Neutrino Physics Course

Lecture XVI

24/6/2022

LMU

Summer 2022

LRSM : versus neutrino mass

$$\begin{pmatrix} \nu \\ e \end{pmatrix}_L \xleftrightarrow[LR = P]{} \begin{pmatrix} \nu \\ e \end{pmatrix}_R$$

$$\Rightarrow \quad \text{if } \exists \nu_L \longrightarrow \exists \nu_R$$

$$\Downarrow$$

$$\boxed{m_\nu \neq 0}$$

- $G_{LR} = SU(2)_L \times SU(2)_R \times U(1)_{B-L}$

- Higgs: $\Delta_L \longleftrightarrow \Delta_R$

$$\Delta_{L(R)} \longrightarrow U_{L(R)} \, \Delta_{L(R)} \, U^{+}_{L(R)}$$

$$(B-L)\Delta = 2\Delta$$

$$\Phi \to U_L \Phi U_R^{+}$$

$$\Downarrow$$

a) $\langle \Delta_R \rangle = \begin{pmatrix} 0 & 0 \\ v_R & 0 \end{pmatrix}$

$$\Rightarrow \boxed{M_{W_R} = g v_R, \quad M_{Z'}^2 = 2 \frac{M_{W_R}^2}{\cos^2\theta_R}}$$

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b) $\langle \Phi \rangle = \begin{pmatrix} v_1 & 0 \\ 0 & v_2 \end{pmatrix}$ $\quad v = |v_1|^2 + |v_2|^2$

$$M_{W_L} = \frac{g}{2} v, \quad \boxed{M_Z = \frac{M_{W_L}}{\cos\theta_W}}$$

$$D_\mu \Phi = \partial_\mu - ig T_L^i A_L^i \Phi +$$

$$+ ig \Phi T_2^R A_i^R \rightarrow W_L, Z$$

$$D_\mu \Delta_R = \partial_\mu - ig [T_R^i, \Delta_R]$$

$$\longrightarrow W_R, Z'$$

<u>Neutral gauge bosons</u>

$$i \bar{f}_L \gamma^\mu D_\mu f_L + \bar{f}_R \gamma^\mu D_\mu f_R$$

$$D_\mu = \cdots - ig T_{3L} \cdots$$

$\uparrow$ neutral gen $T_3, \ldots$

Charged gauge bosons

$$i \bar{f}_R \gamma^\mu D_\mu f_R \to$$

$$i \bar{f}_R \gamma^\mu (-ig)\left(T_{1R} A^R_{\mu 1} + T_{2R} A^R_{\mu 2}\right) f_R$$

$$= \frac{g}{2} \bar{f}_R \begin{pmatrix} 0 & A_{\mu 1} - i A_{\mu 2} \\ A_{\mu 1} + i A_{\mu 2} & 0 \end{pmatrix}_R \gamma^\mu f_R$$

$$f_R = \begin{pmatrix} u_R \\ d_R \end{pmatrix}$$

$$\to \boxed{\frac{g}{\sqrt{2}} \bar{u}_R \gamma^\mu W^+_{\mu R} d_R + h.c.}$$

$$\cdots$$

$$\boxed{W^{\pm}_{\mu R} = \frac{(A_1 \mp i A_2)_{\mu R}}{\sqrt{2}}}$$

$$\Downarrow$$

$$\boxed{M_{W_R} = g\, v_R}$$

$$\boxed{M_{Z_R}^{2} = \frac{M_{W_R}^{2}}{\cos^2\theta_R}\, 2} \qquad \tan\theta_R = \frac{g_{B2}}{g}$$

$$\Downarrow$$

$$\boxed{\sin\theta_R = \tan\theta_W} \longrightarrow \left(\equiv g'/g \right)$$

$$\Downarrow$$

$$\boxed{M_{Z_R} = \sqrt{2}\,\frac{M_{W_R}}{\sqrt{1-\tan^2\theta_W}}}$$

$$\boxed{M_{Z_R} \gtrsim \sqrt{3}\, M_{W_R} \;!?}$$

$$\Downarrow$$

$$\boxed{LRSM \longleftarrow SM}$$

Complete analogy

but

$$\boxed{m_\nu \neq 0}$$

- $L \leftrightarrow R$ in neutral current

$$Z \longleftrightarrow Q_Z = T_{3L} - \underbrace{Q \sin^2\theta_W}_{\text{LR symm.}}$$

$$W \longleftrightarrow \frac{g}{\sqrt{2}} \bar{u}_L d_L W^+ \leftarrow \text{only } L$$

$$\boxed{\begin{array}{ll} \nu: & Z \longleftrightarrow \nu_L \\ & Z' \longleftrightarrow \nu_R \ (?) \end{array}}$$

