Neutrino Physics Course

Lecture XXII

15/7/2022

LMU

Summer 2022

Neutrino Oscillations

$$M_\nu = - M_D^T \frac{1}{M_N} M_D$$

$\Downarrow$ LRSM

$$M_D = i\, M_N \sqrt{1/M_N \, M_\nu} \qquad (M_D \in \mathbb{R})$$

$$y_D = \frac{M_D}{v}$$

input(s)

$$M_\nu = V_L^{*} m_\nu V_L^{\dagger} \qquad (LH)$$

$$M_N = V_R \, m_N \, V_R^T \qquad (RH)$$

$$N_L \propto V_R^*$$

mixings

$$\mathcal{L}_{WK} = \frac{g}{\sqrt{2}} \left(\bar{\nu}_L \gamma_\mu V_L^+ e_L W_{\mu L}^+ + \right.$$

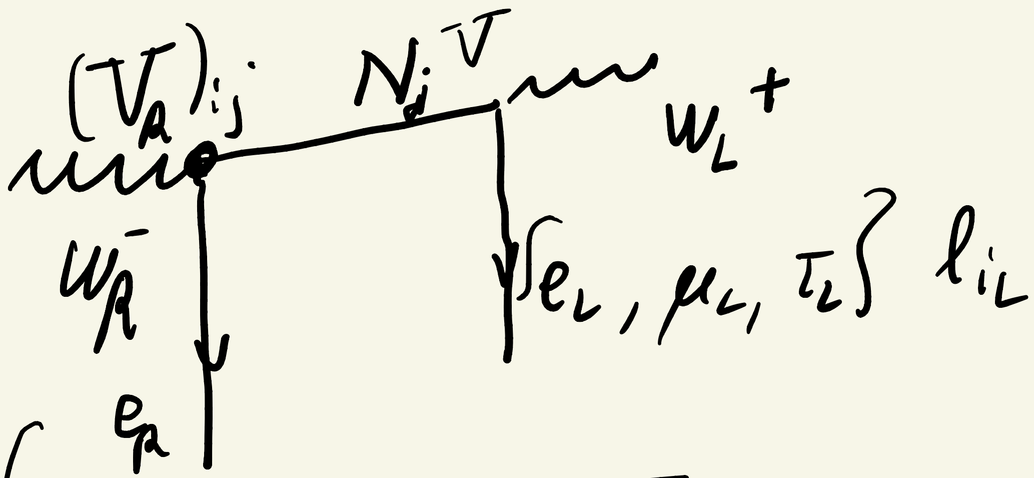
$$\left. \bar{N}_R \gamma^\mu V_R^+ e_R W_{\mu R}^+ \right)$$

mixings

N : $\longrightarrow$ produce it

$$N \to e_R + q + q'$$

$$\left.\begin{array}{l} \to e_L + W_L^+ \\ (e_L)^c + W_L^- \end{array}\right\} \text{Majorana}$$



$$l_{iR} \begin{cases} e_R \\ \mu_R \\ \tau_R \end{cases}$$

$$m_N = E_e + E_{W} + \text{(rest)}$$

$\Longleftrightarrow$ analogy: any decay @ colliders

$\Downarrow$

$V_R \leftarrow$ determine!

$$\delta_R^{--} \longrightarrow l_R^i \, l_R^j$$

$$(M_N)^*_{ij} \quad \text{(cross checks)}$$

$V_L = ?$ $\qquad M_\nu = V_L^* m_\nu V_L^+ \quad (M_\nu = M_\nu^T)$

$\boxed{VV^+ = 1}$ $\qquad$ determine ?

$$\boxed{\mathcal{L}_{WL} = \frac{g}{\sqrt{2}} \bar{\nu}_L \gamma_\mu V_L^+ e_L W_L^{\mu +} + h.c. \qquad (1)}$$

$\bar{\nu}_L \to$ physical states

- $H = H^+$ hermitian (masseigen--)

$$H = U h U^+ \Leftrightarrow H^+ = U h U^+$$

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2 generations

$V_L^\dagger = 2\times 2$ unit.

