

Neutrino Physics Course

Lecture XVI

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24/6/2022

LMU

Summer 2022

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# LRSM : versus neutrino mass

$$\begin{pmatrix} \nu \\ e \end{pmatrix}_L \longleftrightarrow \begin{pmatrix} \nu \\ e \end{pmatrix}_R$$

$LR = P$

$$\Rightarrow \text{if } \exists \nu_L \rightarrow \exists \nu_R$$

$$\boxed{m_\nu \neq 0}$$

- $G_{LR} = SU(2)_L \times SU(2)_R \times U(1)_{B-L}$

- Higgs:  $\Delta_L \leftrightarrow \Delta_R$

$$\Delta_{(L)} \rightarrow U_{(R)} \Delta_{(R)} U_{(R)}^\dagger$$

$$(B-2) \Delta = 2 \Delta$$

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$$\bar{\Phi} \rightarrow U_L \bar{\Phi} U_R^\dagger$$



$$a) \langle \Delta_R \rangle = \begin{pmatrix} 0 & 0 \\ \nu_R & 0 \end{pmatrix}$$

$$\Rightarrow \left[ M_{\nu_R} = g \nu_R, \quad M_{z'}^2 = \frac{2 M_{W_R}^2}{\cos^2 \theta_R} \right]$$

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$$b) \langle \bar{\Phi} \rangle = \begin{pmatrix} \nu_1 & 0 \\ 0 & \nu_2 \end{pmatrix} \quad \nu = |\nu_1|^2 + |\nu_2|^2$$

$$M_{W_2} = \frac{g}{2} \nu, \quad \left[ M_z = \frac{M_{W_1}}{\cos \theta_W} \right]$$

$$D_\mu \Phi = \partial_\mu - ig T_L^i A_\mu^i \Phi + ig \bar{\Phi} T_R^i A_\mu^i \rightarrow \textcircled{W_L, Z}$$

$$D_\mu \Delta_R = \partial_\mu - ig [T_R^i, \Delta_R]$$

$$\rightarrow \textcircled{W_R, Z'}$$

Neutral gauge bosons

$$i \bar{f}_L \gamma^\mu D_\mu f_L + \bar{f}_R \gamma^\mu D_\mu f_R$$

$$D_\mu = \dots - ig T_{3L} \dots$$

$\hat{z}$  neutral gen  $T_3, \dots$



# Charged gauge bosons

$$i \bar{f}_R \gamma^\mu D_\mu f_R \rightarrow$$

$$i \bar{f}_R \partial^\mu (ig) (T_{1a} A_{\mu 1}^R + T_{2R} A_{\mu 2}^R) f_R$$

$$= \frac{g}{2} \bar{f}_R \begin{pmatrix} 0 & A_{\mu 1} - i A_{\mu 2} \\ A_{\mu 1} + i A_{\mu 2} & 0 \end{pmatrix}_R \gamma^\mu f_R$$

$$f_R = \begin{pmatrix} u_R \\ d_R \end{pmatrix}$$

$$\rightarrow \frac{g}{\sqrt{2}} \bar{u}_R \gamma^\mu W_{\mu R}^+ d_R + \text{h.c.} \quad |$$

$$W_{\mu R}^{\pm} = \frac{(A_1 \mp i A_2)_{\mu R}}{\sqrt{2}}$$



$$M_{WR} = g v_R$$

$$M_{ZR}^2 = \frac{M_{WR}^2}{\cos^2 \theta_R} \quad \text{then } \theta_R = \frac{g' B_2}{g}$$