LHC: $\underset{(d)}{p} + \underset{(\bar{u})}{p} \longrightarrow W_R^- \to \bar{u} + d$

jets

$$\boxed{\begin{array}{l}\text{LHC (ATLAS + CMS?)} \\ \Rightarrow \quad M_{W_R} \gtrsim 5\ \text{TeV}\end{array}}$$

$\Downarrow$

LRSM: $M_{Z'} \gtrsim 8$ TeV

LHC: $\boxed{M_{Z'} \gtrsim (1-2)\ \text{TeV}}$

Z $\left(\boxed{Q_Z = T_{3L} - Q \sin^2\theta_W} \right.$

A) W_R int. with Z ?

$\gamma\,(A)$: $\boxed{Q \equiv Q_{em}}$ (photon)

A2 W_R int. with A ?

YES !

A1 $T_{3L}\, W_R^{\pm} = 0$

$$Q\, W_R^{\pm} = \pm\, W_R^{\pm}$$

$\Rightarrow$ YES !

$Z\, W_R^{+} W_R^{-}$: $\propto \sin^2\theta_W$

Neutrino masses and interactions

$$\boxed{L\,\Delta_{L,R} = -2\,\Delta_{L,R}}$$

$$\mathcal{L}_Y = l_L^T\, i\sigma_2\, \Delta_L\, Y_\Delta\, C\, l_L + $$

$$\updownarrow LR$$

$$+\, l_R^T\, i\sigma_2\, \Delta_R\, Y_\Delta\, C\, l_R + h.c.$$

$$\Downarrow \quad \boxed{\langle \Delta_R \rangle = \begin{pmatrix} 0 & 0 \\ v_R & 0 \end{pmatrix}} \quad \Delta L = 2$$

$$\nu_R^T\, C\, Y_\Delta\, v_R\, \nu_R + h.c.$$

$$\uparrow \qquad\qquad \uparrow$$

$$m \qquad\qquad m$$

Majorana mass (matrix)

$\updownarrow$

y_Δ = matrix in gen. space

$$\boxed{M_{\nu_R} = y_\Delta v_R}$$

$\Downarrow$

$$\boxed{C^T = -C}$$

$$\nu_R{}^T C M_{\nu_R} \nu_R = -\nu_R{}^T C^T M_{\nu_R}{}^T \nu_R$$

$$= \nu_R{}^T C M_{\nu_R}{}^T \nu_R$$

$$\Rightarrow \boxed{M_{\nu_R}{}^T = M_{\nu_R}}$$

$$\bullet \ \nu_R^T C M_{\nu_R} \nu_R + h.c. =$$

$$= \nu_R^T C M_{\nu_R} \nu_R + \nu_R^\dagger C^\dagger M_{\nu_R}^* \nu_R^*$$

$$\boxed{N_L \equiv C \bar{\nu}_R^T} = C \gamma_0 \nu_R^*$$

$$= i \gamma_2 \nu_R^*$$

$$N_L = \begin{pmatrix} 0 & i\sigma_2 \\ -i\sigma_2 & 0 \end{pmatrix} \begin{pmatrix} 0 \\ u_R^* \end{pmatrix}$$

$$= \begin{pmatrix} i\sigma_2 u_R^* \\ 0 \end{pmatrix} \Leftarrow \text{LH}$$

$$\Downarrow$$

ν_R = RH neutrino

$$\Updownarrow$$

(anti)