$$V_L^\dagger = K_M\, O$$

$$O = \begin{pmatrix} c & s \\ -s & c \end{pmatrix}, \quad K_M = \begin{pmatrix} 1 & 0 \\ 0 & e^{i\varphi} \end{pmatrix}$$

$$\mathcal{L}_{wk} = \frac{g}{\sqrt{2}} \underbrace{(\bar{\nu}_e\ \bar{\nu}_\mu)}_{\text{weak eigenstates}} \gamma^\mu \begin{pmatrix} e \\ \mu \end{pmatrix} W^+_{\mu L} + h.c. \qquad (2)$$

$$\underbrace{\begin{pmatrix} \nu_e \\ \nu_\mu \end{pmatrix}_L = V_L^\dagger \begin{pmatrix} \nu_1 \\ \nu_2 \end{pmatrix}_L}_{\text{mixing:}}$$

SUN you produce ν_e !

$$U_L^{+} = \begin{pmatrix} c & s \\ -s e^{i\varphi} & c e^{i\varphi} \end{pmatrix}$$

$$\Rightarrow \boxed{\begin{aligned} \nu_e &= \cos\theta\, \nu_1 + \sin\theta\, \nu_2 \\ \nu_\mu &= e^{i\varphi}\left(-\sin\theta\, \nu_1 + \cos\theta\, \nu_2\right) \end{aligned}}$$

$t=0$: $|\nu_e(0)\rangle = c\,|\nu_1\rangle + s\,|\nu_2\rangle$

$|\nu_e(t)\rangle = c\,|\nu_1(t)\rangle + s\,|\nu_2(t)\rangle$

$$|\nu_1(t)\rangle = e^{iE_1 t}|\nu_1\rangle$$

$$|\nu_2(t)\rangle = e^{iE_2 t}|\nu_2\rangle$$

$$E_i = \sqrt{p_i^2 + m_i^2} \simeq p_i + \frac{1}{2}\frac{m_i^2}{p_i} + \cdots$$

$$p_1 \simeq p_2$$

$$\boxed{E_i \simeq p + \frac{1}{2}\frac{m_i^2}{p}}$$

$$P(\nu_e \to \nu_\mu) = |\langle \nu_\mu | \nu_e(t)\rangle|^2$$

$$= \left| e^{-i\varphi} \langle (-s\nu_1 + c\nu_2) | | c e^{iE_1 t} \nu_1 + \right.$$

$$\left. + s e^{iE_2 t} \nu_2 \rangle \right|^2$$

$$\boxed{\langle \nu_i | \nu_j \rangle = \delta_{ij}}$$

$$\hookrightarrow = \left| sc \left(-e^{+iE_1 t} + e^{iE_2 t} \right) \right|^2$$