$$\sin \theta_R = \tan \theta_W \rightarrow \left( \equiv g'/g \right)$$



$$M_{2R} = \sqrt{2} \frac{M_{WR}}{\sqrt{1 - \tan^2 \theta_W}}$$



$$M_{2R} \approx \sqrt{3} M_{WR} \quad ??$$

LR SM  $\leftrightarrow$  SM

Complete ecology

but

$$w_j \neq 0$$

•  $L \leftrightarrow R$  in neutral current

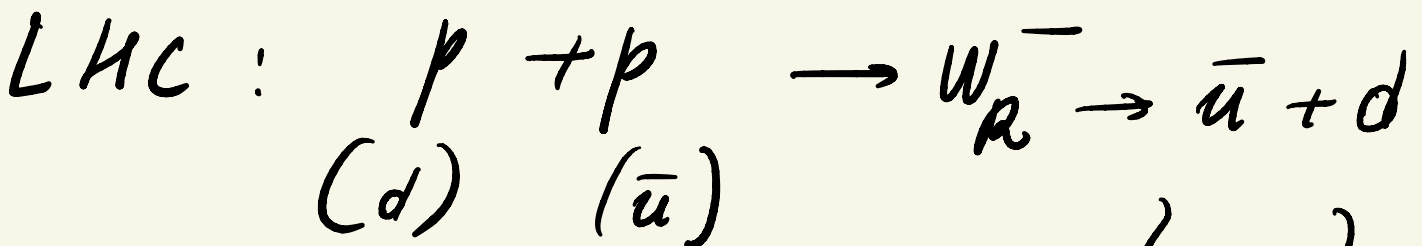
$$Z \leftrightarrow Q_Z = T_{3L} - Q \sin^2 \theta_W$$

LR sym.  
↓

$$W \leftrightarrow \frac{1}{\sqrt{2}} \bar{u}_L d_L W^+ \leftarrow \text{only } L$$

$$\partial: Z \leftrightarrow \nu_L$$

$$Z' \leftrightarrow \nu_R (?)$$



jets

LHC (ATLAS + CMS?)

$$\Rightarrow M_{WA} \gtrsim 5 \text{ TeV}$$

$\Downarrow$

LRSB:  $M_{Z'} \gtrsim 8 \text{ TeV}$

$$\text{LHC: } \left[ M_{Z'} \gtrsim (1-2) \text{ TeV} \right]$$

$$Z \left( Q_Z = T_{3L} - Q \sin^2 \theta_W \right)$$

(A)  $W_R$  int. with  $Z$ ?

$\gamma(A) : \boxed{Q = Q_{ew}} \text{ (photo)}$

(A2)  $W_R$  int. with A?

YES!

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(A1)  $T_{3L} W_R^\pm = 0$

$$Q W_R^\pm = \pm W_R^\pm$$

$\Rightarrow$  YES!

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$$\int W_R^+ W_R^- : \propto \sin^2 \theta_w$$

# Neutrino masses and interactions

$$\mathcal{L}_Y = \bar{l}_L^T i \sigma_2 \Delta_L Y_\Delta C l_L + \bar{l}_R^T i \sigma_2 \Delta_R Y_\Delta C l_R + \text{h.c.}$$

$\Delta_{L,R} = -2 \Delta_{L,R}$

$\updownarrow LR$

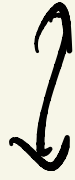
$$\Downarrow \left( \langle \Delta_R \rangle = \begin{pmatrix} 0 & 0 \\ v_R & 0 \end{pmatrix} \right)$$

$\Delta L = 2$

$$\bar{\nu}_R^T C Y_\Delta v_R \nu_R + \text{h.c.}$$

$\uparrow \qquad \qquad \qquad \uparrow$   
 $m_\nu \qquad \qquad \qquad m_\nu$

Majorana mass (matrix)



$Y_{\Delta}$  = matrix in gen. space

$$M_{\nu_R} = Y_{\Delta} \nu_R$$



$$C^T = -C$$

$$\begin{aligned} \nu_R^T C M_{\nu_R} \nu_R &= -\nu_R^T C^T M_{\nu_R}^T \nu_R \\ &= \nu_R^T C M_{\nu_R}^T \nu_R \end{aligned}$$

$$\Rightarrow \boxed{M_{\nu_R}^T = M_{\nu_R}}$$



$$\bullet \psi_R^T C M_{\psi_R} \psi_R + h.c. =$$

$$= \psi_R^T C M_{\psi_R} \psi_R + \psi_R^\dagger C^\dagger M_{\psi_R}^* \psi_R^*$$

$$\boxed{N_L \equiv C \bar{\psi}_R^T} = C \gamma_0 \psi_R^*$$

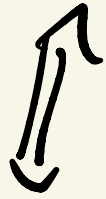
$$= i\sigma_2 \psi_R^*$$

$$N_L = \begin{pmatrix} 0 & i\sigma_2 \\ -i\sigma_2 & 0 \end{pmatrix} \begin{pmatrix} 0 \\ u_R^* \end{pmatrix}$$

$$= \begin{pmatrix} i\sigma_2 u_R^* \\ 0 \end{pmatrix} \Leftrightarrow \text{LM}$$



