2 \tag{2.44}$$

In the case of having a nearly diagonal $V_R \sim 1$, we could then allow $m_N/m^{++}_{\Delta_R}$ to be large and at the same time keep the rates of LFV processes under control. The contribution of Δ_R^{++} becomes important, specially when the heavy neutrino masses are close to M_{W_R}. So for $V_R \sim 1$ we have additionally:

$$m_{\Delta_R^{++}} > 500\,\mathrm{GeV} \times \left(\frac{3.5\mathrm{TeV}}{M_{W_R}}\right)^2 \times \sqrt{\frac{m_{N_1}}{3\,\mathrm{TeV}}} \tag{2.45}$$

2.6.2 Implication for the LR scale

The mass of W_R can not be arbitrarily large if the new physics is to generate substantial rates of $0\nu\beta\beta$. Here we aim to find maximum value of M_{W_R} that could do that [80]. We start by analysing the contribution of the doubly charged scalar Δ_R^{++}:

$$m_\Delta^{ee} = 2\frac{M_W^4}{M_{W_R}^4}(V_R^2)_{ie}\frac{k^2 m_{N_i}}{m^2_{\Delta_R^{++}}} \tag{2.46}$$

It can only be dominant when the heavy neutrino masses m_{N_i} are large. But the LFV processes $\mu \to e^+ee$ and $\mu \to e\gamma$ are proportional to $m_N/m_{\Delta_R^{++}}$ and, as we have seen, when m_N is large the only way to satisfy the experimental constraints is to have a nearly diagonal

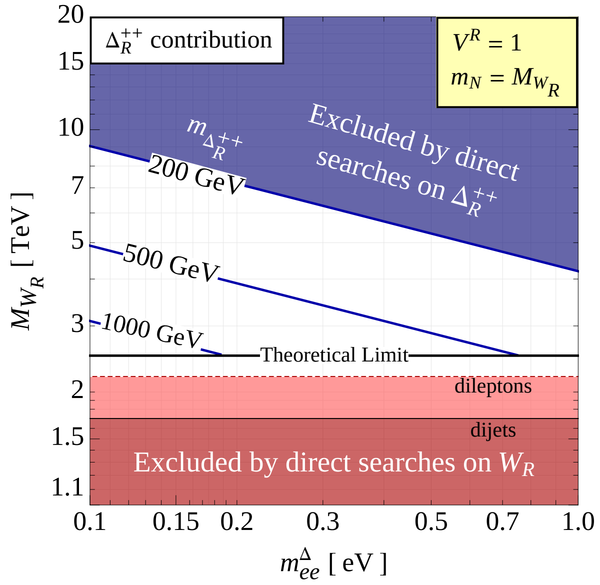

Figure 2.8: Upper bound from $0\nu\beta\beta$ due to the doubly charged scalar Δ_R^{++}

right-handed mixing matrix. Figure 2.8 shows the contribution to the m_Δ^{ee} amplitude for different values of $m_{\Delta_R^{++}}$ and for a fixed value of $m_N = M_{W_R}$ and with V_R diagonal. The limit $m_{\Delta_R}^{++}$ follows from direct collider searches and is expected to increase in the near future, see Table 2.1. If this is the case, and if the direct searches in the collider continue pushing up the limits on M_{W_R}, we can see that the contribution of Δ_R^{++} will lose space and may get ruled out before any evidence of $0\nu\beta\beta$. In any case this contribution is complementary to the contribution of the right-handed heavy neutrinos. The scale of the right-handed boson in any case can not exceed the 5-9 TeV if the rate is to be larger than 0.1eV.

The contribution of the heavy neutrinos becomes important when their masses are close to $k \simeq 100\,\text{MeV}$:

$$m_N^{ee} = \frac{M_W^4}{M_{W_R}^4}(V_R^2)_{ie}\frac{k^2 m_{N_i}}{k^2 + m_{N_i}^2} \tag{2.47}$$

For masses lower and larger than k the amplitude decreases and two values of m_N can give the same amplitude. Figure 2.9 shows the contribution of one N to the m_N^{ee} amplitude for the $(V_R)_{eN} = 1$ which gives the most conservative bound on M_{W_R}. The lower regime in which m_N is lower than ~ 140MeV is disfavored by cosmology as it is explained below.

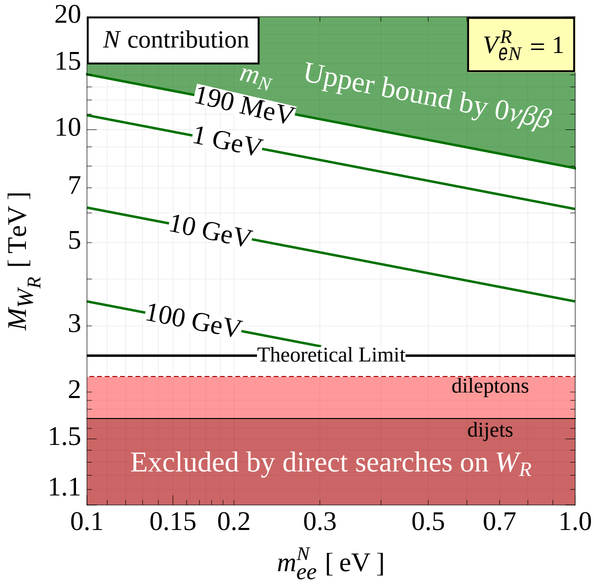

Figure 2.9: Upper bound from $0\nu\beta\beta$ due to the heavy right handed neutrino N

From the figure, we can see that an M_{W_R} of no more than $\sim 10\,\text{TeV}$ can still produce large contribution of $0\nu\beta\beta$. However, there is a strong dependence in the mass hierarchy and the mixing matrix V_R analogous to the flavor dependence of the light neutrino contribution. In principle this rate could vanish itself even with low scale of m_N and M_{W_R}.

