LRSM : SSB

- $G_{LR} = SU(2)_L \times SU(2)_R \times U(1)_{B-L}$

- $\begin{pmatrix} u \\ d \end{pmatrix}_L \longleftrightarrow \begin{pmatrix} u \\ d \end{pmatrix}_R$

  $\begin{pmatrix} \nu \\ e \end{pmatrix}_L \longleftrightarrow \begin{pmatrix} \nu \\ e \end{pmatrix}_R$

$$\boxed{\begin{array}{ccc} \phi(SM) & \subseteq & \Phi(LR) \\ \text{SM doublet} & & \text{LR bi-doublet} \end{array}}$$

$$\updownarrow \qquad\qquad\qquad \updownarrow$$

$$\phi \to U_L \phi \qquad\qquad \Phi \to U_L \Phi U_R^\dagger$$

$$\langle \phi \rangle \simeq M_{W_L} \implies \langle \Phi \rangle \simeq M_{W_L}$$

$$\phi = \begin{pmatrix} \phi^0 \\ \phi^- \end{pmatrix} \Rightarrow \quad \Phi = \begin{pmatrix} \phi_1^0 & \phi_2^+ \\ \phi_1^- & -\phi_2^{0*} \end{pmatrix}$$

$$Y = -1$$

$$(\phi_1, \phi_2)$$

$$\boxed{M_{W_L} = \frac{g}{2}\langle\phi\rangle}$$

$$M_Z = \frac{M_{W_L}}{\cos\theta_W}$$

$$e = g\sin\theta_W$$

$$\updownarrow$$

$$(D_\mu\phi)^+(D^\mu\phi)$$

$$\tilde{\phi}_2 \equiv i\sigma_2\phi_2 = \begin{pmatrix} \phi_2^+ \\ -\phi_2^{0*} \end{pmatrix}$$

$$(\tilde{\phi} \to U_L\tilde{\phi})$$

both are SM doublets

$$\Downarrow$$

$$M_{W_L}^2 = \frac{g^2}{4}\left[\langle\phi_1\rangle^2 + \langle\phi_2\rangle^2\right]$$

$$M_Z = \frac{M_{W_L}}{\cos\theta_W}$$

$$\updownarrow$$

$$(D_\mu\phi_i)^+(D^\mu\phi_i)$$

$(i=1,2)$

$$V = V(\Phi)$$

$$\Phi \Rightarrow \tilde{\Phi} = \sigma_2 \phi^* \sigma_2$$

$$\Downarrow$$

$$\boxed{\tilde{\Phi} \to U_L \tilde{\Phi} U_R^\dagger}$$

<u>invariant</u>

$$\Phi_a \equiv (\Phi, \tilde{\Phi}) \quad (a=1,2)$$

$$\Phi_a \to U_L \Phi_a U_R^\dagger$$

$$\Phi_a^\dagger \to U_R \Phi_a^\dagger U_L^\dagger$$

$$\Rightarrow \quad \Phi_a^+ \Phi_b \to U_R \Phi_a^+ \Phi_b U_R^+$$

$$\Rightarrow \left\{ \begin{aligned} \mathrm{Tr}\, \Phi_a^+ \Phi_b &\to \mathrm{Tr}\, U_R^+ U_R \Phi_a^+ \Phi_b \\ &= \mathrm{Tr}\, \Phi_a^+ \Phi_b \end{aligned} \right.$$

① inv.

② $\underbrace{\mathrm{Tr}\, \Phi_a^+ \Phi_b \Phi_c^+ \Phi_d}_{\text{inv.}}$

③ $\Phi_a \to U_L \Phi_a U_R^+$

$$\det U_{L,R} = 1$$

$$\Downarrow$$

$$\det \Phi_a \to \det U_L \det U_R^\dagger \times \det \Phi$$
$$= \det \Phi$$

$$\boxed{\det \Phi_a = \text{quadratic}}$$ INV

but

$$\boxed{\text{How many independent?}}$$

• (SM) $\phi \longrightarrow U_L \phi$

<u>one</u> inv $= \phi^\dagger \phi$

$\cancel{\phi^T i\sigma_2 \phi, \ - \ - -}$

$$U_L\,\phi = \begin{pmatrix} 0 \\ \varphi \end{pmatrix}$$ **are invariant**

• (LR) $\quad \Phi \longrightarrow \underbrace{U_L\, \Phi\, U_R^{+}}$

$$\downarrow$$

$$\begin{pmatrix} \varphi_1 & 0 \\ 0 & \varphi_2 \end{pmatrix}$$

$$\downarrow$$

$$\underbrace{\begin{pmatrix} e^{i\alpha_L} & 0 \\ 0 & e^{-i\alpha_L} \end{pmatrix}}_{SU(2)_L} \begin{pmatrix} \varphi_1 & 0 \\ 0 & \varphi_2 \end{pmatrix} \underbrace{\begin{pmatrix} e^{-i\alpha_R} & 0 \\ 0 & e^{i\alpha_R} \end{pmatrix}}_{SU(2)_R}$$

