


LR SM: $S S' B$

• $G_{LR} = SU(2)_L \times SU(2)_R \times U(1)_{B-L}$

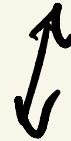
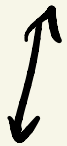
• $\begin{pmatrix} u \\ d \end{pmatrix}_L \longleftrightarrow \begin{pmatrix} u \\ d \end{pmatrix}_R$

$\begin{pmatrix} \nu \\ e \end{pmatrix}_L \longleftrightarrow \begin{pmatrix} \nu \\ e \end{pmatrix}_R$

$\Phi(SM) \subseteq \Phi(LR)$

SM doublet

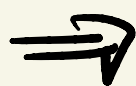
LR bi-doublet



$\Phi \rightarrow U_L \Phi$

$\Phi \rightarrow U_L \Phi U_R^\dagger$

$\langle \Phi \rangle \simeq M_{WL}$



$\langle \Phi \rangle \simeq M_{WL}$

$$\phi = \begin{pmatrix} \phi^0 \\ \phi^- \end{pmatrix} =$$

$$y = -1$$

(ϕ_1, ϕ_2)

$$\Phi = \begin{pmatrix} \phi_1^0 & \phi_2^+ \\ \phi_1^- & -\phi_2^{0*} \end{pmatrix}$$

$$\tilde{\phi}_2 = i\sigma_2 \phi_2 = \begin{pmatrix} \phi_2^+ \\ -\phi_2^{0*} \end{pmatrix}$$

($\tilde{\phi} \rightarrow U_L \tilde{\phi}$)

both are
SM doublets

$$M_{W_L} = \frac{g}{2} \langle \phi \rangle$$

$$M_Z = \frac{M_{W_L}}{\cos \theta_W}$$

$$e = g \sin \theta_W$$

$$(D_\mu \phi)^\dagger (D^\mu \phi)$$

$$M_{W_L}^2 = \frac{g^2}{4} [\langle \phi_1 \rangle^2 + \langle \phi_2 \rangle^2]$$

$$M_Z = \frac{M_{W_L}}{\cos \theta_W}$$

$$(D_\mu \phi_i)^\dagger (D^\mu \phi_i)$$

$(i=1,2)$

$$V = V(\Phi)$$

$$\Phi \Rightarrow \tilde{\Phi} = \sigma_2 \phi^* \sigma_2$$

$$\Downarrow$$
$$\boxed{\tilde{\Phi} \rightarrow U_L \tilde{\Phi} U_R^\dagger}$$

invariant

$$\Phi_a \equiv (\bar{\Phi}, \tilde{\Phi}) \quad (a=1,2)$$

$$\bar{\Phi}_a \rightarrow U_L \bar{\Phi}_a U_R^\dagger$$

$$\Phi_a^\dagger \rightarrow U_R \Phi_a^\dagger U_L^\dagger$$

$$\Rightarrow \Phi_a^\dagger \bar{\Phi}_b \rightarrow U_R \Phi_a^\dagger \bar{\Phi}_b U_R^\dagger$$

$$\begin{array}{l} \Rightarrow \\ \textcircled{1} \end{array} \left. \begin{array}{l} \text{Tr } \Phi_a^\dagger \bar{\Phi}_b \rightarrow \text{Tr } U_R^\dagger U_R \Phi_a^\dagger \bar{\Phi}_b \\ \\ \text{inv.} \end{array} \right\} = \text{Tr } \Phi_a^\dagger \bar{\Phi}_b$$

$$\textcircled{2} \quad \underbrace{\text{Tr } \Phi_a^\dagger \bar{\Phi}_b \bar{\Phi}_c^\dagger \bar{\Phi}_d}_{\text{inv.}}$$

$$\textcircled{3} \quad \Phi_a \rightarrow U_L \Phi_a U_R^\dagger$$

$$\det U_{L,R} = 1$$

\Downarrow

$$\det \bar{\Phi}_a \rightarrow \det U_L \det U_R^T \times \det \bar{\Phi} \\ = \det \bar{\Phi}$$

$$\boxed{\det \bar{\Phi}_a = \text{quadratic}} \quad (\text{INV})$$

but

How many independent?