$$= s^2 c^2 \left| e^{iE_1 t} \left(1 - e^{i\Delta E t} \right) \right|^2$$

$$\Delta E \equiv E_2 - E_1$$

$$= s c^2 \left(1 - e^{i\Delta E t} \right) \left(1 - e^{-i\Delta E t} \right)$$

$$= s^2 c^2 \left(2 - e^{i\Delta E t} - e^{-i\Delta E t} \right)$$

$$= 2 c^2 s^2 \left(1 - \cos \Delta E t \right)$$

$$= 4 c^2 s^2 \sin^2 \frac{\Delta E t}{2}$$

$$\Downarrow$$

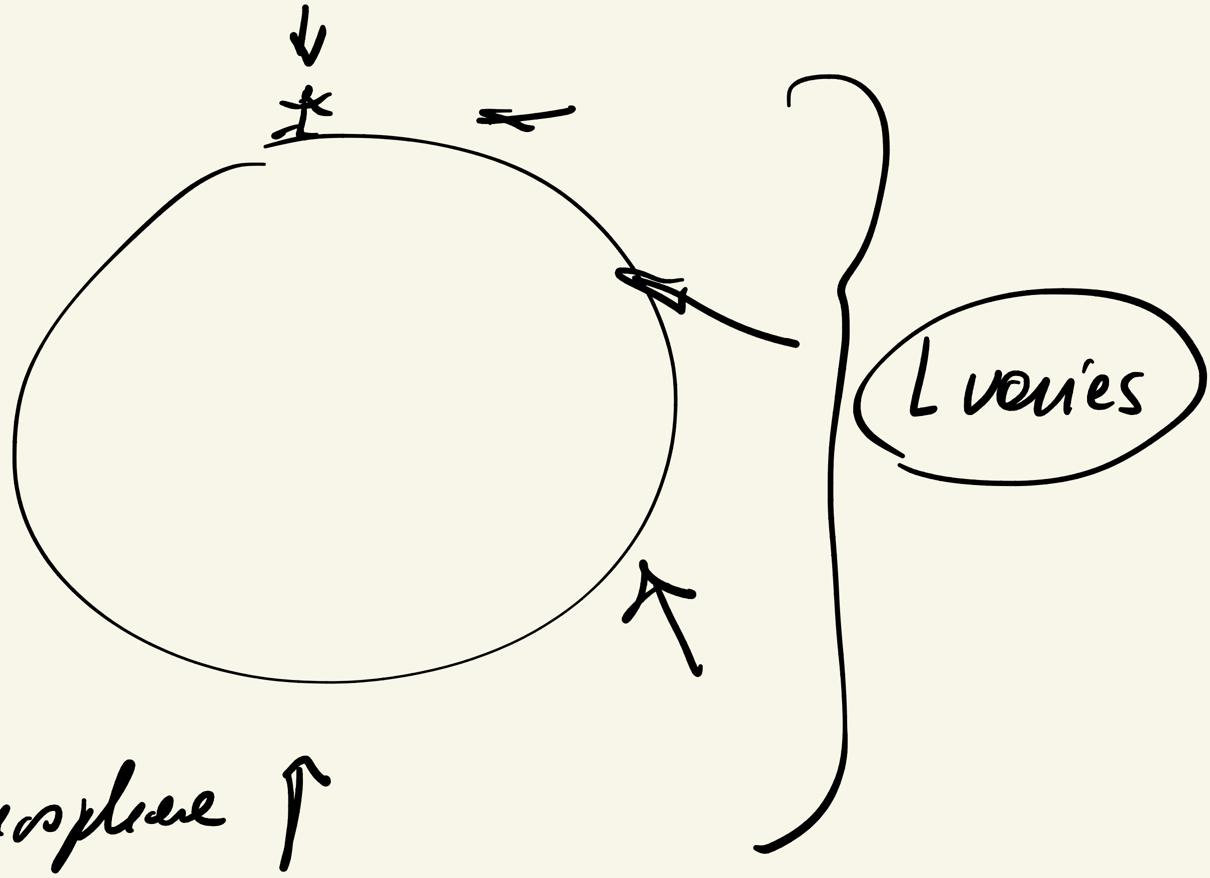
$$P(\nu_e \to \nu_\mu) = \sin^2 2\theta \, \sin^2 \frac{\Delta m^2 L}{4E}$$

$(p \simeq E, \; L \simeq t)$

+ $\Delta m^2 \leq \sigma_{m^2}$ = uncertainty in m^2 diff.

Gribov, Pontecorvo '68

↓ Atmospheric neutrinos



atmosphere ↑

↓

$$\pi^- \longrightarrow e + \bar{\nu}_e$$

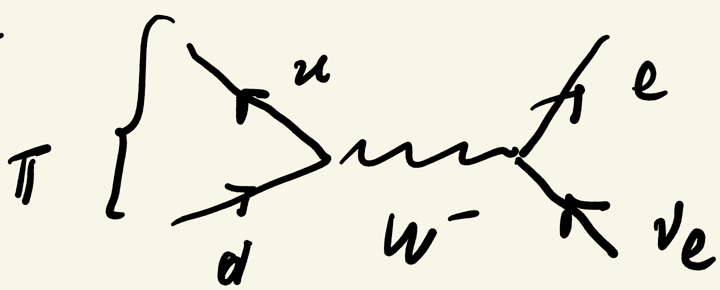
$$\mu + \bar{\nu}_\mu$$

$m_\pi \simeq 140$ MeV

$m_\mu \simeq 100$ MeV

$m_e \simeq$ MeV

$$\pi^- \longrightarrow \bar{u} d$$



$$\mathcal{H}_F \simeq \frac{G_F}{\sqrt{2}} \bar{u}_L \gamma^\mu d_L \; \bar{e}_L \gamma^\mu \nu_L$$