$\nu_R = \text{RH neutrinos}$



$N_L = \text{LH neutral heavy lepton}$  (anti)

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$$\underbrace{\nu_R^T C M_{\nu_R} \nu_R + \nu_R^\dagger C^\dagger M_{\nu_R}^\dagger \nu_R}_{\Delta L = 2} =$$

||

$$\left\{ N_L = C \bar{\nu}_R^T = C \gamma_0 \nu_R^* \text{ (trellis)} \right.$$

$$\Rightarrow \boxed{\nu_R^\dagger \gamma_0 C^T = N_L^T}$$



$$\underline{N_L^T C M_{V_R}^* N_L} = V_R^+ \gamma_0 \overbrace{C^T C}^{\parallel} M_{V_R}^* \\ C \gamma_0 V_R^*$$

$$= V_R^+ \gamma_0 C \gamma_0 M_{V_R}^* V_R^*$$

$$= V_R^+ (-C) \gamma_0^2 M_{V_R}^* V_R^*$$

$$= V_R^+ \overset{\parallel}{C} + M_{V_R}^* V_R^*$$



h.c. of RH was term

if  $M_N = M_{V_R}^*$

$$\nu_R^\dagger C^\dagger M \nu_R^* = N_L^T C M_N N_L$$

Majorana mass  
(matrix) for  $N_L$

SM dispersion

$$\nu = L H \quad (\nu_L)$$

what is the meaning of this?

$$\nu_L \rightarrow (\nu_R^c) \equiv C \bar{\nu}_L^T$$

$$\frac{g}{\sqrt{2}} \bar{\nu}_L \gamma^\mu e_L W_\mu^+ =$$

$$= \frac{g}{\sqrt{2}} \bar{e}_R^c \gamma^\mu \nu_R^c W_\mu^+ \quad \rightarrow$$

$\begin{pmatrix} \nu \\ e \end{pmatrix}_L \leftarrow$  partners of  $e$

$\Leftrightarrow \nu = \text{lepton} = LH$   
in weak int.

$$\nu^c = \text{anti-lepton}$$

$$= RH \text{ in weak int.}$$

# Fermion masses (Dirac)

$$\mathcal{L}_f(\Phi) = \bar{f}_L (\gamma \Phi + \tilde{\gamma} \tilde{\Phi}) f_R + \text{h.c.}$$

$$\Phi = (\phi_1, \tilde{\phi}_2)$$

$$\tilde{\Phi} = (\phi_2, \tilde{\phi}_1)$$

$$\Rightarrow \phi = (v_1 \phi_1 + v_2 \phi_2) N \quad \therefore \langle \phi \rangle \neq 0$$

$$\phi' = (v_2 \phi_1 - v_1 \phi_2) N \quad \therefore \langle \phi' \rangle = 0$$

$$m_{\phi'} = m_H > 10 \text{ TeV}$$

Forget about  $\phi'$

SM Higgs doublet

$$\Downarrow \phi = \begin{pmatrix} 0 \\ v+h \end{pmatrix}$$

SM Higgs boson

$$h \left[ \bar{u}_L \frac{M_u}{v} u_R + \bar{d}_L \frac{M_d}{v} d_R \right] + \text{h.c.}$$

$$\left. \begin{aligned} M_u &= \gamma_u v \\ M_d &= \gamma_d v \end{aligned} \right\} \gamma_u, \gamma_d = f(y, \tilde{y})$$

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$\Downarrow$  anomaly

electron mass  $\leftrightarrow M_D$

(charged lepton  $M_e$ ) +

$$\bar{\nu}_R M_D \nu_L + \bar{\nu}_L M_D^\dagger \nu_R$$

$\hookrightarrow$  (convention) of

Dirac neutrino mass

$$M_D \rightarrow M_n^\dagger$$

but  $\oplus$

$$\nu_R^T C M_{\nu R} \nu_R + h.c.$$





better use  $N_L$ !