The contribution due to the light and heavy mixing, is less eloquent, and goes through the mixing matrix element: Θ_{ee}:

$$m_{\Theta}^{ee} = 2\frac{M_W^2}{M_{W_R}^2} k\Theta^*_{ee} \qquad (\textit{suppressed by} \sim 10^{-1}) \tag{2.48}$$

Nuclear physics calculations are giving a suppression to the naive estimation by a factor of 10% (see [78]). The light-heavy neutrino mixing matrix is given roughly by $\sqrt{\epsilon - m_\nu/m_N}$ with $\epsilon = v_L/v_R$. Thus it becomes important when the vev v_L is large and when the masses of m_N are small. In the left frame of Figure 2.10 we show different values of v_L with the m_N fixed and large in order not to interfere with v_L, in the right frame we fix v_L to zero and see the dependence on light m_N. When v_L is large compared with $M_{W_R} m_\nu/m_N$ there will be cancellations in the seesaw formula (between type I and type II seesaw). This Θ contribution points in a direction which is hard to probe experimentally. The LHC can

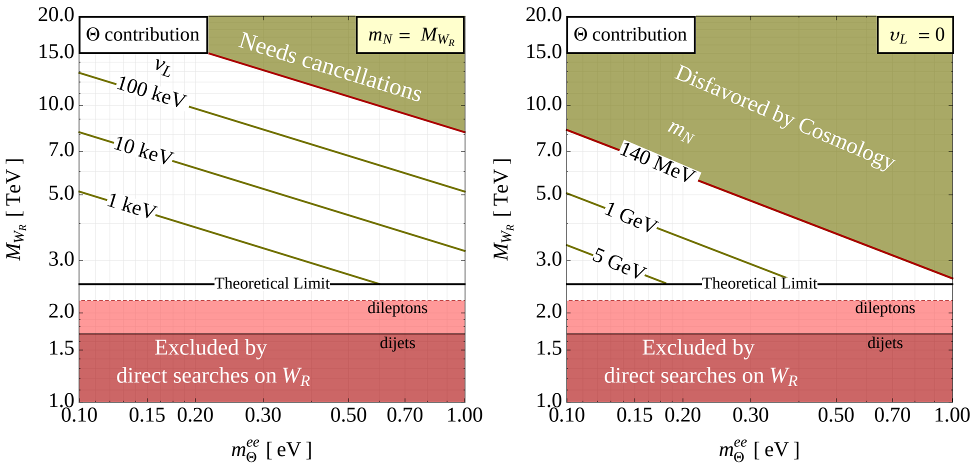

Figure 2.10: Upper bound from $0\nu\beta\beta$ due to the light and heavy neutrino mixing Θ.

indeed test right-handed heavy neutrinos above $10 - 100$GeV (lower values will escape the detector because N will simply not have time to decay). Moreover large m_N give a subdominant contribution because they are already present in the amplitude of m_N^{ee}. The only interesting case is when v_L is large. This can in principle be seen at colliders because the coupling of Δ_L^{++} to the W_L boson is through v_L. Nevertheless its value is bounded from above from electroweak precision tests $v_L \lesssim 10$GeV.

2.6.3 Cosmological constraints on m_N

The cosmological constraint on the masses of the heavy neutrinos requires the life time of N to be smaller than a second $\tau_N \lesssim$ sec. This is needed in order not to ruin the abundance of light elements and the predictions of the BBN. When m_N is larger than the mass of the pion plus a lepton the decay rate of N into $\pi\ell$ will be given by:

$$\begin{aligned}\Gamma(N \to \ell\pi) =& \frac{G_F^2 f_\pi m_N^3}{8\pi} \frac{M_W^4}{M_{W_R}^4} |(V_R^q)_{ud}|^2 |(V_R)_{\ell N}|^2 \\ &\times \left[(1-x_\ell^2)^2 - x_\pi^2(1+x_\ell^2)\right] \left[\left(1-(x_\pi+x_\ell)^2\right)\left(1-(x_\pi-x_\ell)^2\right)\right]^{\frac{1}{2}}\end{aligned} \tag{2.49}$$

where $f_\pi = 130$MeV is the pion decay constant, and $x_{\pi,\ell} = m_{\pi,\ell}/m_N$. In order to satisfy the cosmological constraint we will need generically the lightest heavy neutrinos heavier than the sum of the dacay products:

$$m_N > m_\pi + m_\ell \tag{2.50}$$

The precise limit depends on the leptonic right-handed mixing matrix V_R. In the case of having one more heavy neutrinos lower than (2.50) the only way to avoid the over-closure of the universe is to have them lighter than about few eV [81]. Moreover, if a light N is coupling substantially to electrons a low scale of W_R is disfavoured by supernova constraints [82]. The problem comes from the amount of energy released by the supernova. If N is light it will carry a considerable amount of energy unless W_R is heavy enough. The constraint reads [83]:

$$(V_R)_{eN}\left(\frac{3.5\,\mathrm{TeV}}{M_{W_R}}\right) < 0.02 \tag{2.51}$$

If there is one light N at the eV scale, the other two will have an additional decay channel open. The decay rate of N into a lighter one is given by:

$$\Gamma(N_i \to N_j \ell\ell') = 2\Gamma_\mu \left(\frac{M_W}{M_{W_R}}\right)^4 \left(\frac{m_N}{m_\mu}\right)^5 \left|(V_R)_{\ell i}(V_R^*)_{\ell' j} + (V_R)_{\ell j}(V_R^*)_{\ell' i}\right|^2 \tag{2.52}$$

which numerically translates into a bound on the heavier neutrinos:

$$m_N \gtrsim 140\,\mathrm{MeV}\left(\frac{M_{W_R}}{3.5\,\mathrm{TeV}}\right)^{4/5} \tag{2.53}$$

meaning that again the neutrinos in the heavy regime need to have masses above $\sim$ 100MeV.