↳

$$\langle 0 | \mathcal{H}_{eff} | \pi^- \rangle$$

$$\propto G_F \langle 0 | \bar{u}_L \gamma^\mu d_L | \pi^- \rangle \; \bar{e}_L \gamma^\mu \nu_L$$

$$\propto G_F \langle 0 | \bar{u} \gamma^\mu (1+\gamma_5) d | \pi^- \rangle \text{ -- '' --}$$

$$\propto G_F \left[\cancel{\langle 0 | (\bar{u} \gamma^\mu d) | \pi^- \rangle} + \langle 0 | \bar{u} \gamma^\mu \gamma_5 d | \pi^- \rangle \right] \cdot \text{''}$$

↳ pseudoscalar

$$\langle 0 | \bar{u} \gamma^\mu \gamma_5 | d \rangle = f_\pi \, p^\mu$$

pion decay constant

$$\boxed{f_\pi \simeq 100 \text{ MeV}}$$

$$\mathcal{M} \propto f_\pi \, p^\mu \, \bar{l}_L \gamma^\mu \nu_l \, G_F$$

$$\parallel$$

$$\bar{l} \gamma^\mu \frac{1+\gamma_5}{2} \nu_e$$

$$p_\mu = p_e - p_{\nu_e}$$

(a) $p^\mu \bar{\ell} \gamma_\mu \nu_e = ?$

(b) $p^\mu \bar{\ell} \gamma_\mu \gamma_5 \nu_e = ?$

$$\left(\begin{aligned} p^\mu \bar{f}_1 \gamma^\mu f_2 &= (m_2 - m_1) \bar{f}_1 f_2 \\ p^\mu \bar{f}_1 \gamma^\mu \gamma_5 f_2 &= (m_2 + m_1) \bar{f}_1 \gamma_5 f_2 \end{aligned} \right)$$

$$\Downarrow$$

$m_1 = m_\nu, \quad m_2 = m_\ell$

$\Downarrow$

0

$$\Downarrow$$

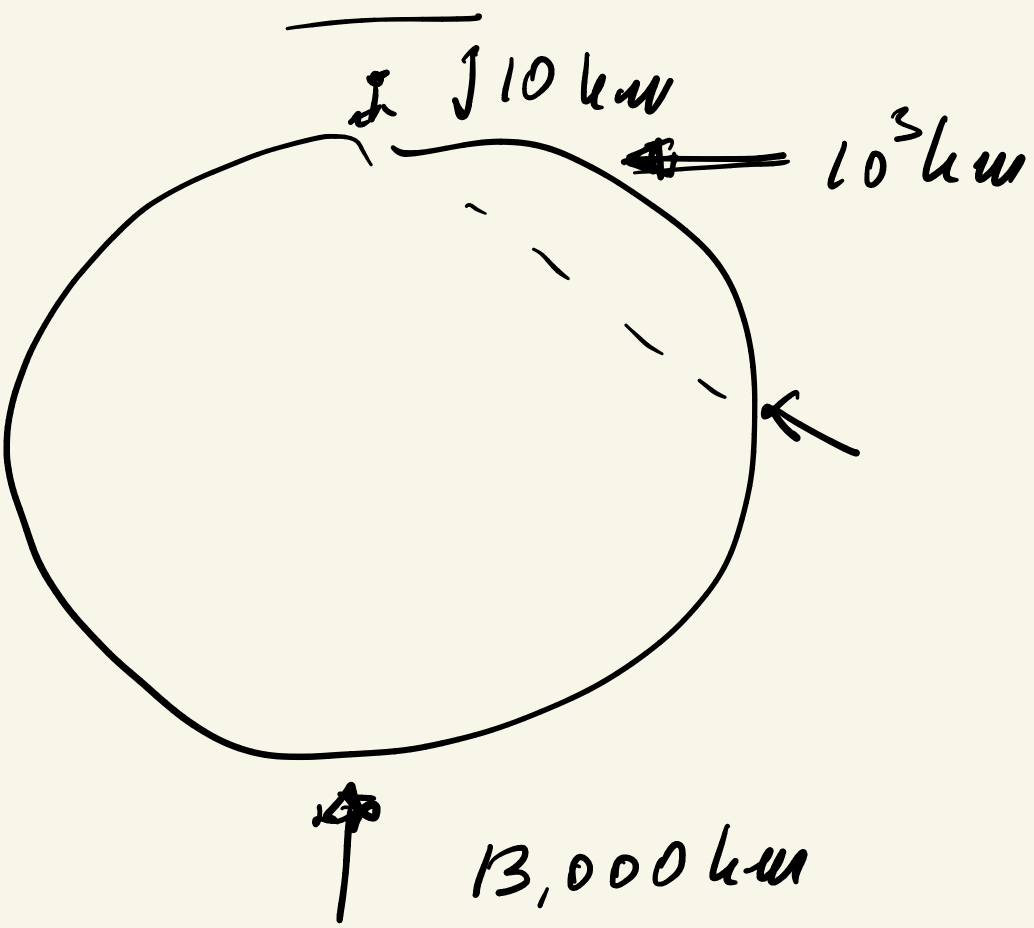
$$\mathcal{M} \propto G_F\, m_\ell\, \bar{\ell}(1+\gamma_5)\nu\, f_\pi$$