• (SM) $\phi \rightarrow U_L \phi$

one inv = $\phi^\dagger \phi$

~~$\phi^\dagger \sigma_2 \phi, \dots$~~

$$U_L \phi = \begin{pmatrix} 0 \\ \varphi \end{pmatrix} \quad \underline{\text{one invariant}}$$

$$\cdot (LR) \Phi \rightarrow U_L \Phi U_R^\dagger$$

$$\begin{pmatrix} \varphi_1 & 0 \\ 0 & \varphi_2 \end{pmatrix}$$

$$\underbrace{\begin{pmatrix} e^{i\alpha_L} & 0 \\ 0 & e^{-i\alpha_L} \end{pmatrix}}_{SU(2)_L} \begin{pmatrix} \varphi_1 & 0 \\ 0 & \varphi_2 \end{pmatrix} \underbrace{\begin{pmatrix} e^{-i\alpha_R} & 0 \\ 0 & e^{i\alpha_R} \end{pmatrix}}_{SU(2)_R}$$



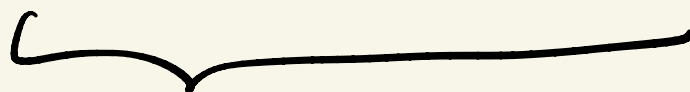
$$= \begin{pmatrix} e^{i(\alpha_L - \alpha_R)} \psi_1 & 0 \\ 0 & e^{-i(\alpha_L - \alpha_R)} \psi_2 \end{pmatrix}$$

$$\psi_1 = r_1 e^{i\theta_1} \dots$$

$$\alpha_L - \alpha_R + \theta_1 = 0$$



$$\phi \rightarrow \begin{pmatrix} r_1 & 0 \\ 0 & r_2 e^{i\theta_2} \end{pmatrix}$$



3 real comp.



3 invariants

$$\begin{array}{ccc} \text{Tr } \Phi^+ \Phi, & \det \Phi, & \det \Phi^+ \\ \parallel & \parallel & \parallel \\ (r_1^2 + r_2^2) & r_1 r_2 e^{i\theta} & r_1 r_2 e^{-i\theta} \end{array}$$

$$\text{Tr } \Phi^+ \tilde{\Phi} \propto \det \Phi$$

PROVE

$$\text{Tr } \Phi^+ \Phi \Phi^+ \Phi \propto \text{what?}$$

$$\text{Tr } \Phi^+ \tilde{\Phi} \Phi^+ \tilde{\Phi} \propto \text{---?}$$

• SM digression

$$(\phi^\dagger \vec{\sigma} \phi) (\phi^\dagger \vec{\sigma} \phi) \propto (\phi^\dagger \phi)$$

$$(\phi^\dagger i \sigma_2 \vec{\sigma} \phi) (\phi^\dagger i \sigma_2 \vec{\sigma} \phi^*) \propto (\phi^\dagger \phi)$$

Bottom line

$$-V(\Phi) = -V(\Phi^\dagger \Phi, \det \Phi, \det \Phi^\dagger)$$

SSB \Downarrow

$$(-\mu^2 \bar{1}_\nu \Phi^\dagger \Phi - \mu'^2 \det \Phi + \text{h.c.})$$

$$\Downarrow$$

$$\langle \Phi \rangle = \begin{pmatrix} v_1 & 0 \\ 0 & v_2 e^{ia} \end{pmatrix}$$

$$= \begin{pmatrix} \langle \phi_1^0 \rangle & 0 \\ 0 & \langle \phi_2^0 \rangle \end{pmatrix}$$

\Downarrow

$$Q_{em} \langle \Phi \rangle = 0$$

at this stage :

SSB \rightarrow charge
conservation

4

$$V = V(a) =$$

$$= A + B \cos a + C \cos 2a$$

Minima:

$$a = 0 \quad (\text{conditions?})$$

$$a \neq 0 \quad (\pi -)$$

To be discussed

$$G_{LR} \longrightarrow G_{SM}$$

$$\langle \Delta_R \rangle$$



$$\left\{ \begin{array}{l} SU(2)_R \text{ triplet (adjoint)} \\ (B-L) \Delta_R = 2 \Delta_R \end{array} \right.$$

$$\Downarrow LR$$

$$(\Delta_L, \Delta_R)$$

Invariants?

$$\Delta = SU(2) \text{ adjoint, } Y' = 2$$

Complex

$$\Delta = \Delta_1 + i \Delta_2$$

"real" = Hermitian objects

$$\left\{ \begin{array}{l} \Delta_i \rightarrow U \Delta_i U^\dagger \quad i=1,2 \\ \text{Tr } \Delta_i = 0, \quad \Delta_i^\dagger = \Delta_i \end{array} \right.$$

$$\text{Tr } \Delta_i^2 \rightarrow \text{Tr } \Delta_i \Delta_j$$

$$\rightarrow \text{Tr } \Delta_i \Delta_j \Delta_u \Delta_e \quad (?)$$

\rightarrow count independent inv.