$$\downarrow$$

$$= \begin{pmatrix} e^{i(\alpha_L-\alpha_R)}\varphi_1 & 0 \\ 0 & e^{-i(\alpha_L-\alpha_R)}\varphi_2 \end{pmatrix}$$

$$\varphi_1 = r_1 e^{i\theta_1} \quad \therefore$$

$$\alpha_L - \alpha_R + \theta_1 = 0$$

$$\Downarrow$$

$$\boxed{\phi \longrightarrow \underbrace{\begin{pmatrix} r_1 & 0 \\ 0 & r_2 e^{i\theta_2} \end{pmatrix}}_{\text{3 real comp.}}}$$

$$\Downarrow$$

$\boxed{\text{3 invariants}}$

$$\boxed{\begin{array}{ccc} \mathrm{Tr}\,\Phi^{+}\Phi\,, & \det\Phi\,, & \det\Phi^{+} \\ \| & \| & \| \\ (r_1^2+r_2^2) & r_1 r_2\, e^{i\theta} & r_1 r_2\, e^{-i\theta} \end{array}}$$

$$\mathrm{Tr}\,\Phi^{+}\tilde{\Phi} \;\propto\; \det\Phi$$

PROVE

$$\mathrm{Tr}\,\Phi^{+}\Phi\,\Phi^{+}\Phi \;\propto\; \text{what ?}$$

$$\mathrm{Tr}\,\Phi^{+}\tilde{\Phi}\,\Phi^{+}\tilde{\Phi} \;\propto\; \text{— " — ?}$$

- SM digression

$$(\phi^\dagger \vec{\sigma} \phi)(\phi^\dagger \vec{\sigma} \phi) \propto (\phi^\dagger \phi)$$

$$(\phi^T i\sigma_2 \vec{\sigma} \phi)(\phi^\dagger i\sigma_2 \vec{\sigma} \phi^*) \propto (\phi^\dagger \phi)$$

Bottom line

$$V(\Phi) = V(\Phi^\dagger \Phi, \det \Phi, \det \Phi^\dagger)$$

SSB $\Downarrow$

$$\left(-\mu^2 \mathrm{Tr}\, \Phi^\dagger \Phi - \mu'^2 \det \Phi + \text{h.c.} \right)$$

$$\Downarrow$$

$$\langle \Phi \rangle = \begin{pmatrix} v_1 & 0 \\ 0 & v_2 e^{ia} \end{pmatrix}$$

$$= \begin{pmatrix} \langle \phi_1^0 \rangle & 0 \\ 0 & \langle \phi_2^0 \rangle \end{pmatrix}$$

$$\Downarrow$$

$$\boxed{Q_{em} \langle \Phi \rangle = 0}$$

at this stage:

SSB $\longrightarrow$ charge conservation

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$$V = V(a) =$$

$$= \underbrace{A + B\cos a + C\cos 2a}$$

Minima:

$a = 0$ (conditions?)

$a \neq 0$ (+| −)

$\boxed{\text{To be discussed}}$

$$G_{LR} \longrightarrow G_{SM}$$

$$\langle \Delta_R \rangle$$

$\uparrow$

$$\begin{cases} SU(2)_R \text{ triplet (adjoint)} \\ (B-L)\, \Delta_R = 2\, \Delta_R \end{cases}$$

$\Downarrow$ LR

$$(\Delta_L, \quad \Delta_R)$$

<u>Invariants?</u>

Δ = SU(2) adjoint, $Y' = 2$

complex

$$\Delta = \underbrace{\Delta_1 + i\,\Delta_2}_{\text{"real" = Hermitian objects}}$$

$$\begin{cases} \Delta_i \longrightarrow U \Delta_i U^+ & i = 1,2 \\ \mathrm{Tr}\,\Delta_i = 0\,, \quad \Delta_i^+ = \Delta_i & \end{cases}$$

$$\mathrm{Tr}\,\Delta_i^2 \longrightarrow \mathrm{Tr}\,\Delta_i \Delta_j$$

$$\rightarrow \mathrm{Tr}\,\Delta_i \Delta_j \Delta_k \Delta_l \qquad (?)$$

→ count independent inv.

