Neutrino Physics Course

Lecture XXIII

19/7/2022

LMU

Summer 2022

Neutrino Oscillations (II)

$$\begin{pmatrix}\nu_e\\ e\end{pmatrix}, \quad \begin{pmatrix}\nu_\mu\\ \mu\end{pmatrix}$$

physical states: e, μ

$$\nu_e = \nu_1 c + \nu_2 s \qquad c \equiv \cos\theta$$

$$\nu_\mu = -\nu_1 s + \nu_2 c \qquad s \equiv \sin\theta$$

$$\nu_i(t) = e^{i(E_i t - p_i x)}\,\nu_i$$

$$\Downarrow$$

$$P(\nu_e \to \nu_\mu) = \sin^2 2\theta \, \sin^2 \frac{\Delta m^2 L}{4E}$$

maximal $\theta = 45^0$

iff :

$$\Delta m^2 \ll \sigma_{m^2}$$

inherent uncertainty
of measurement

$$E^2 = p^2 + m^2$$

$$\sigma_{E^2} \simeq E\,\sigma_E , \quad \sigma_{p^2} \simeq p\,\sigma_p$$

$$\delta E^2 \sim \delta p^2$$

$\Downarrow$

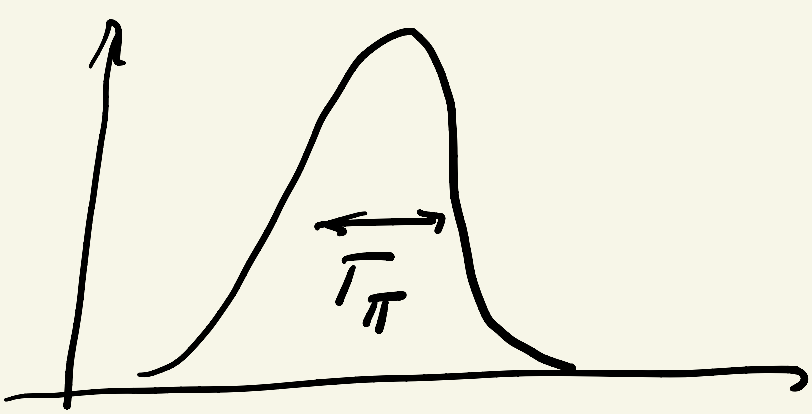
$$\boxed{\sigma_m^2 \simeq E\,\sigma_E}$$

error on E ?

- <u>atmospheric neutrinos</u>

$$\pi^- \longrightarrow \mu^- + \bar{\nu}_\mu$$

$$E \simeq m_\pi \,, \quad \sigma_E$$



$$\begin{cases} \Gamma_\pi \simeq 10^{-17}\,\mathrm{GeV} \simeq 10^{-8}\,\mathrm{eV} \\ m_\pi \simeq 100\,\mathrm{MeV} \simeq 10^{8}\,\mathrm{eV} \end{cases}$$

$$\Downarrow$$

$$\sigma_{m^2} \simeq E\,\Gamma_\pi \simeq m_\pi \Gamma_\pi$$

$$\Rightarrow \quad \sigma_{m^2} \simeq 10^{-8}\,\mathrm{GeV}^2 \simeq \mathrm{eV}^2$$

but

$$\boxed{\Delta m_A^2 \simeq 10^{-3}\,\mathrm{eV}^2 \ll \mathrm{eV}^2 \simeq \sigma_{m^2}}$$

$$\boxed{\text{ATM:} \quad \nu_\mu \to \nu_\tau}$$

$$P(\nu_\mu \to \nu_\mu) = 1 - P(\nu_\mu \to \nu_\tau)$$

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we took:

$$\nu \equiv \psi_\nu = e^{ipx} = e^{i(Et - \vec{p}\vec{x})}$$

free wave

but

particles are wave packets

$$\psi_i(p) = e^{i(E_i t - \vec{p}\cdot\vec{x})} \; e^{-\frac{(\vec{p}-\vec{p}_0)^2}{\sigma_p^2}}$$

