Neutrino Physics Course

Lecture XX

8/7/2022

LMU

Summer 2022

Untangling seesaw

LRSM

1. $\exists\, \nu_R \Rightarrow m_\nu \neq 0$

2. ν_R = Majorana $= N_R$

$$\left(N_L = C \bar{\nu}_R^{\,T} \right)$$

$$M_N \gtrsim M_W \qquad \left(M_N \gg M_D \right)$$

3. seesaw

$$M_\nu = - M_D^T \frac{1}{M_N} M_D$$

low E hadron collide

$\Downarrow$ untangling seesaw

$$M_D = i\, M_N \sqrt{\frac{1}{M_N}\, M_\nu}$$

$$\Downarrow$$

$$\boxed{\theta_{\nu N} = i \sqrt{\frac{1}{M_N}\, M_\nu}}$$

$$\Downarrow$$

$$\Gamma(N \to e W^+) = \Gamma(N \to \bar{e} W^-)$$

$$\propto \theta_{\nu N}^2\, m_N$$

rest frame:

$$m_N = E_e + E_W$$

$$\vec{p}_N = 0 = \vec{p}_e + \vec{p}_W$$

$$m_N = \sqrt{p^2 + m_e^2} + \sqrt{p^2 + M_W^2}$$

$$m_e = 0$$

$$\Rightarrow (m_N - p)^2 = p^2 + M_W^2$$

$$\Downarrow$$

$$m_N^2 - M_W^2 = 2 p m_N$$

$$\left(p = \frac{m_N^2 - M_W^2}{2 m_N} \right)$$

$$\mathcal{M}_{\nu_L} = V_L^* m_\nu V_L^\dagger \quad m_\nu > 0$$

$$\mathcal{M}_{\nu_R} = V_R^* m_N V_R^\dagger \quad m_N > 0$$

$$\Updownarrow \qquad \boxed{M_N = V_R m_N V_R^T}$$

$$\nu_L^T \mathcal{M}_\nu \nu_L = \nu_L'^T m_\nu \nu_L'$$

physical states

(eigenstates)

$$\Downarrow$$

$$\nu_L^T V_L^* m_\nu V_L^\dagger \nu_L = \nu_L'^T m_\nu \nu_L'$$

$$\Downarrow \qquad \nu_L \equiv \begin{pmatrix} \nu_1 \\ \nu_2 \\ \nu_3 \end{pmatrix}_L$$

$$\nu_L' = V_L^\dagger \nu_L$$

$$\Downarrow$$

$$\nu_L = V_L \nu_L'$$

$$\Downarrow$$

$$\frac{g}{\sqrt{2}} \bar{\nu}_L \gamma^\mu e_L W_\mu^+ = \frac{g}{\sqrt{2}} \bar{\nu}_L V_L^+ \gamma^\mu e_L W_\mu^+$$

in the basis of
diagonal lepton states
(charged)

$$\boxed{V_{leptonic} = V_{PMNS} = V_L^+}$$

$$\updownarrow$$

$$V_{quark} = V_{CKM} = U_{Lu}^+ U_{Ld} \to U_{Lu}^+$$

$$(V_{ud} = 1 \leftrightarrow U_{\nu e} = 1)$$

$$\boxed{V_{lept}\,,\ V_{quark} = \text{unitary}}$$

- $$\boxed{V_{2\times 2} = K1\ O\ K2} \qquad (n_g = 2)$$

$$O^T O = 1 \Rightarrow O = \begin{pmatrix} c & s \\ -s & c \end{pmatrix} \qquad \begin{array}{l} c \equiv \cos\theta \\ s \equiv \sin\theta \end{array}$$

$$K1 = \begin{pmatrix} 1 & 0 \\ 0 & e^{i\varphi} \end{pmatrix}, \quad K2 = \begin{pmatrix} e^{i\alpha} & 0 \\ 0 & e^{i\beta} \end{pmatrix}$$

$n\times n$: $\quad V^{+} V = 1 \qquad$ (n^2-equations)

$$\#\ \text{of}\ V = 2n^2 - n^2 = n^2$$

(real)