N_L = LH neutral heavy lepton

$$\underbrace{\nu_R^T C \, M_{\nu_R} \, \nu_R}_{\boxed{\Delta L = 2}} + \nu_R^\dagger C^\dagger M_{\nu_R}^* \nu_R =$$

$$\parallel$$

$$\left\{ N_L = C \bar{\nu}_R^{\,T} = C \gamma_0 \nu_R^* \right.$$ (transpose)

$$\Rightarrow \boxed{\nu_R^\dagger \gamma_0 C^T = N_L^T}$$

$$\Downarrow$$

$$\underline{N_L^T C M_{\nu_R}^* N_L} = \nu_R^+ \gamma_0 \overbrace{\underline{C^T C}}^{=} M_{\nu_R}^* C \gamma_0 \nu_R^*$$

$$= \nu_R^+ \gamma_0 C \gamma_0 M_{\nu_R}^* \nu_R^*$$

$$= \nu_R^+ (-C) \gamma_0^2 M_{\nu_R}^* \nu_R^*$$

$$\boxed{= \nu_R^+ C^+ M_{\nu_R}^* \nu_R^*}$$

h.c. of RH mass term

if $\boxed{M_N = M_{\nu_R}^*}$

$$\boxed{\nu_R^{\dagger} C^{\dagger} M_{\nu_R}^{*} \nu_R^{*} = \underbrace{N_L^{T} C M_N N_L}_{\substack{\text{Majorana mass} \\ \text{(matrix) for } N_L}}}$$

<u>SM digression</u>

$\nu = LH \quad (\nu_L)$

what is the meaning of this?

$$\nu_L \longrightarrow (\nu^c)_R \equiv C \bar{\nu}_L^{T}$$

$$\frac{g}{\sqrt{2}} \overline{\nu_L} \gamma^\mu e_L \, W_\mu^+ =$$

$$= \frac{g}{\sqrt{2}} \overline{e^c_R} \gamma^\mu \nu^c_R \, W_\mu^+$$

$\binom{\nu}{e}_L \leftarrow$ partner of e

$\Leftrightarrow$ $\nu =$ lepton $=$ LH
in weak int.

$\nu^c =$ anti - lepton

$=$ RH in weak int.

Fermion masses (Dirac)

$$\mathcal{L}_Y(\Phi) = \bar{f}_L\left(Y\Phi + \tilde{Y}\tilde{\Phi}\right) f_R + \text{h.c.}$$

$$\Phi = (\phi_1, \tilde{\phi}_2)$$

$$\tilde{\Phi} = (\phi_2, \tilde{\phi}_1)$$

$$\Rightarrow \phi = (v_1 \phi_1 + v_2 \phi_2) N \quad \therefore \langle\phi\rangle \neq 0$$

$$\phi' = (v_2 \phi_1 - v_1 \phi_2) N \quad \therefore \langle\phi'\rangle = 0$$

$$\boxed{m_{\phi'} = m_H > 10\,\text{TeV}}$$

Forget about ϕ'

SM Higgs doublet

$$\Downarrow \quad \phi = \begin{pmatrix} 0 \\ v + h \end{pmatrix}$$

SM Higgs boson

$$h\left[\bar{u}_L \frac{M_u}{v} u_R + \bar{d}_L \frac{M_d}{v} d_R\right] + \text{h.c.}$$

$$\left.\begin{aligned} M_u &= Y_u v \\ M_d &= Y_d v \end{aligned}\right\} \quad Y_u, Y_d = f(Y, \tilde{Y})$$