$i = 1, 2$ $\qquad E_i = \sqrt{\vec{p}^2 + m_i^2}$

$$\boxed{\psi_i(x) = \int \frac{d^3p}{(2\pi)^3}\ \psi_i(p)}$$

space - time

$(d=1)$

$$\psi_i(x) = \int \frac{dp}{2\pi}\ e^{i(E_i t - px)}\ e^{-\frac{(p-p_0)^2}{\sigma_p^2}}$$

$$\cancel{E_i = \sqrt{p^2 + m_i^2} \simeq p + \frac{1}{2}\frac{m_i^2}{p}}$$

$$\cancel{\simeq p + \frac{1}{2}\frac{m_i^2}{E}}$$

$$\boxed{\int_{-\infty}^{+\infty} |\psi(x)|^2 dx = 1}$$

$$E_i(p) = E_i(p_0) + \left.\frac{\partial E_i}{\partial p}\right|_{p_0} (p-p_0) + \cdots$$

$$\cong E_i(p_0) + \frac{\cancel{2}p_0}{\cancel{2}\sqrt{p_0^2+m_i^2}}(p-p_0)$$

$$\simeq E_i(p_0) + \underbrace{\frac{p_0}{E_i}}_{\displaystyle \equiv v_i}(p-p_0)$$

$$\Downarrow$$

$$\phi \equiv E_i t - p x =$$

$$= E_i(p_0)\,t + v_i\,(p-p_0)\,t$$

$$- p_0 x - (p-p_0)\,x$$

$$= \underbrace{\left(E_i(p_0)\,t - p_0 x\right)}_{\phi_0^i} + (v_i t - x)(p - p_0)$$

$$+ \cdots$$

$$\Downarrow \quad \boxed{y \equiv x - v_i t}$$

$$\psi(x) \propto e^{i\phi_0^i} \int dp\, e^{-i(p-p_0)y}\, e^{-\frac{(p-p_0)^2}{\sigma_p^2}}$$

$$\propto e^{i\phi_0^i} \int du\, e^{-iyu}\, e^{-u^2/\sigma_p^2}$$

$$u \longrightarrow u - \frac{i}{2}\,y\,\sigma_p \equiv v$$

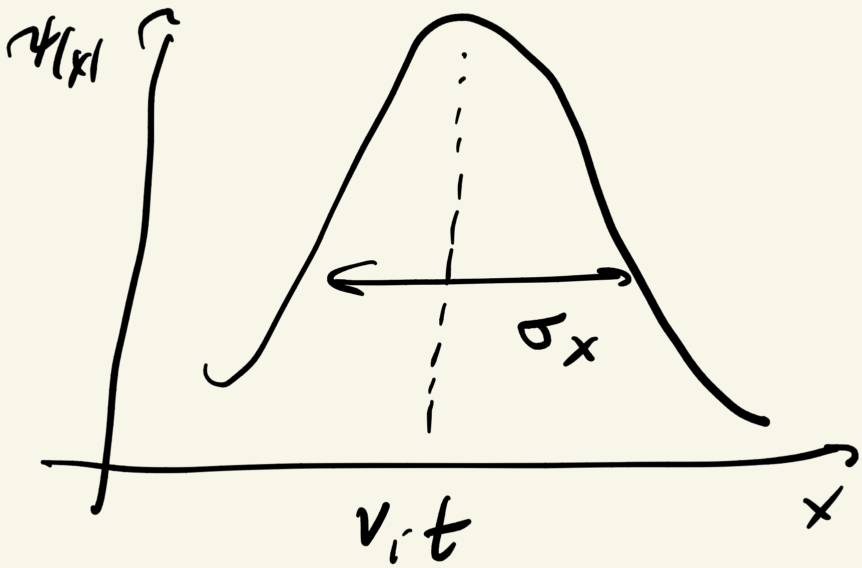
$$\Downarrow$$

$$\psi_i(x) \propto e^{i\phi_0^i} \underbrace{\int_{-\infty}^{+\infty} dv\, e^{-v^2/\sigma_p^2}}_{\#}\, e^{-\frac{y^2\sigma_p^2}{4}}$$