2×2 ! $\longrightarrow 4 = 1$ angle $+ 3$ phases

• $V_{3\times 3} = K2 \boxed{O_{23}\, K_D\, O_{13}\, K_D^{+}\, O_{12}} K3$

$$K2 = \begin{pmatrix} 1 & & 0 \\ & e^{i\varphi_1} & \\ 0 & & e^{i\varphi_2} \end{pmatrix} \qquad K3 = \begin{pmatrix} e^{i\alpha} & & 0 \\ & e^{i\beta} & \\ 0 & & e^{i\gamma} \end{pmatrix}$$

$$K_D = \begin{pmatrix} 1 & & 0 \\ & 1 & \\ 0 & & e^{i\delta} \end{pmatrix}$$

Keung, Chau

$$O_{12} = \begin{pmatrix} c_{12} & s_{12} & 0 \\ -s_{12} & c_{12} & 0 \\ 0 & 0 & 1 \end{pmatrix}$$

$$O_{13} = \begin{pmatrix} c_{13} & 0 & s_{13} \\ 0 & 1 & 0 \\ -s_{13} & 0 & c_{13} \end{pmatrix}$$

$$\Downarrow$$

$$K_\delta O_{13} K_\delta^\dagger = \begin{pmatrix} c_{13} & 0 & s_{13} e^{i\delta} \\ 0 & 1 & 0 \\ -s_{13} e^{-i\delta} & 0 & c_{13} \end{pmatrix}$$

As many phases as possible outside

$$\Updownarrow$$

rotate phase of Dirac

particles

$$\boxed{\begin{aligned} m_D\, \bar{f}_L f_R &\to m_D\, \bar{f}_L f_R \\ f_L, f_R &\to e^{i\alpha} f_L, f_R \end{aligned}}$$

· quarks = Dirac

$V_{CKM} \to$ 9 elements =

= 3 angles + 6 phases

$$\Downarrow$$

6 phases = outside

n generations $\Rightarrow$ # angles ?

$$O_{n\times n}\, O_{n\times n}^T = \mathbb{1}$$

$$\hookrightarrow \frac{1}{2} n(n-1) = \text{angles}$$

$$\underset{\uparrow}{n^2} = \underbrace{\frac{1}{2} n(n-1)}_{\text{angles}} + \underbrace{\frac{1}{2} n(n+1)}_{\text{phases}}$$

unitary

- $n = 2 \longrightarrow 3$
- $n = 3 \longrightarrow 6$

# of physical phases

$$\frac{1}{2} n(n+1) - 2n \quad ???$$

$n = 3 \longrightarrow 0$

$$u_L \to e^{i\alpha_u} u_L \to e^{i\alpha_u} u_L$$

$$c_L \to e^{i\alpha_c} c_L \to e^{i\alpha_u} e^{i(\alpha_c - \alpha_u)} c_L$$

$$t_L \to e^{i\alpha_t} t_L \to e^{i\alpha_u} e^{i(\alpha_t - \alpha_u)} t_L$$

- - - - -

$$d_L \to e^{i\alpha_d} d_L \to e^{i\alpha_u} e^{i(\alpha_d - \alpha_u)} d_L$$

$$s_L \to e^{i\alpha_s} s_L \to e^{i\alpha_u} \; - - - \; s_L$$

$$b_L \to e^{i\alpha_b} b_L \to e^{i\alpha_u} \; - - - \; b_L$$

$$\Downarrow$$

$$\left[(\bar{u}\,\bar{c}\,\bar{t})_L \begin{pmatrix} d \\ s \\ b \end{pmatrix}_L \longrightarrow \right]$$

α_n cancels

$$\Downarrow$$

$$\frac{1}{2} n(n+1) - (2n-1) = \# \text{ phases}$$

$$= \frac{n^2 - n - 4n + 2}{2} = \frac{n^2 - 3n + 2}{2} = \frac{(n-1)(n-2)}{2}$$

- $n = 2 \longrightarrow 0$
- $n = 3 \longrightarrow 1 \ (K_D)$

$$\Downarrow$$

$$\boxed{V_{CKM} = O_{23}\, K_D\, O_{13}\, K_D^{+}\, O_{12}}$$

KM phase = physical

$\delta \simeq 45^{0}$

$\theta_{12} \simeq 13^{0}$, $\theta_{23} \simeq 10^{-2}$, $\theta_{13} \simeq 4\times 10^{-3}$

quark world

$\boxed{n=2}$

$$V_{2\times 2} = K1\; O\; K2$$

$$= \underbrace{K1\, K_\delta}_{K1'}\; O\; \underbrace{K_\delta^{+}\, K2}_{K2'}$$

rotate away

$$\boxed{V_{2\times 2} \longrightarrow O_c}$$

<u>Leptons</u>

- $\boxed{n=2}$

$$V_{lept} = K1 \; O \; K2 = V_L^+$$

$$\bar{\nu} \; V_{lept} \; \gamma^\mu e \; W_\mu^+ =$$

$$= \bar{\nu} \; K1 \; O \; K2 \, e \; W_\mu^+$$

$\nu =$ Majorana

$K2 \; e = e'$

$\Downarrow$

no K2 phases

$\Downarrow$ but K1 stays

$$\boxed{V_{lept} = \begin{pmatrix} 1 & 0 \\ 0 & e^{i\varphi} \end{pmatrix} O_{lept}}$$

