SM + neutrino mass

LRSM

$\downarrow$

Type I + II seesaw

- Type I : $\exists\ \nu_R$

$\Downarrow$

$$N_L = C\bar{\nu}_R^T$$

$\Downarrow$

$$M_{\nu N} = \begin{pmatrix} 0 & M_D \\ M_D^T & M_N \end{pmatrix}$$

$$(\text{def.})\ N_L^T C M_D \nu_L \equiv \boxed{\overline{\nu_R} M_D \nu_L}$$

$$\boxed{\text{if } M_N \gg M_D}$$

$$\Downarrow$$

$$\boxed{M_\nu = - M_D^T \frac{1}{M_N} M_D \qquad \boxed{M_D \equiv Y_D v_{SM}}}$$

$$\Downarrow$$

$$M_\nu \propto Y_D^T \frac{1}{M_N} Y_D \, v_{SM}^2$$

Seesaw: only SM states are "light" ($\sim M_W$)

- ν mass in SM

$$\ell_L \equiv \begin{pmatrix} \nu \\ e \end{pmatrix}_L ; \quad \phi \; ; \; e_R$$

$Y(\ell) = -1$

$Y(\phi) = +1$

$$(\ell_L \, \ell \, \phi \, \phi)$$

$Y: (-1) + (-1)$ $\longrightarrow$ $d=5$

$\Downarrow$ Weinberg '79

$$\frac{(\ell \ell \phi \phi)}{\Lambda} \quad (d=4)$$

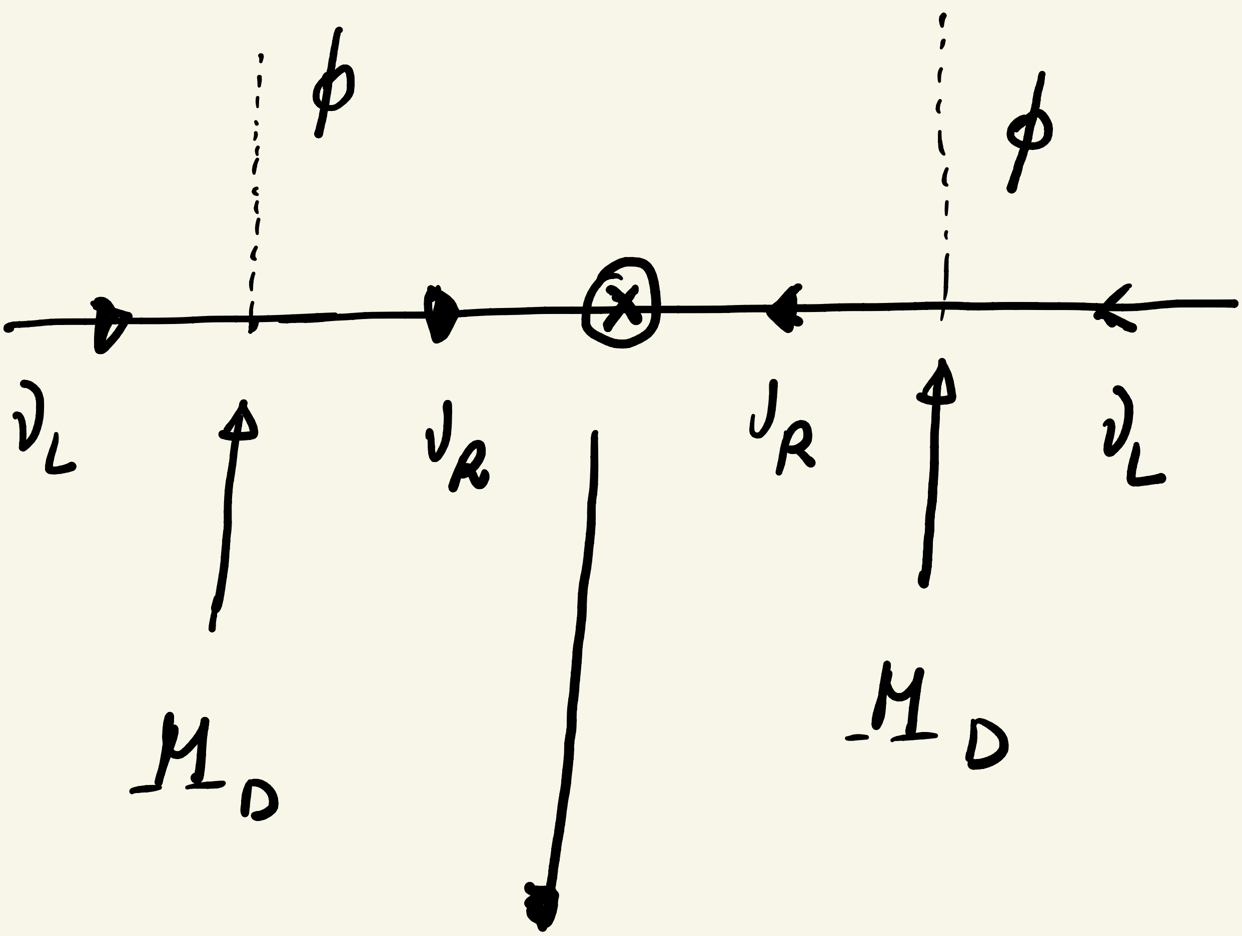
$$\Rightarrow \frac{\nu \nu \langle \phi \rangle \langle \phi \rangle}{\Lambda}$$