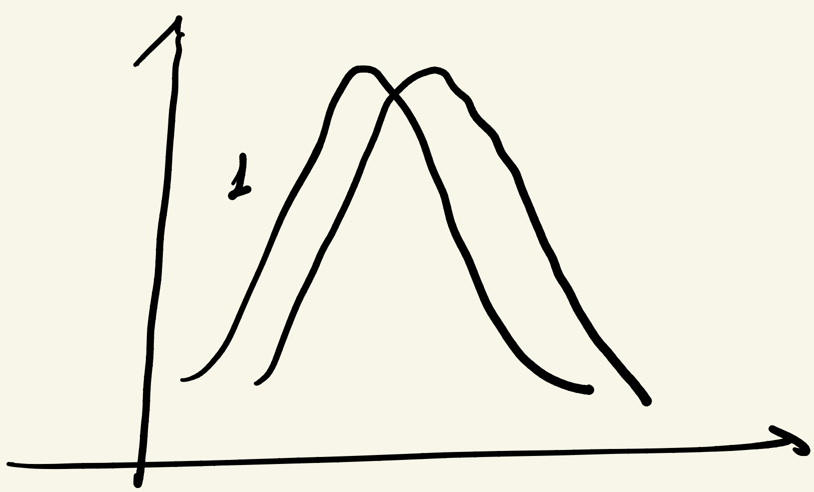
dim.!

$$\Downarrow$$

$$\boxed{\psi_i(x) \propto e^{i\phi_0^{(i)}}\, e^{-\frac{(x-v_it)^2}{4\sigma_x^2}} \qquad \sigma_x \equiv 1/\sigma_p}$$



<u>coherence</u>



time

coherence loss ⟺

well defined states

↓ go back to oscillation

$$P(\nu_\mu \to \nu_\tau) = \sin^2 2\theta \, \sin^2 \frac{\Delta\phi}{2}$$

$$\phi_i = E_i t - p x$$

↓

$$\Delta\phi = \Delta E\, t - \Delta p\, x$$

(a) $$\Delta E = \frac{\partial E}{\partial p}\Delta p + \frac{\partial E}{\partial m^2}\Delta m^2$$

$$\Downarrow$$

$$\Delta\phi = \left(\frac{p}{E}t - x\right)\Delta p + \frac{\Delta m^2}{2E}t$$

$$= (vt - x)\,\Delta p + \frac{\Delta m^2}{2E}t$$

$$(vt - x) \ll \sigma_x$$

$$\Delta\phi^{(a)} = \underbrace{\frac{\Delta m^2}{2E}t}_{O(1)} + \underbrace{\left(\ll \Delta p\,\sigma_x\right)}_{\Delta p/\sigma_p}$$

$$\simeq \frac{\Delta m^2}{2E}L \qquad \longrightarrow 0, \ (\Delta p/\sigma_p < 1)$$

leading term

(b) $$\Delta \phi = \Delta E t - \Delta p x$$

$$p = \sqrt{E^2 - m^2}$$

$$\Delta p = \frac{\partial p}{\partial E} \Delta E + \frac{\partial p}{\partial m^2} \Delta m^2$$

$$= \frac{E}{p} \Delta E + \frac{1}{2p} \Delta m^2$$

$$\Downarrow$$

$$\Delta \phi^{(b)} = \left(t - \frac{E}{p} x\right) \Delta E - \frac{1}{2p} \Delta m^2 x$$

$$= \underbrace{\left(t - \frac{x}{v}\right)}_{\text{small (as before)}} \Delta E - \frac{1}{2p} \Delta m^2 x$$