2.7 Interplay between the LHC and Neutrinoless double beta decay.

There is a region in the (M_{W_R}, m_N) plane in which the same parameter space is being explored simultaneously by direct searches at the LHC and by indirect searches on $0\nu\beta\beta$

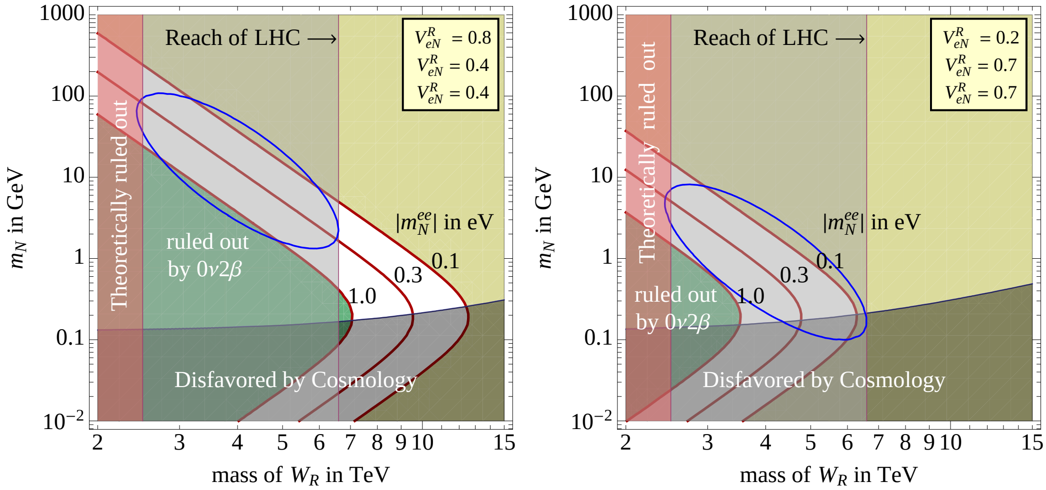

Figure 2.11: Contours of m_N^{ee} in the (M_{W_R}, m_N) plane. Illustrated for large (left frame) and small (right frame) couplings to the electron $(V_R)_{eN}$.

[80]. The common region is highlighted with a blue ellipse in Figure 2.11. There we show two different choices of the coupling of the lightest N to the electron. The reach of LHC is thus covering most of the space in which m_N^{ee} is dominating the rate of $0\nu\beta\beta$. There is only a small window with $7\,\text{TeV} < M_{W_R} < 15\,\text{TeV}$ and $300\,\text{MeV} < m_N < 10\,\text{GeV}$ in which we could encounter a large rate of $0\nu\beta\beta$ due to m_N^{ee}, but no sign of W_R at the LHC. In Figure 2.11 we have also shown the theoretical constraint on the mass of $M_{W_R} > 2.6\,\text{TeV}$ coming from the right-handed contribution to K_L-K_S mass difference [29]. From the two values that m_N can have to produce the same amplitude m_N^{ee} one of them is disfavored by cosmology and only those with masses larger than $\sim 100\,\text{MeV}$ can contribute to the decay rate. This increases the chance for searches at the LHC as the same-sign lepton channel prefers heavy neutrino decaying inside the detector.

2.8 Illustrative example: Type II seesaw dominance

As we have seen, we know nothing about the V+A leptonic mixing angles. There is hope that LHC can probe the mixing angles together with the heavy neutrino masses. This will allow a complete reconstruction of the heavy neutrino mass matrix M_N. Therefore no harm will be done by going ahead with a definite mixing matrix. We could, for example,

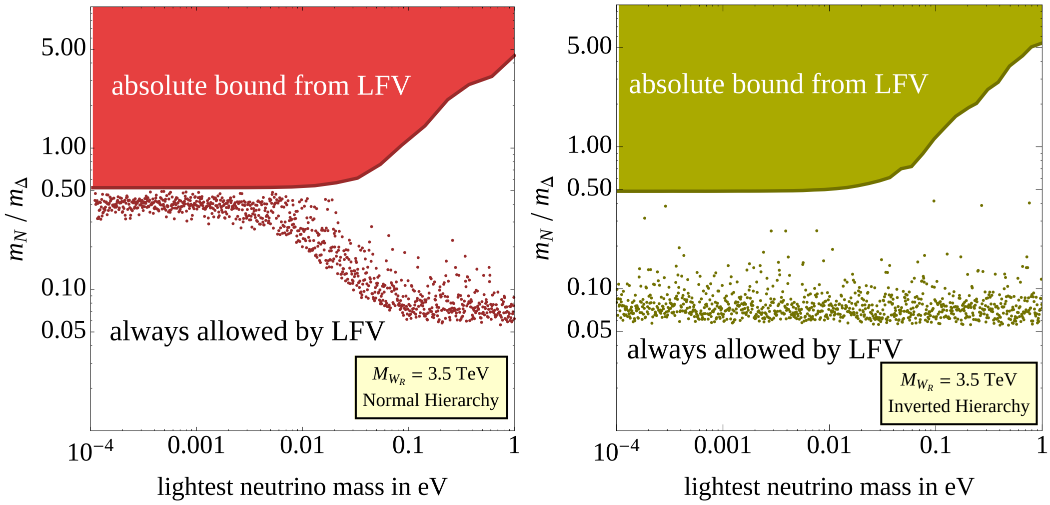

Figure 2.12: Combined bounds on $m_N^{heaviest}/m_{\Delta^{++}}$ from LFV. The dots show the most probable upper bounds resulting for different mixing angles and phases. They were varied in the inteval $\{\theta_{12}, \theta_{23}, \theta_{13}\} = \{31°\text{-}38°, 36°\text{-}55°, 7°\text{-}10°\}$, $[0, 2\pi]$ respectively. The plot scales as $M_{W_R}/3.5\text{TeV}$