Majorana

$\boxed{\bullet \quad n = 3}$

$i = (12, 23, 13)$

$$\boxed{V_{lept} = \underbrace{K_2}_{2 \text{ Majorana}} \, V(\theta_i, \delta)}$$

Probing angles and phases

$$M_N = V_R \, m_N \, V_R^T$$

ν_R is defined:

$$\boxed{\overline{N}_R \gamma^\mu V_R^\dagger e_R W_{\mu R}^+}$$

- $n_g = 2 \Rightarrow$ $\boxed{V_{lept}(R) = V_R^\dagger = K\,1\,O}$

$OO^T = 1$

looking for Majorana

$$\mathcal{L}_Y(\Delta) = l_R^T C\, Y_\Delta\, \Delta_R\, l_R + h.c.$$

$$l_R \equiv \begin{pmatrix} \nu \\ e \end{pmatrix}_R , \quad \Delta_R \Rightarrow \begin{pmatrix} 0 & \delta_R^{++} \\ v_R & 0 \end{pmatrix}$$

$$\mathcal{L}_Y(\Delta) \Rightarrow \nu_R^T C\, y_\Delta\, \nu_R\, v_R \; +$$

$$+\, e_R^T C\, y_\Delta\, e_R\, \delta_R^{++} + h.c.$$

$$\Downarrow$$

$$\boxed{M_{\nu_R} = M_N^* = \quad y_\Delta v_R}$$

$$\Rightarrow \mathcal{L}_{int}(++) = e_R^T C\, \frac{M_N^*}{v_R}\, e_R\, \delta_R^{++} + h.c.$$

$$(C^T = C)$$

$$C = i\sigma_2 \gamma_0 = \begin{pmatrix} i\sigma_2 & 0 \\ 0 & -i\sigma_2 \end{pmatrix}$$

$$\Downarrow$$

$$\Gamma\left(\delta_R^{--} \to e_i e_j\right)$$

stands for e, μ, τ

$$\left(M_N^* = V_R^* m_N V_R^+\right)_{ij}$$

$$\boxed{V_R^+ = k\, O} \qquad k = \begin{pmatrix} 1 & 0 \\ 0 & e^{i\varphi} \end{pmatrix}$$

$$\Downarrow$$

$$\boxed{M_N^* = O_R^T k\, m_N k\, O_R}$$

$$\Downarrow$$

$$O_R = \begin{pmatrix} c & s \\ -s & c \end{pmatrix}_R$$

$$M_N^* = \begin{pmatrix} c & -s \\ s & c \end{pmatrix}_R \begin{pmatrix} m_1 & 0 \\ 0 & m_2 e^{2i\varphi} \end{pmatrix} \begin{pmatrix} c & s \\ -s & c \end{pmatrix}_R$$

$$= \begin{pmatrix} c^2 m_1 + s^2 m_2 e^{2i\varphi} & cs\,(m_2 e^{2i\varphi} - m_1) \\ cs\,(m_2 e^{2i\varphi} - m_1) & s^2 m_1 + c^2 m_2 e^{2i\varphi} \end{pmatrix}_R$$

<u>Process</u>

$$\Gamma(\delta^{--} \to e\mu) \propto c_R^2 s_R^2 \left| m_2 e^{2i\varphi} - m_1 \right|^2$$

<u>deg. case</u>: $m_2 = m_1 = m$

$$\Gamma(\delta^{--} \to e\mu) \propto m^2 \left| e^{2i\varphi} - 1 \right|^2$$

$$\propto \ldots (1 - \cos 2\varphi) \propto \sin^2 \varphi$$

• quarks : $V_q(L) = V_q(R)$

• leptons : $V_\ell(L) \neq V_\ell(R)$

$$\underline{M}_\nu \propto M_D^T \frac{1}{M_N} M_D \qquad \underline{M} \propto Y_\Delta$$

ψ_M

$$\boxed{\psi_M = \psi_L + C\overline{\psi}_L^T}$$

$$\mathcal{L}(\psi_M) = i\,\overline{\psi}_M \gamma^\mu \partial_\mu \psi_M - m_M \overline{\psi}_M \psi_M$$