$$\Rightarrow \quad \Delta\phi^{(b)} = -\frac{1}{2E}\Delta m^2 L$$

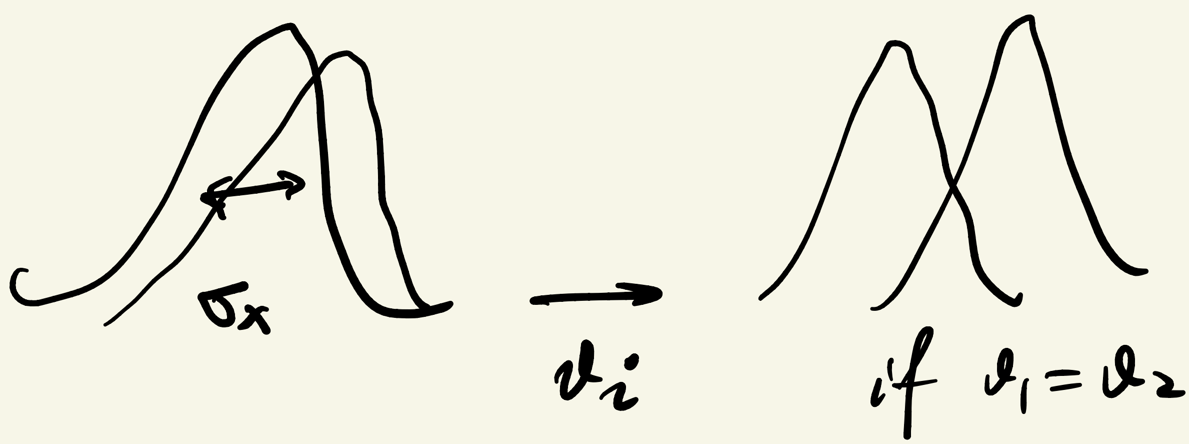
$$= -\Delta\phi^{(a)}$$

$$\Downarrow$$

$$\boxed{\sin^2\Delta\phi^{(a)} = \sin^2\Delta\phi^{(b)}}$$

but

do they remain coherent?



$$\boxed{t_{coh} = \text{time when you lose coherence}}$$

but: $v_1 \neq v_2$

$$\Rightarrow \underbrace{t_{coh}\, \Delta v \simeq \sigma_x}_{\text{coherence loss}}$$

$$\boxed{l_{coh} \simeq t_{coh}} \quad (v_i \simeq 1)$$

oscillation!

$$\boxed{l_{coh} \gg L}$$

$$E = \sqrt{p^2 + m^2} \simeq p + \frac{m^2}{2p} + \cdots$$

guess: $\Delta v \simeq \dfrac{\Delta m^2}{E^2}$

$$E \simeq E v + \frac{m^2}{2p}$$

$$E(1-v) \simeq \frac{m^2}{2p}$$

$$E \Delta v \simeq \frac{\Delta m^2}{2E}$$

$$\boxed{\Delta v \simeq \frac{\Delta m^2}{E^2}}$$

$$\Downarrow$$

$$\boxed{l_{coh} \hat{=} \frac{E^2}{\Delta m^2} \sigma_x}$$

$$\Downarrow$$

$$\boxed{l_{coh} \simeq \frac{E^2}{\Delta m^2} \frac{1}{\sigma_E}}$$

- **ATM** $\quad \Delta m^2_A \simeq 10^{-3}\, eV^2$

$$E \simeq m_\pi \simeq 10^{-1}\, GeV = 10^8\, eV$$

$$\sigma_E \simeq \Gamma_\pi \simeq 10^{-8}\, eV \simeq 10^{-17}\, GeV$$

$$\Downarrow$$

$$l_{coh} \simeq \frac{10^{16}}{10^{-3}}\, 10^{17}\, GeV^{-1}$$

$$GeV^{-1} \simeq 10^{-14}\, cm$$

$$\Downarrow$$

$$\boxed{l_{coh} \simeq 10^{22}\, cm} \quad / \quad L \simeq 10^3\, km$$

$\simeq 10^8 \mathrm{cm}$)

- ATM: $\sigma_{m^2} \simeq eV^2$

$e-\mu$: $\Delta m^2 \simeq m_\mu^2 \simeq 10^{16} eV^2$

$\boxed{\text{NO oscillations}}$

- <u>W decay</u> ?

$$W^- \longrightarrow l + \bar{\nu}_e \qquad (m_e = m_{\nu_e} = 0)$$

(1) $\sigma_{m^2} \simeq E\,\sigma_E \simeq M_W \Gamma_W$

$$\Gamma_W \simeq \alpha M_W \simeq GeV$$

$$\sigma_W^2 \simeq 10^2\,\mathrm{GeV}^2 \gg \Delta m^2_{e\mu} \simeq 10^{-2}\,\mathrm{GeV}^2$$

$$\Downarrow$$

$$\boxed{e-\mu \text{ oscillations ???!!!}}$$

(ii) $$l_{coh} \simeq \frac{E^2}{\Delta m^2}\frac{1}{\Gamma_W} \simeq \frac{M_W^2}{m_\mu^2}\frac{1}{\Gamma_W}$$

$$\Gamma_W = \alpha M_W$$

$$\Rightarrow \quad l_{coh} \simeq \frac{1}{\alpha}\frac{M_W}{m_\mu^2} \simeq 100\,\frac{100}{10^{-2}}\,\mathrm{GeV}^{-1}$$

$$\simeq 10^{+6}\,10^{-14}\,\mathrm{cm}$$

$$\Rightarrow \boxed{l_{coh} \simeq 10^{-8}\,\mathrm{cm}} \quad \text{NO}$$