imagine to have $V_R = V_L^*$ analogous to what happens in the quark sector where the Left-Right symmetry does imply, up to phases, that $V_R^q = V_L^{q*}$. Then one would get after some simplifications the following total decay rate of $0\nu\beta\beta$:

$$\Gamma_{0\nu\beta\beta} \propto \left|(V_L)_{ei}^2 m_{\nu_i}\right|^2 + \frac{M_W^8}{M_{W_R}^8}\left|(V_L)_{ei}^2 \frac{k^2}{m_{N_i}}\right|^2 + 4|\epsilon|\frac{M_W^4}{M_{W_R}^4}\left(|V_L|_{ei}^2 s_i\right)^2 k^2 \cdot 10^{-2} \tag{2.54}$$

However, without the right-handed neutrino masses we are only able to draw generic conclusion. To go a step further, we consider the case in which the light neutrino mass matrix is generated mostly by ϵ. This is the so called type-II seesaw scenario. In this case we have a proportionality between the light and heavy neutrino mass matrices:

$$M_\nu \simeq \epsilon\, M_N \tag{2.55}$$

This automatically implies that the left and right-handed mixing matrices are equal (up to a complex phase):

$$V_R = \pm e^{-\frac{i\theta_L}{2}} V_L^* \tag{2.56}$$

with θ_L the spontaneous phase of $\langle \Delta_L^0 \rangle$. The light and heavy neutino masses are proportional:

$$\frac{m_{\nu_1}}{m_{N_1}} = \frac{m_{\nu_2}}{m_{N_2}} = \frac{m_{\nu_3}}{m_{N_3}} = |\epsilon| \tag{2.57}$$

To check this scenario, one needs to measure V_R, m_{N_i} and ϵ. The last quantity ϵ will be tiny and can be translated into a v_L of the order of few keV. This v_L can allow for the production of Δ_L^{++} through gauge interaction in colliders. But in any case, if the masses are proportional and the mixing angles are the same it will be a strong indication in favor of the type II scenario and not in the general case where these relations are a merely coincidence.

With the masses and mixing defined as in (2.56) and (2.57), we can then use all the information available from neutrino oscillations presented in Table 2.2. The only free variables are then the lightest neutrino mass, the Dirac and the Majorana phases. Also two kinds of hierarchies are allowed, depending on the sign of Δm_{13}^2. In Figure 2.12 we show a numerical analysis for the bound arising from $\mu \to e^+ee$, $\mu \to e\gamma$, muon conversion in Au, and also the rare leptonic tau decay. The Majorana phases are chosen in such a way to minimize the rate of all these LFV processes simultaneously. The particular values of the left mixing angles, specially the large θ_{13} make the limit arising from $\mu \to e^+ee$ alone already very robust. In some points where the Majorana phases can make the branching ratio small there will be another LFV process with different flavor dependence which put a limit on $m_N/m_{\Delta^{++}}$. However, for degenerate neutrino masses a proper choice of the Majorana phases can make the LFV rates small enough for any $m_N/m_{\Delta^{++}}$.

The decay rate of $0\nu\beta\beta$ is then given by:

$$\Gamma_{0\nu\beta\beta} \propto |m_{\nu+N}^{ee}|^2 = (V_L)_{ei}^2 (V_L^*)_{ej}^2 \left(m_{\nu_i} m_{\nu_j} + \frac{M_W^8}{M_{W_R}^8} \frac{|\epsilon|^2 k^4}{m_{\nu_i} m_{\nu_j}} \right) \tag{2.58}$$

where have introduced the the effective mass parameter $|m_{\nu+N}^{ee}|$ [11]. This quantity supersedes the standard matrix element m_ν^{ee} in the parameter space accessible to LHC. In Figure 2.13 we show $|m_{\nu+N}^{ee}|$ for a fixed value of ϵ as a function of the lightest neutrino

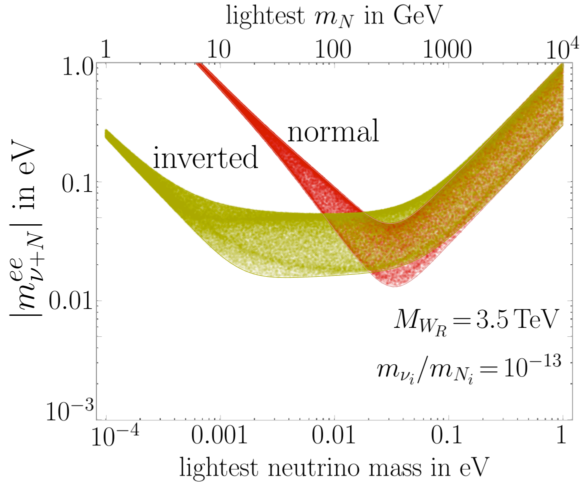

Figure 2.13: Effective $0\nu\beta\beta$ mass parameter $|m^{ee}_{\nu+N}|$, a measure of the total $0\nu\beta\beta$ rate including contribution from both left and right-handed currents.

mass. The effect of the heavy neutrinos is that of inverting the role of the hierarchies. Contrary to the canonical light neutrino contribution the new physics has more chance to occur for highly hierarchical neutrinos. The normal hierarchy being ahead of the inverted hierarchy in a wide region of the parameter space. The combination of light and heavy neutrinos implies that the total rate of $0\nu\beta\beta$ can not vanish for any choice of the Majorana phases. The upper axis of Figure 2.13 shows the range of the lightest m_N. When it is below 100 GeV it would lead to interesting displaced vertices at the LHC [29]. This examples shows, that $0\nu\beta\beta$ may be naturally governed by the new physics, and this may eventually be confirmed by the LHC. Thus it is possible that the light neutrino masses give only a subleading contribution to the decay rate. This demonstrates that the theorem of [84] in which a non-zero $0\nu\beta\beta$ rate can be translated into a non-zero neutrino Majorana mass has no practical purpose. Another example was provided by the minimal supersymmetric standard model [85].