$$\Delta_1 \rightarrow \begin{pmatrix} a & 0 \\ 0 & -a \end{pmatrix}, \quad a \in \mathbb{R}$$

$$\Delta_1 \rightarrow U \Delta_1 U^\dagger$$

with:

$$\Delta_2 = \begin{pmatrix} b & r e^{i\theta} \\ r e^{-i\theta} & -b \end{pmatrix}, \quad b \in \mathbb{R}$$

but, after $\Delta_1 = \begin{pmatrix} a & 0 \\ 0 & -a \end{pmatrix}$

let $U = \begin{pmatrix} e^{i\alpha} & 0 \\ 0 & e^{-i\alpha} \end{pmatrix}$

$$\Delta_1 \rightarrow U \Delta_1 U^\dagger = \Delta_1 \quad (\text{inv.})$$

(symmetry in T_3 direction)

$$SU(2) \simeq SO(3)$$

$$\hookrightarrow U(1) = SO(2)$$

$$\Delta_2 \rightarrow \begin{pmatrix} e^{i\alpha} & 0 \\ 0 & e^{-i\alpha} \end{pmatrix} \begin{pmatrix} b & r e^{i\theta} \\ r e^{-i\theta} & -b \end{pmatrix} \begin{pmatrix} e^{-i\alpha} & 0 \\ 0 & e^{i\alpha} \end{pmatrix}$$

$$= \begin{pmatrix} e^{i\alpha} b & r e^{i(\alpha+\theta)} \\ r e^{-i(\alpha+\theta)} & -b e^{-i\alpha} \end{pmatrix} \begin{pmatrix} e^{-i\alpha} & 0 \\ 0 & e^{i\alpha} \end{pmatrix}$$

$$= \begin{pmatrix} b & r e^{i(2\alpha+\theta)} \\ r e^{-i(2\alpha+\theta)} & -b \end{pmatrix}$$

$$2\alpha + \theta \quad \therefore$$

$$\Delta_2 = \begin{pmatrix} b & r \\ r & -b \end{pmatrix}$$

$$\Delta = \begin{pmatrix} a+ib & ir \\ ir & -a-ib \end{pmatrix}$$

$$\Delta = \begin{pmatrix} z & ir \\ ir & -z \end{pmatrix}$$

3 invariants

$$\underbrace{\text{Tr } \Delta^+ \Delta,}_{\text{SU(2) x U(1) inv.}}$$

$$\underbrace{\text{Tr } \Delta^2, (\text{Tr } \Delta^2)^*}_{= \text{Tr } \Delta^{+2}} = \text{Tr } \Delta^{+2}$$

3 invariants

vector language

$$V_1 = \begin{pmatrix} 0 \\ 0 \\ v_1 \end{pmatrix} \Big\} \text{so(2)} \quad V_2 = \begin{pmatrix} 0 \\ v_3 \\ v_2 \end{pmatrix}$$

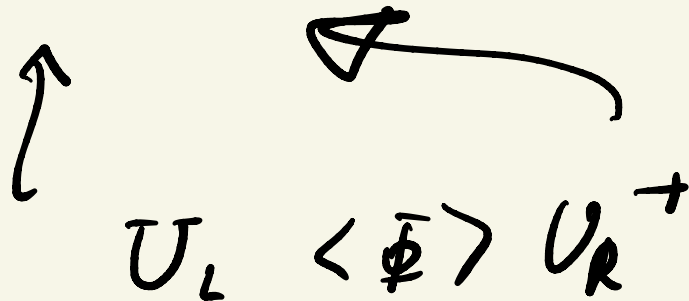
3 terms \Rightarrow 3 invariants

$$T_1 \Delta^+ \Delta \Delta^+ \Delta = f(T_1 \Delta^+ \Delta, T_1 \Delta^2, T_1 \Delta^{+2})$$

* * Prove * *

S S R of G L R

$$\langle \Phi \rangle = \begin{pmatrix} v_1 & 0 \\ 0 & v_2 e^{i\theta} \end{pmatrix}$$

$$U_L \langle \Phi \rangle U_R^+$$


but now $\langle \Delta_R \rangle = \text{arbitrary}$

⇓

In general Qem is
not preserved

①

$$G_{LR} \xrightarrow{\langle \Delta_R \rangle} G_{SM}$$

$$\Delta_R = \begin{pmatrix} \delta_R^+ & \delta_R^{++} \\ \delta_R^0 & -\delta_R^+ \end{pmatrix}$$