d=5 operator

$$\Rightarrow \boxed{m_\nu \simeq \frac{v_{SM}^2}{\Lambda}}$$

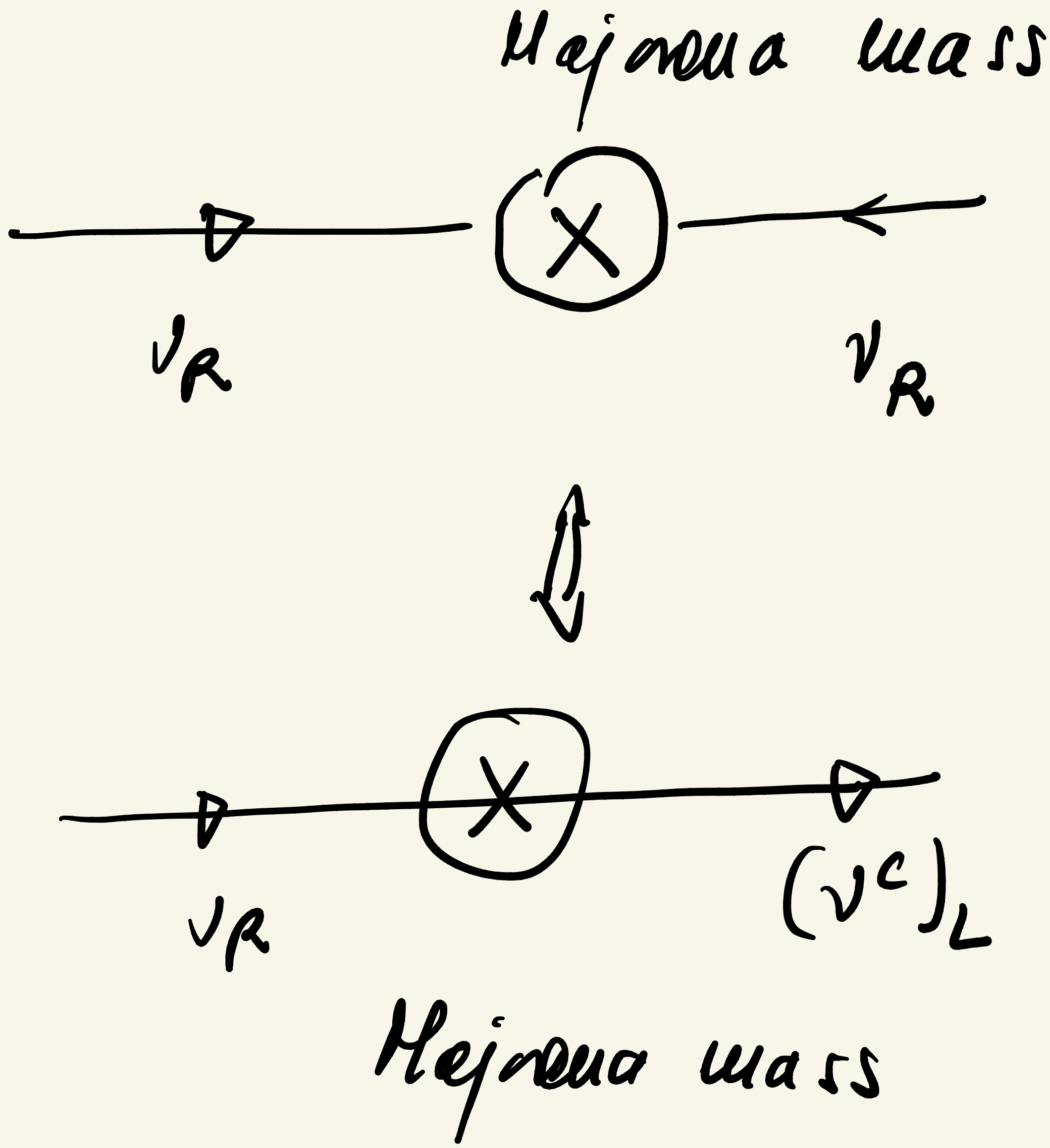
$$\boxed{\text{seesow} = \text{UV completion of } d=5}$$