2.9 Electron electric dipole moment

A non zero value of EDM will signal the violation of P and T (or CP) symmetry. For a review see [86]. The experimental constraints for the electric dipole moment of charged leptons are shown in Table 2.10. For the electron, the contribution of the Standard Model appears at 4 loop order with an approximate value of [87]:

$$d_e^{\mathrm{SM}} \simeq 10^{-27}\frac{e}{2m_e} \tag{2.59}$$

And thus it is far from being relevant. In the Left-Right model, on the contrary, we encounter a non-zero value already at the 1 loop level. To perform the calculation we need the effective interaction $l_1 \to l_2\gamma$. For this, we define the vertex Γ^μ as:

$$\langle f_2(p_2)|J_\mu^{em}|f_1(p_1)\rangle = \overline{u_2}(p_2)\Gamma^\mu u_1(p_1) \tag{2.60}$$

The conservation of electromagnetic current implies that Γ^μ depends only in four form factors:

$$\Gamma^\mu = \left(q^2\gamma^\mu - q^\mu \not{q}\right)(F_1 + F_2\gamma_5) + i\sigma^{\mu\nu}q_\nu(F_3 + F_4\gamma_5) \tag{2.61}$$

with $q = p_1 - p_2$. When the initial and final particles are the same, the coefficient F_3 and F_4 are real and correspond to the electric and magnetic dipole moments, respectively. Using complex quantities, we could write the effective dipole interactions as:

$$H = \overline{\ell}\,\sigma_{\mu\nu}\big[\sigma\gamma_L + \sigma^*\gamma_R\big]\ell F^{\mu\nu} \tag{2.62}$$

In that case we have:

$$d = \mathrm{Re}(\sigma) \qquad \text{electric dipole moment} \tag{2.63}$$

$$\mu = \mathrm{Im}(\sigma) \qquad \text{magnetic dipole moment} \tag{2.64}$$

We can use the form factors of $\mu \to e\gamma$ already calculated in (2.15) and simply take the imaginary part. There we found that the only complex form factor was given by the mixed amplitude σ_{LR}. This corresponds to the diagram in which the W_L boson and the

charged lepton	$\|d\|$
e	$< 1 \times 10^{-16} \dfrac{e}{2m_e}$
μ	$< 1 \times 10^{-6} \dfrac{e}{2m_\mu}$
τ	$< 2 \times 10^{-2} \dfrac{e}{2m_\tau}$

Table 2.10: Limits on charged leptons electric dipole moments

heavy neutrinos are propagating in the loop. The rest of the diagrams are real and do not contribute to the EDM. The amplitude σ was found to be:

$$\sigma = \sigma_{LR} = -\frac{eG_F}{4\sqrt{2}\pi^2} M_N V_R^* F_2(t) V_R^T \xi \Theta \tag{2.65}$$

The EDM of the electron d_e is simply the imaginary part of σ_{ee}, and can be written after some simplifications in the following way:

$$d_e = \frac{eG_F}{4\sqrt{2}\pi^2} \frac{M_W^2}{M_{W_R}^2} \sin(2\beta) \mathrm{Im} \left[e^{ia} (V_R)_{ei} (V_L^*)_{ej} F_2(t_i) (M_D^{\text{phys}})_{ij} \right] \tag{2.66}$$

Similar expression can be derived for the muon and tau leptons by simply replacing the index e by μ and τ in the formula above. The flavor dependence is controlled by the left and right-handed mixing angles and also by the neutrino Dirac mass matrix which we reproduce here for convenience:

$$M_D^{\text{phys}} = V_R^\dagger M_N \sqrt{\epsilon - M_N^{-1} M_\nu} V_L \tag{2.67}$$

Therefore apart from the overall factor $\sin(2\beta)$, which fixes the overall scale of the EDM, with the data from LHC we could be able to predict the ratio $d_e : d_\mu : d_\tau$ between different EDM [21]:

$$\frac{d_\alpha}{d_\beta} = \frac{\sum_{ij} \mathrm{Im} \left[e^{ia} (V_R)_{\alpha i} (V_L^*)_{\alpha j} F_2(t_i) (M_D^{\text{phys}})_{ij} \right]}{\sum_{ij} \mathrm{Im} \left[e^{ia} (V_R)_{\beta i} (V_L^*)_{\beta j} F_2(t_i) (M_D^{\text{phys}})_{ij} \right]} \tag{2.68}$$

From (2.66) we can write also:

$$d_e \simeq 10^{-16}\frac{e}{2m_e} \times \left(\frac{3.5\text{TeV}}{M_{W_R}}\right)^2 \sin(2\beta)\text{Im}\left[e^{ia}(V_R)_{ei}(V_L^*)_{ej}F_2(t_i)\frac{(M_D^{\text{phys}})_{ij}}{\text{MeV}}\right] \tag{2.69}$$

Using the experimental constraint on the electron EDM from Table 2.10 we can see that, for a $W_R \simeq 3.5\,\text{TeV}$, the experiment on the electron EDM are probing Dirac masses of order:

$$m_D \sin(2\beta) \sim \text{MeV} \tag{2.70}$$

This complements the limit from $0\nu\beta\beta$ in (2.44). The additional dependence on β can be seen from a positive point of view. In principle if d_e is measured and m_N and V_R are determined from the collider, we could predict β as well. It can then be used to determine d_μ, d_τ and also the W_L-W_R mixing angle $\xi \simeq \sin 2\beta(M_W^2/M_{W_R}^2)$. A direct test of this mixing would then have to wait for further experiments.

