

Neutrino Physics Course

Lecture XX

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8/17/2022

LMU

Summer 2022


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# Untangling see saw

## LRSM

1.  $\exists \nu_R \Rightarrow m_\nu \neq 0$
2.  $\nu_R = \text{Majorana} = N_R$   
 $(N_L = c \bar{\nu}_R^T)$   
 $M_N \gtrsim M_W \quad (M_N \gg M_D)$
3.  see saw

$$M_\nu = - M_D^T \frac{1}{M_N} M_D$$

  
low E

  
hadron collide

⇓ untagging section

$$\underline{M}_D = i \underline{M}_N \sqrt{\frac{1}{M_N}} M_D$$

⇓

$$\Theta_{\nu N} = i \sqrt{\frac{1}{M_N}} M_{\nu}$$

⇓

$$\Gamma(N \rightarrow e W^+) = \Gamma(N \rightarrow \bar{e} W^-)$$
$$\propto \Theta_{\nu N}^2 M_N$$

rest frame:

$$U_N = E_e + E_w$$

$$\vec{P}_N = 0 = \vec{p}_e + \vec{p}_w$$

$$U_N = \sqrt{p^2 + m_e^2} + \sqrt{p^2 + M_w^2}$$

$$m_e = 0$$

$$\Rightarrow (U_N - p)^2 = p^2 + M_w^2$$



$$U_N^2 - M_w^2 = 2pU_N$$

$$\left( p = \frac{U_N^2 - M_w^2}{2U_N} \right)$$

$$\left. \begin{aligned} M_{\nu_L} &= V_L^* M_{\nu} V_L^{\dagger} \\ M_{\nu_R} &= V_R^* M_N V_R^{\dagger} \end{aligned} \right\} \begin{aligned} m_{\nu} &> 0 \\ m_N &> 0 \end{aligned}$$

$$\Downarrow \quad \boxed{M_N = V_R M_N V_R^T}$$

$$v_L^T M_{\nu} v_L = v_L'^T M_{\nu} v_L'$$

↑  
physical states  
(eigenstates)

⇓

$$v_L^T V_L^* M_{\nu} V_L^{\dagger} v_L = v_L'^T M_{\nu} v_L'$$

⇓

$$v_L' = V_L^{\dagger} v_L$$

$$v_L \equiv \begin{pmatrix} \nu_1 \\ \nu_2 \\ \nu_3 \end{pmatrix}_L$$

$$\Downarrow$$

$$\nu_L = V_L \nu_L'$$

$$\Downarrow$$

$$\frac{g}{\sqrt{2}} \bar{\nu}_L \gamma^\mu e_L W_\mu^+ = \frac{g}{\sqrt{2}} \bar{\nu}_L \left( V_L^\dagger \right) \gamma^\mu e_L W_\mu^+$$

in the basis of

diagonal lepton states  
(charged)

$$V_{\text{leptonic}} = V_{\text{PMNS}} = V_L^\dagger$$

$$V_{\text{quark}} = V_{\text{CKM}} = U_{Ld}^\dagger U_{Ld} \rightarrow U_{Ld}^\dagger$$

$$(U_{L0} = 1 \Leftrightarrow U_{Le} = 1)$$

$$V_{\text{left}}, V_{\text{right}} = \text{unitary}$$

$$V_{2 \times 2} = k_1 \quad \parallel \quad k_2 \quad (u_g = 2)$$

$$O^T O = 1 \Rightarrow \begin{pmatrix} c & s \\ -s & c \end{pmatrix}$$

$$c \equiv \cos \theta$$

$$s \equiv \sin \theta$$

$$k_1 = \begin{pmatrix} 1 & 0 \\ 0 & e^{i\varphi} \end{pmatrix}, \quad k_2 = \begin{pmatrix} e^{i\alpha} & 0 \\ 0 & e^{i\beta} \end{pmatrix}$$

$$u \times u: \quad V^+ V = 1 \quad (u^2\text{-equations})$$

$$\# \text{ of } V = 2u^2 - u^2 = u^2$$

(real)

$2 \times 2$ :  $\rightarrow 4 = 1 \text{ angle} + 3 \text{ phases}$

•  $V_{3 \times 3} = K_2 \left( \begin{array}{ccccc} & & & & \\ & & & & \\ & & & & \\ O_{23} & K_0 & O_{13} & K_0^\dagger & O_{12} \\ & & & & \end{array} \right) K_3$

$\parallel$

$$\begin{pmatrix} 1 & & 0 \\ & e^{i\varphi_1} & \\ 0 & & e^{i\varphi_2} \end{pmatrix}$$

$$\begin{pmatrix} e^{i\alpha} & & 0 \\ & e^{i\beta} & \\ 0 & & e^{i\gamma} \end{pmatrix}$$

$$K_0 = \begin{pmatrix} -1 & & 0 \\ & 1 & \\ 0 & & e^{i\delta} \end{pmatrix}$$

$$O_{12} = \begin{pmatrix} c_{12} & s_{12} & 0 \\ -s_{12} & c_{12} & 0 \\ 0 & 0 & 1 \end{pmatrix}$$