$\Downarrow$ analogy

electron mass $\longleftrightarrow$ M_d

(charged lepton M_e) $\qquad +$

$$\overline{\nu_R}\, M_D\, \nu_L + \overline{\nu_L}\, M_D^{\dagger}\, \nu_R$$

$\longrightarrow$ (convention) of

Dirac neutrino mass

$$M_D \longrightarrow M_u^{\dagger}$$

but $\quad +$

$$\nu_R^T\, C\, M_{\nu_R}\, \nu_R + h.c.$$

$$\Downarrow$$

$$\boxed{\text{better use } N_L\text{!}}$$

$$\nu_L = C\overline{\nu_R}^T$$

$$\Downarrow$$

$$\overline{\nu_R}\, C^T = N_L^T \,/\, C$$

$$\Rightarrow \quad \overline{\nu_R}\, C^T C = N_L^T C$$

$$\Downarrow \quad C^T C = 1$$

$$\boxed{\overline{\nu_R} = N_L^T C}$$

$$\Downarrow$$

$$\overline{\nu_R}\, M_D \nu_L = N_L^T C\, M_D \nu_L$$

Dirac

$(\Delta L = 0)$

N_L = anti - lepton

$(\Delta L = 0)$

$$\oplus \; \underbrace{N_L^T C M_N N_L}_{\Delta L = 2}$$

$\Downarrow$

$\boxed{\nu - N \text{ mass system}}$

$\downarrow$ $\boxed{\text{More constrained}}$

$$N_L^T C\, M_D\, \nu_L = \frac{1}{2}\, N_L^T C\,(M_D + M_D)\,\nu_L$$

$$= \frac{1}{2} N_L^T C\, \nu_L + \boxed{\frac{1}{2}\, \nu_L^T (-1)\, C^T M_D^T\, N_L}$$

- $N^{i\,T}_{L\,\alpha}\, C_{\alpha\beta}\, M_D^{ij}\, \nu^j_\beta = -\, \nu^j_\beta\, C_{\alpha\beta}\, M_D^{ij}\, N_\alpha^i$

($\alpha, \beta =$ Lorentz index

$i, j =$ gen. -"-)

$$= -\,\nu_\beta^{j\,T}\, C^T_{\beta\alpha}\, M_D^{T\,ji}\, N_\alpha^i$$

$$= -\,\nu_L^T\,(-C)\, M_D^T N = \nu_L^T C\, M_D^T N$$

$$\boxed{\begin{aligned} N_L^T C \mathcal{M}_D \nu_L &= \frac{1}{2} N_L^T C \mathcal{M}_D \nu_L \\ &+ \frac{1}{2} \nu_L^T C \mathcal{M}_D^T N_L \\ &\text{Dirac mass terms} \end{aligned}}$$

$$+ \frac{1}{2} N_L^T \mathcal{M}_N C N_L \qquad + h.c.$$

masses of N

new (normalization)

rescaling

$$\Downarrow$$

ν_L $\qquad$ N_L

$$\boxed{M_{\nu N} = \begin{array}{c c} \nu_L \\ N_L \end{array} \begin{pmatrix} 0 & M_D^T \\ M_D & M_N \end{pmatrix}}$$

$$M_N = M_{\nu_R}^* = (y_\Delta v_R)^*$$

$$\boxed{M_N = y_\Delta^* v_R}$$

$$\boxed{M_D = y_D v}$$

$$\left.\begin{array}{l} M_{W_R} = g\, v_R \\ M_{W_L} = g\, v\, \frac{1}{2} \end{array}\right\} \Rightarrow \boxed{v_R \gg v}$$

$$\Downarrow$$

$$\boxed{M_N \gg M_D}$$ (seesaw scenario)

$$\boxed{\text{NATURAL?}}$$

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e: $\mathcal{L}_D = i\bar{e}\gamma^\mu \partial_\mu e - m\bar{e}e$

$$\Downarrow$$

$$\not{p} e = m e$$

$$\Downarrow$$

$$p^2 = m^2 \Rightarrow \boxed{m = m_e}$$

$$\underbrace{l_L^T C\, i\sigma_2\, \Delta_L\, l_L + L \leftrightarrow R}_{\text{(def. of } \Delta_L)}$$

$$\cancel{q_L^T C\, i\sigma_2\, \Delta_L\, q_L}$$

$$\cancel{B-L}$$

$$l_R^T \begin{pmatrix} 0 & 1 \\ 0 & 0 \end{pmatrix} \begin{pmatrix} 0 & 0 \\ v_R & 0 \end{pmatrix} C\, l_R$$

$$= l_R^T \begin{pmatrix} v_R & 0 \\ 0 & 0 \end{pmatrix} C\, l_R$$

$$= \nu_R^T \nu_R C \nu_R$$

only $\nu_R \longrightarrow$ mass

$$\Phi \longrightarrow U_L \, \Phi \, U_R^\dagger$$

SU(2) matrices

$$\boxed{\det U_L = \det U_R = 1}$$