2.9.1 Neutrino dipole moments

For Majorana particles only the transition dipole moments are different from zero. What happens is that the dipole and magnetic moment of the neutrino are cancelled by equally opposite moments coming from the antineutrino.

The calculation of the transition dipole moments goes in the same way as before. We can again make use of the general formulas in [50]. The only difference is in the number of diagrams. We need the diagrams with neutrinos as well as with antineutrinos. Transition amplitudes μ_{ij} and d_{ij} are given by [88, 21]:

$$\begin{aligned}\mu_{ij} = \frac{3eG_F i}{16\sqrt{2}\pi^2}\Big\{&(m_{\nu_i}+m_{\nu_j})\,\text{Im}\big[(V_L)_{ki}(V_L^*)_{kj}\big]\frac{m_{\ell_k}^2}{M_W^2}\\ &+\frac{16}{3}m_{\ell_k}\text{Im}[\xi(\Theta^*V_L^*)_{ki}(V_L^*)_{kj}+\xi^*(\Theta V_L)_{kj}(V_L)_{ki}]\Big\}\end{aligned} \tag{2.71a}$$

$$\begin{aligned}d_{ij} = \frac{3eG_F}{16\sqrt{2}\pi^2}\Big\{&(m_{\nu_j}-m_{\nu_i})\text{Re}\big[(V_L)_{ki}(V_L^*)_{kj}\big]\frac{m_{\ell_k}^2}{M_W^2}\\ &+\frac{16}{3}m_{\ell_k}\text{Re}\big[\xi(\Theta^*V_L^*)_{ki}(V_L^*)_{kj}-\xi^*(\Theta V_L)_{kj}(V_L)_{ki}\big]\Big\}\end{aligned} \tag{2.71b}$$

One can check that when $i = j$ these quantities vanish as they should. The nice feature is that the new contribution is not suppressed by the light neutrino masses. The new physics through the left-right mixing is expected to dominate over the light neutrino masses by a factor of roughly:

$$\sim 10^{10} \sin(2\beta)\Theta \left(\frac{3.5\text{TeV}}{M_{W_R}}\right)^2 < 10^5 \sin(2\beta) \tag{2.72}$$

Here on the last step, we have used the generic constraint (2.41) from $0\nu\beta\beta$ on the mixing matrix Θ. So unless the angle β is very close to zero the dominant contribution will come from the left-right mixing. However, the experimental limit on the magnetic moment of neutrinos is rather weak [89]:

$$\mu_{12} < 10^{-10} \frac{e}{2m_e} \tag{2.73}$$

and at the present does not provide new constraints. More important, once more we will be able to predict its value up to the overall factor $\sin\beta$ with the outcome from LHC on V_R and m_N. As we have said in the previous section, if we combine the searches from LHC with the measurement of the electron EDM we could shed light also on the β angle and fully predict the neutrino transition dipole moments.

Conclusion

The left-right model offers a natural answer to one of the most puzzling facts about neutrino masses. The smallness of their mass scale is connected with the scale of right-handed gauge currents. With the lepton number violated in both places we are left with two complementary manifestations of the Majorana nature of neutrinos, namely, neutrinoless double beta decay ($0\nu\beta\beta$) and the KS production of the same-sign charged lepton pairs at colliders. Both processes will signal the violation of lepton number by two units. But the low energy manifestation of this large scale in which the parity is restored, can also show up in $0\nu\beta\beta$. We have shown quantitatively in this thesis that it can even dominate the total decay rate. Moreover, it can do it in the parameter space in which the KS signal is in turn within reach of the LHC, thus doubling the physical impact of the scale of the right-handed gauge currents.

Moreover we have also shown how the low scale of lepton flavor violating processes such as $\mu \to 3e$, $\mu \to 3\gamma$ and the μ-e conversion in the nuclei, can be used to assist the searches of the scalar particles and right-handed neutrinos in the collider. For example, one should expect in general masses of the right-handed neutrinos much below the masses of the doubly charged scalar particles. This was shown quantitatively for a particular scenario in which the neutrino masses are generated mostly by the vacuum expectation value of the left scalar triplet. For the same case, we also illustrated graphically the interconnection between $0\nu\beta\beta$ and the KS signal outlined above. We found that the neutrino mass hierarchy in $0\nu\beta\beta$ can not be extracted with a simple measurement of the total decay rate. Obviously this is not the best place to look for oscillation parameters.

On the contrary, we argued and repeatedly emphasized throughout this thesis, that the best place to look for the leptonic right-handed mixing parameters is the LHC through the KS lepton number (and also lepton flavor) violating signal.

In the scenario in which the right-handed neutrino mass matrix is determined from the LHC we showed that one can in turn predict the coupling of light and heavy neutrinos to the Higgs boson. We found a definite expression for the Dirac mass matrix depending on oscillation parameters and high energy collider observables. It was precisely the left-right symmetry which eliminated the unphysical parameters present in generic type I seesaw, that allowed us to invert the seesaw formula in favor of the Dirac mass matrix in a definite way. This is of fundamental importance now that we have entered in the era of the Higgs boson. The light-heavy neutrino mixing matrix is proportional to the Dirac mass matrix and therefore, with the outcome of the LHC, one will be able to quantify at the same time all the low energy manifestation depending on this small mixing.