$$u_L = C \bar{v}_R^T$$

$$\Downarrow$$
$$\bar{v}_R C^T = N_L^T / C$$

$$\Rightarrow \bar{v}_R C^T C = N_L^T C$$

$$\Downarrow C^T C = 1$$

$$\bar{v}_R = N_L^T C$$

$\Downarrow$

$$\bar{v}_R M_D v_L = N_L^T C M_D v_L$$

Dirac  
( $\Delta L = 0$ )

$N_L = \text{anti-lepton}$   
( $\Delta L = 0$ )

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⊕  $N_L^T C M_N N_L$   
 $\Delta L = 2$



$\nu$ -N mass system

↓ More careful

$$N_L^T C M_D v_L = \frac{1}{2} N_L^T C (M_D + M_D) v_L$$

$$= \frac{1}{2} N_L^T C v_L + \left[ \frac{1}{2} v_L^T (-1) C^T M_D^T N_L \right]$$

$$\bullet N_{\alpha}^i L^T C_{\alpha\beta} M_D^{ij} v_{\beta}^j = - v_{\beta}^j C_{\alpha\beta} M_D^{ij} N_{\alpha}^i$$

$$\left( \begin{array}{l} \alpha, \beta = \text{Lorentz index} \\ \varepsilon_{ij} = \text{gen. } -11- \end{array} \right)$$

$$= - v_{\beta}^j C_{\beta\alpha}^T M_D^{Tji} N_{\alpha}^i$$

$$= - v_L^T (-c) M_D^T N = v_L^T C M_D^T N$$



$$N_L^T C M_D \nu_L = \left(\frac{1}{2}\right) N_L^T C M_D \nu_L + \left(\frac{1}{2}\right) \nu_L^T C M_D^T N_L$$

Dirac mass terms

$$+ \left(\frac{1}{2}\right) N_L^T M_N C N_L$$

+ h.c.

new (normalization) masses of  $N$

↳ rescaling



$\nu_L$

$N_L$

$$\underline{M}_{\nu N} = \nu_L \begin{pmatrix} 0 & \underline{M}_D^T \\ \underline{M}_D & \underline{M}_N \end{pmatrix}$$

$$\underline{M}_N = \underline{M}_{\nu R}^* = (\gamma_D \nu_R)^*$$

$$\underline{M}_N = \gamma_\Delta^* \nu_R$$

$$\underline{M}_D = \gamma_D \nu$$

$$\underline{M}_{\nu R} = \gamma \nu_R$$

$$\underline{M}_{\nu L} = \gamma \nu \frac{1}{2}$$

$$\Rightarrow \nu_R \gg \nu$$



$$M_N \gg M_D \quad (\text{seesaw scenario})$$

NATURAL?

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$$e: \mathcal{L}_0 = \bar{e} \gamma^\mu \partial_\mu e - m \bar{e} e$$



$$\not{p} e = m e$$



$$p^2 = m^2 \Rightarrow \boxed{m = m_e}$$

$$l_L^T C i \sigma_2 \Delta_L l_L + l \leftrightarrow R$$

↓  
(def. of  $\Delta_L$ )

~~$$q_L^T C i \sigma_2 \Delta_L q_L$$~~

~~$$B-L$$~~

$$l_R^T \begin{pmatrix} 0 & 1 \\ 0 & 0 \end{pmatrix} \begin{pmatrix} 0 & 0 \\ v_R & 0 \end{pmatrix} C l_R$$

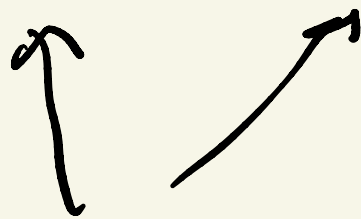
$$= l_R^T \begin{pmatrix} v_R & 0 \\ 0 & 0 \end{pmatrix} C l_R$$

$$= \nu_R^T \nu_R C \nu_R$$

only  $\nu_R \rightarrow$  mass

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$$\Phi \rightarrow U_L \Phi U_R^\dagger$$



SU(2) matrices

$$\det U_L = \det U_R = 1$$