$$\Delta_1 \longrightarrow \begin{pmatrix} a & 0 \\ 0 & -a \end{pmatrix}, \quad a \in R$$

$$\Delta_1 \to U \Delta_1 U^+$$

with:

$$\Delta_2 = \begin{pmatrix} b & re^{i\theta} \\ re^{-i\theta} & -b \end{pmatrix}, \quad b \in R$$

but, after $\Delta_1 = \begin{pmatrix} a & 0 \\ 0 & -a \end{pmatrix}$

let $U = \begin{pmatrix} e^{i\alpha} & 0 \\ 0 & e^{-i\alpha} \end{pmatrix}$

$$\Delta_1 \to U \Delta_1 U^+ = \Delta_1 \quad \text{(inv.)}$$

(symmetry in T_3 direction)

$$\boxed{\begin{array}{l} SU(2) \simeq SO(3) \\ \hookrightarrow \; U(1) = SO(2) \end{array}}$$

$$A_2 \to \begin{pmatrix} e^{i\alpha} & 0 \\ 0 & e^{-i\alpha} \end{pmatrix} \begin{pmatrix} b & re^{i\theta} \\ re^{-i\theta} & -b \end{pmatrix} \begin{pmatrix} e^{-i\alpha} & 0 \\ 0 & e^{i\alpha} \end{pmatrix}$$

$$= \begin{pmatrix} e^{i\alpha} b & re^{i(\alpha+\theta)} \\ re^{-i(\alpha+\theta)} & -be^{-i\alpha} \end{pmatrix} \begin{pmatrix} e^{-i\alpha} & 0 \\ 0 & e^{i\alpha} \end{pmatrix}$$

$$= \begin{pmatrix} b & re^{i(2\alpha+\theta)} \\ re^{-i(2\alpha+\theta)} & -b \end{pmatrix}$$

$$2\alpha + \theta \quad \therefore$$

$$\Downarrow$$

$$\Delta_2 = \begin{pmatrix} b & r \\ r & -b \end{pmatrix}$$

$$\Downarrow$$

$$\Delta = \begin{pmatrix} a+ib & ir \\ ir & -a-ib \end{pmatrix}$$

$$\boxed{\Delta = \begin{pmatrix} z & ir \\ ir & -z \end{pmatrix}}$$

$$\Downarrow$$

$$\boxed{\text{3 invariants}}$$

$$\underbrace{\mathrm{Tr}\,\Delta^{+}\Delta,}_{\substack{SU(2)\times U(1)\\ \text{inv.}}} \qquad \underbrace{\mathrm{Tr}\,\Delta^{2},\ \left(\mathrm{Tr}\,\Delta^{2}\right)^{*} = \mathrm{Tr}\,\Delta^{+2}}_{3\ \text{invariants}}$$

<u>Vector language</u>

$$\underbrace{V_1 = \left(\begin{array}{c} \left.\begin{array}{c} 0 \\ 0 \end{array}\right\} SO(2) \\ \hline v_1 \end{array}\right), \qquad V_2 = \left(\begin{array}{c} 0 \\ v_3 \\ \hline v_2 \end{array}\right)}_{3\ \text{terms}\ \Rightarrow\ 3\ \text{invariants}}$$

$$\boxed{\mathrm{Tr}\,\Delta^+\Delta\,\Delta^+\Delta = f\left(\mathrm{Tr}\,\Delta^+\Delta,\ \mathrm{Tr}\,\Delta^2,\ \mathrm{Tr}\,\Delta^{+2}\right)}$$

** Prove **

SSB of G_{LR}

$$\langle\Phi\rangle = \begin{pmatrix} v_1 & 0 \\ 0 & v_2 e^{i\theta} \end{pmatrix}$$

$$U_L \langle\Phi\rangle U_R^+$$

but now $\langle\Delta_R\rangle$ = arbitrary

$\Downarrow$

$\boxed{\text{the general Qem is } \underline{\text{not}} \text{ preserved}}$

(1) $\boxed{G_{LR} \xrightarrow[\langle \Delta_R \rangle]{} G_{SM}}$

$$\Delta_R = \begin{pmatrix} \delta_R^{+} & \delta_R^{++} \\ \delta_R^{0} & -\delta_R^{+} \end{pmatrix}$$