Appendix A

Scalar Potential

The most general scalar potential invariant under $\mathcal{G}_{LR}$ and the $\mathcal{C}$ symmetry in which $\Phi \leftrightarrow \Phi^T$ and $\Delta_L \leftrightarrow \Delta_R^*$ can be written as follows:

$$
\begin{aligned}
V = &-\mu_1^2 \operatorname{Tr}|\Phi|^2 - \mu_2^2 \det\Phi - \mu_2^{*2} \det\Phi^* - \mu_3^2\big(\operatorname{Tr}|\Delta_L|^2 + \operatorname{Tr}|\Delta_R|^2\big) \\
&+ \lambda_1(\operatorname{Tr}|\Phi|^2)^2 + \lambda_2(\det\Phi)^2 + \lambda_2^*(\det\Phi^*)^2 + \lambda_3|\det\Phi|^2 \\
&+ (\lambda_4 \det\Phi + \lambda_4^* \det\Phi^*)\operatorname{Tr}|\Phi|^2 \\
&+ \rho_1\Big[(\operatorname{Tr}|\Delta_L|^2)^2 + (\operatorname{Tr}|\Delta_R|^2)^2\Big] + \rho_2\Big(|\det\Delta_L|^2 + |\det\Delta_R|^2\Big) \\
&+ \rho_3 \operatorname{Tr}|\Delta_L|^2 \operatorname{Tr}|\Delta_R|^2 + \rho_4 \det\Delta_L \det\Delta_R^* + \rho_4^* \det\Delta_L^* \det\Delta_R \\
&+ (\alpha_1 \operatorname{Tr}|\Phi|^2 + \alpha_2 \det\Phi + \alpha_2^* \det\Phi^*)(\operatorname{Tr}|\Delta_L|^2 + \operatorname{Tr}|\Delta_R|^2) \\
&+ \alpha_3(\operatorname{Tr}\Phi\Phi^\dagger\Delta_L\Delta_L^\dagger + \operatorname{Tr}\Phi^\dagger\Phi\Delta_R\Delta_R^\dagger) \\
&+ \beta_1 \operatorname{Tr}\Phi\Delta_R\Phi^\dagger\Delta_L^\dagger + \beta_1^* \operatorname{Tr}\Phi^\dagger\Delta_L\Phi\Delta_R^\dagger \\
&+ \beta_2 \operatorname{Tr}\Phi^c\Delta_R\Phi^\dagger\Delta_L^\dagger + \beta_2^* \operatorname{Tr}\Phi^{c\dagger}\Delta_L\Phi\Delta_R^\dagger \\
&+ \beta_3 \operatorname{Tr}\Phi\Delta_R\Phi^{c\dagger}\Delta_L^\dagger + \beta_3^* \operatorname{Tr}\Phi^\dagger\Delta_L\Phi^c\Delta_R^\dagger
\end{aligned}
$$

The minimum of the potential is attain when:

$$
\langle\Phi\rangle = \begin{pmatrix} v\cos b & 0 \\ 0 & -ve^{-ia}\sin b \end{pmatrix}, \langle\Delta_L\rangle = \begin{pmatrix} 0 & 0 \\ v_L e^{i\delta} & \end{pmatrix}, \langle\Delta_R\rangle = \begin{pmatrix} 0 & 0 \\ v_R & 0 \end{pmatrix} \tag{A.1}
$$

under the conditions that $V'_v = V'_b = V'_a = V'_{v_R} = V'_{v_L} = V'_\delta = 0$. The first four will give us $\mu_1, \mu_2, \mu_2^*, \mu_3$ in term of v, v_R. Using these values in fifth equation we will find $v_L \propto v^2/v_R$. The sixth equation gives a relation between phases. To get the mass spectrum we can use the Higgs mechanism, and rotate the charge fields in the following way:

$$\begin{pmatrix} \phi_1^- \\ \phi_2^- \\ \Delta_R^- \end{pmatrix} \simeq \begin{pmatrix} \cos b & -\sin b & -\frac{M_W}{M_{W_R}} e^{-ia} \sin b \\ e^{ia} \sin b & e^{ia} \cos b & -\frac{M_W}{M_{W_R}} \cos b \\ \frac{M_W}{M_{W_R}} e^{ia} \sin 2b & \frac{M_W}{M_{W_R}} e^{ia} \cos 2b & 1 \end{pmatrix} \begin{pmatrix} G_L^- \\ H^- \\ G_R^- \end{pmatrix} \tag{A.2}$$

where G_L^- and G_R^- are the unphysical scalars which are absorbed by W_L and W_R respectively. In the same way, we can rotate the neutral scalars as follows:

$$\begin{pmatrix} \mathrm{Im}(\phi_1^0) \\ \mathrm{Im}(e^{-ia}\phi_2^0) \\ \mathrm{Im}(\Delta_R^0) \end{pmatrix} \simeq \frac{1}{\sqrt{2}} \begin{pmatrix} \cos b & -\sin b & \frac{M_Z}{M_{Z'}} \frac{\sqrt{c_{2W}}}{c_W^2} \cos b \\ \sin b & \cos b & \frac{M_Z}{M_{Z'}} \frac{\sqrt{c_{2W}}}{c_W^2} \sin b \\ -\frac{M_Z}{M_{Z'}} \frac{\sqrt{c_{2W}}}{c_W^2} \sin 2b & 0 & 1 \end{pmatrix} \begin{pmatrix} G_L^0 \\ H_\phi^0 \\ G_R^0 \end{pmatrix} \tag{A.3}$$

where the G_L^0 and G_R^0, are the unphysical scalars absorbed by Z and Z' respectively. In Table A.1 are show the masses of all the physical scalars. The heavy doublet H_ϕ is mostly φ_2, which is the notation used in the text.