$K_{\text{emg}},$   
 $\text{Chau}$



$$O_{13} = \begin{pmatrix} c_{13} & 0 & s_{13} \\ 0 & 1 & 0 \\ -s_{13} & 0 & c_{13} \end{pmatrix}$$

$$K_D O_{13} K_D^\dagger = \begin{pmatrix} c_{13} & 0 & s_{13} e^{i\delta} \\ 0 & 1 & 0 \\ -s_{13} e^{-i\delta} & 0 & c_{13} \end{pmatrix}$$

As many phases as possible  
outside




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rotate phase of Dirac

particles

$$M_D \bar{f}_L f_R \rightarrow M_D \bar{f}_L f_R$$

$$f_L, f_R \rightarrow e^{i\alpha} f_L, f_R$$

• quarks = Dirac

$$V_{CKM} \rightarrow 9 \text{ elements} =$$

$$= 3 \text{ angles} + 6 \text{ phases}$$



$$6 \text{ phases} = \text{outside}$$

$u$  generators  $\Rightarrow \neq$  angles?

$$O_{u \times u} O_{u \times u}^T = \mathbb{1}$$

$$\hookrightarrow \frac{1}{2} u(u-1) = \text{angles}$$

$$u^2 = \frac{1}{2} u(u-1) + \frac{1}{2} u(u+1)$$

$\uparrow$

angles

phases

unitary

•  $u=2 \rightarrow 3$

•  $u=3 \rightarrow 6$

# of physical phases

$$\frac{1}{2} u(u+1) - 2u \quad ? \quad ? \quad ?$$

$$u=3 \longrightarrow \textcircled{1}$$

$$u_L \rightarrow e^{idu} u_L \rightarrow e^{idu} u_L$$

$$c_L \rightarrow e^{idc} c_L \rightarrow e^{idu} e^{i(dc-du)} c_L$$

$$t_L \rightarrow e^{idt} t_L \rightarrow e^{idu} e^{i(dt-du)} t_L$$

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$$d_L \rightarrow e^{idd} d_L \rightarrow e^{idu} e^{i(dd-du)} d_L$$

$$s_L \rightarrow e^{ids} s_L \rightarrow e^{idu} \dots s_L$$

$$b_L \rightarrow e^{idb} b_L \rightarrow e^{idu} \dots b_L$$



$$\left[ (\bar{u} \bar{c} \bar{t})_L \right] \begin{pmatrix} d \\ s \\ b \end{pmatrix}_L \longrightarrow \left[ \quad \right]$$

$\alpha_n$  cancels



$$\frac{1}{2} u(u+1) - (2u-1) = \# \text{ pluses}$$

$\parallel$

$$\frac{u^2 - u - 4u + 2}{2} = \frac{u^2 - 5u + 2}{2} = \frac{(u-1)(u-2)}{2}$$

$\parallel$

$\cdot u=2 \rightarrow 0$

$\cdot u=3 \rightarrow 1 \quad (k_0)$



$$V_{CKM} = O_{23} K_D O_{13} K_D^\dagger O_{12}$$

KM phase = physical

$$\delta \approx 45^\circ$$

$$\theta_{12} \approx 13^\circ, \quad \theta_{23} \approx 10^{-2}, \quad \theta_{13} \approx 4 \times 10^{-3}$$

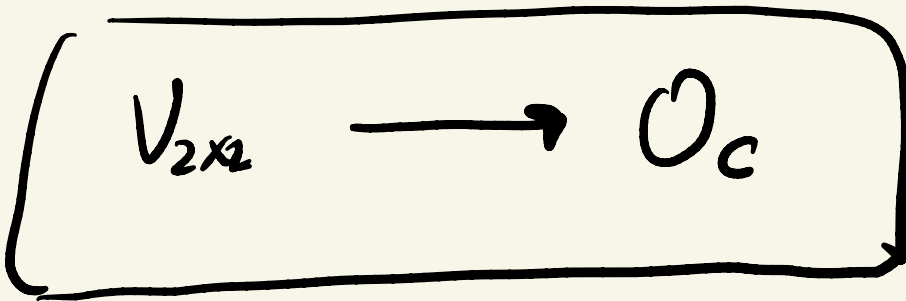
quark world

$$n=2$$

$$V_{2 \times 2} = K_1 O K_2$$

$$= \underbrace{K_1 K_8}_{K_1'} O \underbrace{K_8^\dagger K_2}_{K_2'}$$

rotate away



Leptons

•  $\boxed{n=2}$

$$V_{\text{lept}} = K_1 O K_2 = V_L^T$$

↑

$$\bar{\psi} V_{\text{lept}} \gamma^\mu e W_\mu^+ =$$

$$= \bar{\psi} K_1 O K_2 e W_\mu^+$$

$v = \text{Majorana}$

$$k_2 e = e'$$

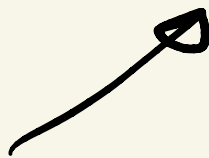


no  $k_2$  phases



but  $k_1$  stays

$$V_{\text{lept}} = \begin{pmatrix} 1 & 0 \\ 0 & e^{i\varphi} \end{pmatrix} O_{\text{lept}}$$