since $\langle \Delta_L \rangle = 0 \Rightarrow$

$$V = V(\Delta_R)$$

$$V(\Delta_R) = \boxed{-\mu^2_{\Delta/2} \, Tr \, \Delta_R^\dagger \Delta_R}$$

$$\cancel{- \mu'^2 \, Tr \, \Delta_R^2} \quad (\cancel{B-L})$$

$$\boxed{+ \frac{\lambda}{4} \left(Tr \, \Delta_R^\dagger \Delta_R \right)^2 + \; + \lambda'/4 \, Tr \, \Delta_R^2 \, Tr \, \Delta_R^{\dagger 2}}$$

$$\cancel{+ \, Tr \, \Delta_R^\dagger \Delta_R \, Tr \, \Delta_R^2} \quad (\cancel{B-L})$$

where by rotation:

$$\langle \Delta_R \rangle = \begin{pmatrix} z & iv \\ iv & -z \end{pmatrix}$$

$$\langle \Delta_R^\dagger \rangle = \begin{pmatrix} z^* & -iv \\ -iv & -z^* \end{pmatrix}$$

$$\mathrm{Tr}\, \Delta_R^\dagger \Delta_R = (|z|^2 + r^2)\, 2$$

$$\mathrm{Tr}\, \Delta_R^2 = (z^2 - r^2)\, 2$$

$$\Downarrow$$

$$\bar{V} = -\mu_\Delta^2 (|z|^2 + r^2) + \lambda (|z|^2 + r^2)^2$$

$$+ \lambda' \underbrace{(z^2 - r^2)(z^{*2} - r^2)}_{\lambda' A A^*} \quad \therefore (AA^* > 0)$$

$$f(|z|^2 + r^2) \Rightarrow \boxed{\lambda' \text{ is a judge}}$$

a) $\boxed{\lambda' > 0} \Rightarrow z = r$

b) $\underline{\lambda' < 0} \Rightarrow z = 0$ or $r = 0$

$$z = 0 \Rightarrow \langle \Delta_a \rangle = \begin{pmatrix} 0 & 1 \\ 1 & 0 \end{pmatrix} ri$$

$$r = 0 \Rightarrow \langle \Delta_a \rangle = \begin{pmatrix} 1 & 0 \\ 0 & -1 \end{pmatrix} z$$

b) $\neq$ GOOD

$\Downarrow$

chose a)

$$\boxed{\langle \Delta_a \rangle = \begin{pmatrix} 1 & i \\ i & -1 \end{pmatrix} v \qquad v \in \mathbb{R}}$$

$\boxed{\lambda' > 0}$ $\Uparrow$

global minimum

$\Downarrow$

$$\underbrace{\begin{pmatrix} 1 & i \\ i & -i \end{pmatrix}}_{\text{good}} \propto U_R \begin{pmatrix} 0 & 0 \\ 1 & 0 \end{pmatrix} U_R^+$$

$\boxed{\text{PROVE}}$

find U_R

$\Downarrow$

$$\langle \Delta_R \rangle = \begin{pmatrix} 0 & 0 \\ \vartheta_R & 0 \end{pmatrix}$$ is a global minimum

$$\bar{V} = -\frac{\mu_\Delta^2}{2}\left(\mathrm{Tr}\,\Delta_L^\dagger\Delta_L + L\leftrightarrow R\right)$$

$$+\frac{\lambda}{4}\left[\left(\mathrm{Tr}\,\Delta_L^\dagger\Delta_L\right)^2 + L\leftrightarrow R\right]$$

$$+\frac{\lambda'}{4}\left[\mathrm{Tr}\,\Delta_L^2\,\mathrm{Tr}\,\Delta_L^{\dagger 2} + L\leftrightarrow R\right]$$

$$+\frac{\rho_1}{4}\,\mathrm{Tr}\,\Delta_L^\dagger\Delta_L\,\mathrm{Tr}\,\Delta_R^\dagger\Delta_R$$

$$+\frac{\rho_2}{4}\left(\mathrm{Tr}\,\Delta_L^2\,\mathrm{Tr}\,\Delta_R^{\dagger 2} + L\leftrightarrow R\right)$$

$$\therefore\quad \rho > \lambda \Rightarrow \langle\Delta_L\rangle = 0,$$

$$\langle\Delta_R\rangle \neq 0$$

summary

$$\bar{V} = f(|z|^2 + r^2)$$

$$+ \lambda' \underbrace{A A^*}_{>0}$$

$$A = z^2 - r^2$$

$$\Rightarrow \lambda' > 0 \longrightarrow A = 0$$

$$\Downarrow$$

$$z = r$$

$$\Updownarrow$$

$$\bar{V}_{SM} = \frac{\lambda}{4}\left(\phi^{\dagger}\phi - v^2\right)^2$$

if $\lambda > 0 \Rightarrow \langle \phi^{+} \phi \rangle = v^2$