$$\Gamma(\pi^- \to \ell\, \bar{\nu}_\ell) \simeq G_F^2 f_\pi^2 m_\ell^2 m_\pi / 8\pi$$

$$\Rightarrow \frac{\Gamma(\pi^- \to \mu\bar{\nu}_\mu)}{\Gamma(\pi^- \to e\,\bar{\nu}_e)} \simeq \left(\frac{m_\mu}{m_e}\right)^2 \simeq 10^4$$

$$\Rightarrow \Gamma(\pi^- \to \mu\bar{\nu}_\mu) \simeq 10^{-10}\, 10^{-2}\, 10^{-2}\, 10^{-1} / 100 \ \text{GeV}$$

$$\simeq 10^{-17} \ \text{GeV}$$



$$\left\langle \sin^2 \frac{\Delta m^2 L}{4E} \right\rangle \simeq 1/2$$

$$P(\nu_\mu \to \nu_\tau) \simeq \frac{1}{2} \sin^2 2\theta \simeq \frac{1}{2} \text{ (exp)}$$

no excess of ν_e

⇓

$$\theta_A \simeq 45$$

$$\sin^2 \frac{\Delta m^2 L}{4E} \simeq O(1)$$

$$\shortparallel$$

$$\pi/2 \simeq O(1)$$

$\to 10^3\,km$?

$$\frac{\Delta m^2 L_{osc}}{E} \simeq 1$$

↑ GeV

$$\Delta m^2 \simeq \frac{GeV}{10^3\,km} \simeq \frac{GeV}{10^8\,cm}$$

$$GeV^{-1} \simeq 10^{-14}\,cm$$

proton size

$$\Delta m_A^2 \simeq 10^{-22}\,GeV^2 \simeq 10^{-4}\,eV^2$$

dust settles $\longrightarrow 10^{-3}$

$$\boxed{\Delta m_A^2 \simeq 10^{-3}\,eV^2 \simeq 10^{-21}\,GeV^2}$$

$\nu_\mu \rightarrow \nu_\tau$ (?) $\longleftarrow$ PROVEN later

$\neq$

ν_e

- SUN: $\nu_e \rightarrow \nu_\mu \Rightarrow \Delta m_\odot^2 \simeq 10^{-5}\,eV^2$

$\theta_\odot \simeq 30^\circ$

$$n \rightarrow p + e + \bar{\nu}_e$$

$$\left\{ \begin{array}{l} \bar{\nu}_e + p \rightarrow n + \bar{e} \\ \\ \bar{\nu}_\mu + p \rightarrow n + \bar{\mu} \end{array} \right\}$$

Atmosphere

$$\pi^- \rightarrow \mu^- + \bar{\nu}_\mu$$

$$\hookrightarrow e + \nu_\mu + \bar{\nu}_e$$

Bahcall

(computes # ν_e
coming out of sun)

Next time:

$\Delta m^2 \ll \sigma m^2$

↑
measurement
uncertainty

$$\mathcal{H}_{eff} \simeq G_F \, J_\mu^{w} \bar{J}^{\mu w}$$

$$\simeq G_F \, T_{\mu\nu}^{w} \bar{T}^{\mu\nu}$$

$$\simeq G_F \, S \bar{S}$$

all possible

$$J_{\mu L}^{w} \bar{J}^{\mu L}$$