Majorana



$$\bullet \mu = \zeta$$

$$i = (12, 23, 13)$$



$$V_{\text{left}} = \underbrace{k 2}_{2 \text{ Majorana}} V(\theta_i, \delta)$$

Probing angles and  
phases

$$M_N = V_R M_N V_R^T$$

$V_R$  is defined :

$$\bar{N}_A \gamma^\mu V_R^\dagger e_R W_{\mu R}^\dagger$$

•  $n_f = 2 \Rightarrow$

$$V_{\text{left}}(R) = V_R^\dagger$$

$$= K \perp 0$$

$$\dagger 0 0^T = 1$$

looking for Majorana

$$\mathcal{L}_4(\Delta) = l_R^T C \gamma_\Delta \Delta_R l_R + \text{h.c.}$$

$$l_R \equiv \begin{pmatrix} \nu \\ e \end{pmatrix}_R, \quad \Delta_R \Rightarrow \begin{pmatrix} 0 & \delta_R^{++} \\ \nu_R & 0 \end{pmatrix}$$

$$\mathcal{L}_Y(\Delta) \Rightarrow \left\{ \begin{aligned} & \psi_R^T C \gamma_\Delta \psi_R \psi_R + \\ & + \ell_R^T C \gamma_\Delta \ell_R \delta_R^{++} + \text{h.c.} \end{aligned} \right.$$

⇓

$$M_{\psi_R} = M_N^* = \gamma_\Delta \psi_R$$

$$\Rightarrow \mathcal{L}_{\text{int}}(++) = \ell_R^T C \frac{M_N^*}{\psi_R} \ell_R \delta_R^{++} + \text{h.c.}$$

( $C^T = C$ )

$$C = i\sigma_2 \gamma_0 = \begin{pmatrix} i\sigma_2 & 0 \\ 0 & -i\sigma_2 \end{pmatrix}$$

$$\Gamma(\delta_R \rightarrow e_i, e_j)$$

stands for  $e, \mu, \tau$

$$(M_N^* = V_R^* M_N V_R^\dagger)_{ij}$$

$$V_R^\dagger = k O$$

$$k = \begin{pmatrix} 1 & 0 \\ 0 & e^{i\varphi} \end{pmatrix}$$

$$M_N^* = O_R^T k M_N k O_R$$



$$O_R = \begin{pmatrix} c & s \\ -s & c \end{pmatrix}_R$$

$$\begin{aligned}
 -M_N^* &= \begin{pmatrix} c & -s \\ s & c \end{pmatrix}_R \begin{pmatrix} m_1 & 0 \\ 0 & m_2 e^{2i\varphi} \end{pmatrix} \begin{pmatrix} c & s \\ -s & c \end{pmatrix}_R \\
 &= \begin{pmatrix} c^2 m_1 + s^2 m_2 e^{2i\varphi} & cs (m_2 e^{2i\varphi} - m_1) \\ cs (m_2 e^{2i\varphi} - m_1) & s^2 m_1 + c^2 m_2 e^{2i\varphi} \end{pmatrix}_R
 \end{aligned}$$

Process

$$\Gamma(\delta^- \rightarrow e\mu) \propto c_R^2 s_R^2 |m_2 e^{2i\varphi} - m_1|^2$$

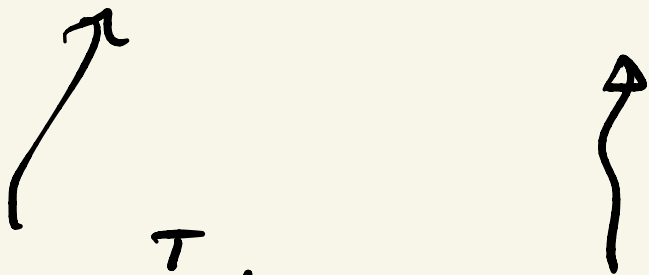
degenerate case:  $m_2 = m_1 = m$

$$\Gamma(\delta^- \rightarrow e\mu) \propto m^2 |e^{2i\varphi} - 1|^2$$

$$\propto (1 - \cos 2\varphi) \propto \sin^2 2\varphi$$

• *quark* :  $V_e(L) = V_e(R)$

• *lepton* :  $V_e(L) \neq V_e(R)$



$\underline{M}_\nu \propto M_D \frac{1}{M_N} M_D$        $\underline{M} \propto Y_\Delta$

$\psi_M$

$\psi_M = \psi_L + c \bar{\psi}_L^T$

$\mathcal{L}(\psi_M) = i \bar{\psi}_M \gamma^\mu \partial_\mu \psi_M - m_M \bar{\psi}_M \psi_M$