"we integrate out N (ν_R)"



$$\frac{\not{k} + m_{\nu_R}}{\cancel{k^2} - m_{\nu_R}^2}$$

(ignoring the motion of ν_R)



limit: $h = 0$ $\Longleftrightarrow$ integrate out ν_R

seesaw $\Longleftrightarrow$ Fermi

integrate ν_R integrate W

$$m_\nu = -\frac{m_D^2}{m_N}$$

$$\frac{G_F}{\sqrt{2}} = \frac{g^2}{8 M_W^2}$$

$\boxed{\text{Different (equivalent) approaches}}$

Type I seesaw

$\boxed{\text{(a) } \nu_L, \nu_R \Rightarrow \text{mass matrix}}$

(b) diagrammatic ν mass
($\nu - h$ interaction)

(c) $\mathcal{L} = \cancel{i\bar{\nu}_R\gamma^\mu\partial_\mu\nu_R} - \frac{m_R}{2}\nu_R^T C \nu_R$

$- \bar{\nu}_R m_D \nu_L - \bar{\nu}_L m_D \nu_R \leftarrow$

$+ i\bar{\nu}_L\gamma^\mu\partial_\mu\nu_L$ $\quad (m_N = m_R^*)$

$\Leftrightarrow m_R \gg m_D$

$\Rightarrow \boxed{\text{solve for } \nu_R}$

$$\frac{\partial\mathcal{L}}{\partial\nu_R} = 0 \Rightarrow \boxed{\nu_R = f(\nu_2)}$$

$$\Downarrow$$

bottom line = Weinberg

$(d=5)$

$$\frac{\ell\,\ell\,\phi\,\phi}{\Lambda} \to \nu\,\nu\,\frac{\phi_0\,\phi_0}{\Lambda}$$

$$\underbrace{T_3 = +\frac{1}{2} + \frac{1}{2} = 1}_{SU(2)_L \text{ triplet}}$$

$(\phi\phi = \text{triplet})$

$$2 \times 2 = 3 + \cancel{1}$$

ν mass = SU(2) triplet

$$\nu_{L_i}^T \, C \, \nu_{L_j} =$$

$$\underbrace{= -\nu_{L_j}^T C^T \nu_{L_i} = \nu_{L_j}^T C \, \nu_{L_i}}_{\text{symmetric}}$$

Type II seesaw

$$\ell \; \ell \; \underbrace{(\phi \; \phi)}_{\text{Triplet, } Y = 2}$$

$$\Downarrow$$

$$\boxed{\begin{array}{l} \Delta \longrightarrow U \Delta U^{+} \\ Y(\Delta) = 2 \end{array}}$$

$$\mathcal{L}_Y(\Delta) = \ell_L^T C \, Y_\Delta \, i\sigma_2 \, \Delta \, \ell_L + h.c.$$

$$\Downarrow$$

$$\boxed{m_\nu = Y_\Delta \langle \Delta \rangle}$$

- Δ must be heavy

$$\Delta = \begin{pmatrix} \delta^+ & \delta^{++} \\ \delta^0 & -\delta^+ \end{pmatrix}$$

$$\langle \Delta \rangle = \langle \delta_0 \rangle$$

δ_0 = light ??

$$m_{++} \gtrsim 500 \text{ GeV}$$

$$\Rightarrow \quad m_0 \gtrsim 500 \text{ GeV}$$

$$\boxed{\text{SU(2) breaking} \simeq 100 \text{ GeV}} \Uparrow$$

$$\Rightarrow m_\Delta^2 \, \mathrm{Tr}\, \Delta^+ \Delta \quad \therefore \quad m_\Delta \gtrsim 500 \text{ GeV}$$

$$\underbrace{+ \; \phi^T i\sigma_2 \Delta^* \phi}_{\text{crucial}} + h.c.$$

$$\cancel{\phi^T i \sigma_2 \Delta^* \phi} \longrightarrow$$

$$\cancel{\phi^T U^T i\sigma_2 U^* \Delta^* U^T U \phi}$$

$\Big\}$ NOT inv.