Original field	Physical field	mass quared
Light doublet	h^0_ϕ	$v^2\big[4\lambda_1+(\lambda_3+\mathrm{Re}(\lambda_2 e^{-ia}))\sin^2 2b-2\mathrm{Re}(\lambda_4 e^{-ia})\sin 2b\big]$
Heavy doublet	H^0_ϕ A^0_ϕ H^+_ϕ	$\alpha_3 v_R^2\sec 2b+v^2\big(\lambda_3+\mathrm{Re}(\lambda_2 e^{-2ia})\big)\cos^2 2b$ $\alpha_3 v_R^2\sec 2b+v^2\big(\lambda_3-\mathrm{Re}(\lambda_2 e^{-2ia})\big)$ $\alpha_3 v_R^2\sec 2b+\alpha_3 v^2\cos 2b$
Left triplet	H^0_L A^0_L Δ^+_L Δ^{++}_L	$(\rho_3-2\rho_1)v_R^2$ $(\rho_3-2\rho_1)v_R^2$ $(\rho_3-2\rho_1)v_R^2+\frac{1}{2}\alpha_3 v^2\cos 2b$ $(\rho_3-2\rho_1)v_R^2+\alpha_3 v^2\cos 2b$
Right triplet	H^0_R Δ^{++}_R	$4\rho_1 v_R^2$ $\rho_2 v_R^2+\alpha_3 v^2\cos 2b$

Table A.1: Physical scalar particles

Appendix B

The square root of a matrix

Here we follow the method of [91]. To find the square root of a matrix A one first needs to find the matrix X and J such that:

$$A = XJX^{-1} \tag{B.1}$$

where J is called the normal form of A. The matrix J is a block diagonal matrix made out of Jordan blocks:

$$J = \mathrm{diag}(J_1, \cdots, J_i), \qquad J_i = \lambda_i I_i + E_i \tag{B.2}$$

Here I_i is a diagonal matrix and E_i a matrix which has ones just above the diagonal and zeros elsewhere, both matrices have the dimension of the block. For non-singular matrices, the square root of J is given by [1]:

$$\sqrt{J_i} = \lambda_i^{\frac{1}{2}} I_i + \frac{1}{2}\lambda^{\frac{1}{2}-1} E_i + \frac{1}{2!}\frac{1}{2}\Big(\frac{1}{2} - 1\Big)\lambda^{\frac{1}{2}-1} E_i^2 + \cdots \tag{B.3}$$

One needs the first m terms of the series for each $m \times m$ block.

[1]This formula fails when $\lambda_i = 0$ but $E_i \neq 0$.

The square root of A is then:

$$\sqrt{A} = XY\sqrt{J}Y^{-1}X^{-1} \tag{B.4}$$

where Y is a matrix which commutes with J and contains arbitrary continuous parameters. It can be shown that when all λ_i are different from each other, the matrix Y has a block diagonal form and then if it commutes with J it commutes also with $\sqrt{J}$. Therefore *when all λ_i are different the square root of a $n \times n$ matrix contains only discrete solutions.*[2]

We now show focus in some special cases.

• Consider the case the matrix A is diagonalizable. This means all E_i are zero. If all λ_i are different from each other (zeros are also possible). The square root is:

$$\sqrt{A} = X\mathrm{diag}(\pm\sqrt{\lambda_1}, \cdots, \pm\sqrt{\lambda_n})X^{-1} \tag{B.5}$$

• Roots of the unity matrix. In this case $X = 1$, $J = 1$ and Y is an arbitrary matrix. Then $\sqrt{I} = Y\mathrm{diag}(\pm 1, \cdots \pm 1)Y$. For example in 2 dimensions we have:

$$\sqrt{I}_{2\times 2} = \begin{pmatrix} y_{11} & y_{12} \\ y_{21} & y_{22} \end{pmatrix} \begin{pmatrix} 1 & 0 \\ 0 & -1 \end{pmatrix} \begin{pmatrix} y_{11} & y_{12} \\ y_{21} & y_{22} \end{pmatrix}^{-1} = \begin{pmatrix} a & b \\ \dfrac{1-a^2}{b} & -a \end{pmatrix}$$

where $a = (y_{11}y_{22} + y_{12}y_{21})/\mathrm{det}Y$ and $b = -2y_{11}y_{22}/\mathrm{det}Y$ are the independent complex parameters.

In the same way, in 3 dimensions one has for example $\sqrt{I} = Y\mathrm{diag}(1, 1, -1)Y$, and after some redefinitions one arrives at:

$$\sqrt{I}_{3\times 3} = \frac{1}{a+b+c} \begin{pmatrix} a & -(b+c)x & -(b+c)y \\ -(a+c)x^{-1} & b & -(a+c)x^{-1}y \\ -(a+b)y^{-1} & -(a+b)xy^{-1} & c \end{pmatrix}$$

[2]The singular case $\lambda_i = 0$ but $E_i \neq 0$ does not always have a root but when it does it contains continuous parameters because Y will not commute with $\sqrt{J}$.

with a, b, c, x, y arbitrary complex parameters.

In our physical case, we have:

$$\Theta = \sqrt{\epsilon - M_N^{-1} M_\nu} \tag{B.6}$$

The arbitrary nature of the square root comes from the matrix Y in (B.4) which commutes with J. If this matrix does not commute with $\sqrt{J}$ there will be arbitrary parameters. We have seen that for non-singular matrices, this would require at least two identical eigenvalues. But this requirement is far from being physical because even the slightest perturbation will destroy the degeneracy. The singular case is also disfavoured because ϵ is already in the main diagonal and only a perfect cancellations would make the matrix singular. Therefore, we conclude that the continuous solutions of the square root should not be taken seriously.

Obviously M_D does not have to be perfectly symmetric. It is enough if it is mostly symmetric. A small asymmetry can always be generated radiatively but provide it is small our formula is still valid.

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