Since $\langle \Delta_L \rangle = 0 \Rightarrow$

$$V = V(\Delta_R)$$

$$V(\Delta_R) = \boxed{-\mu_{\Delta/2}^2 \text{Tr} \Delta_R^\dagger \Delta_R}$$

~~$$-\mu'^2 \text{Tr} \Delta_R^2 \quad (\text{B-t})$$~~

$$+ \boxed{\frac{\lambda}{4} (\text{Tr} \Delta_R^\dagger \Delta_R)^2} +$$

$$+ \lambda' \frac{1}{4} \text{Tr} \Delta_R^2 \text{Tr} \Delta_R^{\dagger 2}$$

~~$$+ \text{Tr} \Delta_R^\dagger \Delta_R \overline{\text{Tr} \Delta_R^2} \quad (\text{B-t})$$~~

where by rotation:

$$\langle \Delta_R \rangle = \begin{pmatrix} z & iv \\ iv & -z \end{pmatrix}$$

$$\langle \Delta_R^\dagger \rangle = \begin{pmatrix} z^* & -iv \\ -iv & -z^* \end{pmatrix}$$

$$\bar{T}_1 \Delta_R^T \Delta_R = (|z|^2 + v^2) z$$

$$\bar{T}_v \Delta_R^2 = (z^2 - v^2) z$$

$$\bar{V} = -\mu_0^2 (|z|^2 + v^2) + \lambda (|z|^2 + v^2)^2$$

$$+ \lambda' (z^2 - v^2) (z^{*2} - v^2)$$

$$\lambda' A A^* \therefore (A A^* > 0)$$

$$f(|z|^2 + v^2) \Rightarrow \lambda' \text{ is a judge}$$

$$a) \lambda' > 0 \Rightarrow z = v$$

$$b) \underline{\lambda' < 0} \Rightarrow z=0 \text{ or } r=0$$

$$z=0 \Rightarrow \langle \Delta_a \rangle = \begin{pmatrix} 0 & 1 \\ 1 & 0 \end{pmatrix} v i$$

$$r=0 \Rightarrow \langle \Delta_a \rangle = \begin{pmatrix} 1 & 0 \\ 0 & -1 \end{pmatrix} z$$

$$b) \neq 6000$$



choose a)

$$\langle \Delta_a \rangle = \begin{pmatrix} 1 & i \\ i & -1 \end{pmatrix} v$$

$v \in \mathbb{R}$

$$\boxed{\lambda' > 0}$$

global
minimum

$$\begin{pmatrix} 1 & i \\ i & -i \end{pmatrix}$$

\propto

$$U_R \begin{pmatrix} 0 & 0 \\ 1 & 0 \end{pmatrix} U_R^+$$

good

PROVE

find U_R

$\langle A_R \rangle = \begin{pmatrix} 0 & 0 \\ U_R & 0 \end{pmatrix}$ is a global minimum

$$\begin{aligned}
\bar{V} = & -\frac{\mu_0^2}{2} (\text{Tr } \Delta_L^\dagger \Delta_L + L \leftrightarrow R) \\
& + \frac{\lambda}{4} [(\text{Tr } \Delta_L^\dagger \Delta_L)^2 + L \leftrightarrow R] \\
& + \frac{\lambda'}{4} [\text{Tr } \Delta_L^2 \text{Tr } \Delta_L^{\dagger 2} + L \leftrightarrow R] \\
& + \frac{p_1}{4} \text{Tr } \Delta_L^\dagger \Delta_L \text{Tr } \Delta_R^\dagger \Delta_R \\
& + \frac{p_2}{4} (\text{Tr } \Delta_L^2 \text{Tr } \Delta_R^{\dagger 2} + L \leftrightarrow R)
\end{aligned}$$

$$\begin{aligned}
\therefore p > \lambda & \Rightarrow \langle \Delta_L \rangle = 0, \\
& \langle \Delta_R \rangle \neq 0
\end{aligned}$$

summe

$$\bar{V} = f(|z|^2 + v^2)$$

$$+ \lambda' \underbrace{A A^*}_{> 0}$$

$$A = z^2 - v^2$$

$$\Rightarrow \lambda' < 0 \longrightarrow A = 0$$

$$\Downarrow$$
$$z = v$$



$$V_{JM} = \frac{\lambda}{4} (\phi + \phi - v^2)^2$$

$$\text{if } \lambda > 0 \Rightarrow \langle \phi^+ \phi \rangle = \alpha^2$$