$\Downarrow$ instead

$$\mu\, \phi^T i\sigma_2\, \Delta^\dagger \phi \;+ h.c. \quad (INV)$$

$$+\; m_\Delta^2\, \mathrm{Tr}\, \Delta^\dagger \Delta$$

$$\hookrightarrow \mu\, \phi_0^2\, \delta_0^* + m_\Delta^2\, \delta_0 \delta_0^* + \cdots$$

$$\Rightarrow \frac{\partial V}{\partial \delta_0^*} = \mu\, \phi_0^2 + m_\Delta^2\, \delta_0 + \cdots$$

$$\Downarrow$$

$$\boxed{\langle \delta_0 \rangle = -\frac{\mu\, v_{SM}^2}{m_\Delta^2}}$$

seesaw picture

- why $+\, m_\Delta^2 \, \mathrm{Tr}\, \Delta^\dagger \Delta$???

why <u>not</u> $-m_\Delta^2 \, \mathrm{Tr}\, \Delta^\dagger \Delta \; +$

$$+ \lambda_\Delta \left(\mathrm{Tr}\, \Delta^\dagger \Delta \right)^2$$

$$\Downarrow$$

$$\langle \delta_0 \rangle^2 = \frac{m_\Delta^2}{\lambda_\Delta} \, , \quad \langle \delta_0 \rangle \ll v_{SM}$$

$\Rightarrow m_\Delta \ll \ldots$

$$\boxed{\text{Higgs mechanism ???}}$$

$$\boxed{m_h^2 \simeq \lambda v^2}$$

$\Downarrow$

$$\boxed{\Delta \text{ is } \underline{\text{not}} \text{ Higgsed}}$$

$$\boxed{d=5}$$

- $\underbrace{l^T C\, i\sigma_2\, l}_{\substack{\shortparallel \\ 0 \text{ for } 1 \text{ gen}}}\ \underbrace{\phi^T i\sigma_2\, \phi}_{\substack{\shortparallel \\ 0}}$

- $\underbrace{(l^T i\sigma_2\, \phi)}_{}\, C\, (\phi^T i\sigma_2\, l)$

$$\underbrace{\text{SU(2) singlet; } Y=0\text{; fermion}}_{\sim\, \nu_R\ (N)}$$

- $\underbrace{\left(\ell^T i\sigma_2 \vec{\sigma} C \ell\right)}\ \underbrace{\left(\phi^T i\sigma_2 \vec{\sigma}\phi\right)}$

SU(2) triplet; −2 (2); bosons

- $\underbrace{\left(\ell^T i\sigma_2 \vec{\sigma}\phi\right)} C \left(\phi^T i\sigma_2 \vec{\sigma}\ell\right)$

$\boxed{\text{SU(2) triplet, } Y=0\text{; fermions}}$

new

bottom line

$$\boxed{\begin{array}{l} 3 \quad d=5 \quad \text{operators} \\ * \quad \text{RELATED} \quad *** \end{array}}$$

$$\phi = \begin{pmatrix} \phi^+ \\ \phi^0 \end{pmatrix}, \quad l = \begin{pmatrix} \nu \\ e \end{pmatrix}$$

$\Downarrow$ guess

Type III seesaw

$\underline{SM} + T_F$ ($y=0$, triplet fermion)

Type I: $N M_D \nu$

$$\Uparrow$$

$$N \; y_D \langle \phi \rangle \nu$$

$$\Uparrow$$

$$N \; y_D \, \phi \, \nu$$

$$\Uparrow$$

$$N \; y_D (\phi \;\; \ell)$$

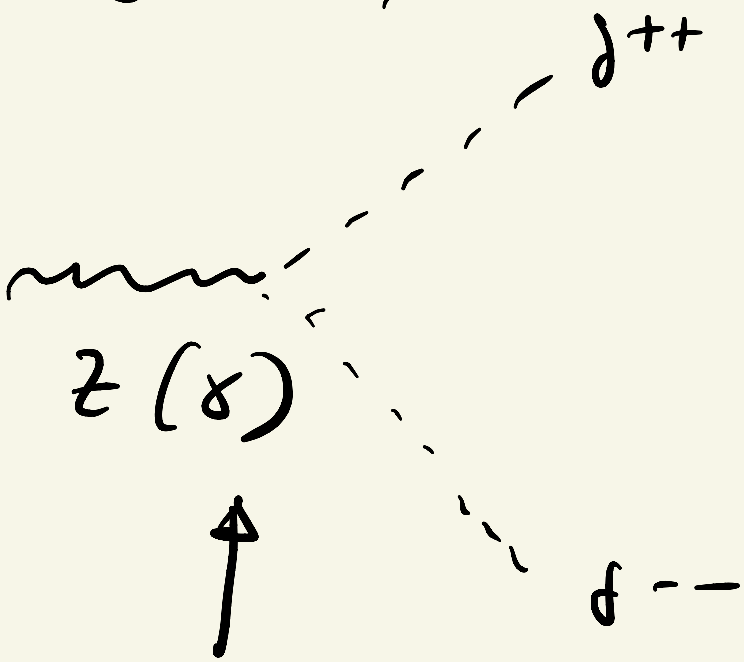
Higgs

lepton doublet

$$2 \times 2 = 1 + 3$$

N $\qquad$ T_F

δ^{++}



$Q = \pm 2$

$\boxed{\text{Exam: } 3(12) - 10(12)}$

midnight