Neutrino Physics Course

Lecture X

3/6/2021

LMU

Summer 2022

Left-Right Symmetric Model

(LRSM)

- Imagine LR symm. world : P

$$q_L \equiv \begin{pmatrix} u \\ d \end{pmatrix}_L \longleftrightarrow \begin{pmatrix} u \\ d \end{pmatrix}_R \equiv q_R$$

$$\ell_L \equiv \begin{pmatrix} \nu \\ e \end{pmatrix}_L \longleftrightarrow \begin{pmatrix} \nu \\ e \end{pmatrix}_R \equiv \ell_R$$

$$\Rightarrow \boxed{m_\nu \neq 0}$$

but

$$(f = q, \ l)$$

$$\mathcal{L}_Y = \bar{f}_L \left(-M_f + y_T T \right) f_R + h.c.$$

$$f_{L,R} \longrightarrow U f_{L,R}$$

$\hookrightarrow SU(2)$

$$\Rightarrow \quad T \to U T U^+$$

$\rightarrow$ adjoint scalar (triplet)

$$T = T^+, \quad \mathrm{Tr}\, T = 0$$

$$v_T \equiv \langle T \rangle \neq 0$$

$$\Rightarrow \quad M_Z = 0, \; M_W = g\, v_T$$

<u>all hell breaks loose!</u>

<u>What if P = conserved?</u>

$$\Downarrow$$

$$g \equiv g_L = g_R$$

$$G_{LR} = SU(2)_L \times SU(2)_R \quad ?$$

$$\Downarrow$$

NOT enough

$$Q_{em} = T_{3L} + T_{3R} \quad ?$$

$$\begin{pmatrix} u \\ d \end{pmatrix}_L \quad \| \quad \begin{pmatrix} u \\ d \end{pmatrix}_R$$

matter

$$\underbrace{\begin{pmatrix} \nu \\ e \end{pmatrix}_L}_{\vec{W}_L} \quad \| \quad \underbrace{\begin{pmatrix} \nu \\ e \end{pmatrix}_R}_{\vec{W}_R}$$

$\boxed{\text{NO masses}}$

$$\boxed{M_{W_R} \gg M_{W_L}}$$

Again:

$$Q = \underbrace{T_{3L} + T_{3R}}_{\text{all multiplets (same)}} \quad ?$$

$\longrightarrow$ same prediction

$$\Downarrow$$

- $Q_\nu = Q_u \,, \quad Q_e = Q_d$

- $Q = \pm 1/2$

$$\Uparrow$$

$$\vec{T}_L f_R = \vec{T}_R f_L = 0$$

$$T_{3L} f_L = T_{3R} f_R = \pm 1/2$$

$$\Downarrow$$

$$Q = T_{3L} + T_{3R} + \frac{Y'}{2}$$

better: $\frac{Y'}{2} = Q - T_{3L} - T_{3R}$

$\uparrow$ exp $\qquad$ $\uparrow$ theory

$\Updownarrow$ $\qquad$ gauge group

$$\boxed{\begin{array}{llll} G_{LR} = & SU(2)_L \times & SU(2)_R \times & U(1)_{\bar{Y}} \\ g \equiv g_L & = & g_R & \bar{g} \end{array}}$$

$$\# \text{ of g.b.} = \underset{(L)}{3} + \underset{(R)}{3} + \underset{(1)}{1}$$

$$\Downarrow$$

$$\boxed{\begin{array}{c} W_L^+,\ W_L^-,\ \underset{(Z)}{Z_L} \ + \ W_R^+,\ W_R^-,\ \underset{(Z')}{Z_R} \\[1em] + \\[1em] A \ (\text{photon}) \end{array}}$$

$$\Downarrow$$

$$\underbrace{M_{W_R} \simeq M_{Z_R}}_{M_R} \gg \underbrace{M_{W_L} \simeq M_{Z_L}}_{M_L \equiv M_W}$$

Pati, Salam 1974

Mohapatra, G.S. – 1975

modern: 1979

$\boxed{m_\nu \neq 0}$ CURSE '70s

$\boxed{\text{Higgsing}}$

(i) S.M. Higgs ϕ (doublet)

= where?

$$\mathcal{L}_Y = \bar{f}_L \; Y \; \Phi \; f_R \quad + \text{h.c.}$$

$$f_L \to U_L f_L, \quad f_R \to U_R f_R$$

$$\left(U_L = e^{i\vec{\theta}_L \cdot \vec{T}}, \quad U_R = e^{i\vec{\theta}_R \cdot \vec{T}} \right.$$

$$\left. \vec{T} \equiv \vec{\sigma}/2 \right)$$

$$\Downarrow$$

$$\boxed{\Phi \longrightarrow U_L \, \Phi \, U_R^{\dagger}}$$

$$\Downarrow$$

Φ = matrix

(bi-doublet)

(SM Higgs: $\phi \longrightarrow U \phi$

$$T_i \phi = \frac{\sigma_i}{2} \phi$$

$$U = e^{i T_i \theta_i} = 1 + i T_i \theta_i + \dots \Big)$$

$$\Phi \to U_L \, \Phi \, U_R^{\dagger}$$

$$U_L = 1 + i \vec{\theta}_L \cdot \vec{T} + \dots$$

$$U_R = 1 + i \vec{\theta}_R \cdot \vec{T} + \dots$$

$$\boxed{\hat{T} \Phi = \frac{\vec{\sigma}}{2} \Phi - \Phi \frac{\vec{\sigma}}{2}}$$

- $Q = T_{3L} + T_{3R} + \frac{\bar{Y}}{2}$

  $\bar{Y} = ?$

$f_L : \quad T_{3R} f_L = 0$

$$\frac{\bar{Y}}{2} f_L = \underbrace{(Q - T_{3L})}\, f_L$$

$$= Y'/2 \; f_L$$

$$\Rightarrow \bar{Y} f_L = Y' f_L$$

SM hypercharge

$\therefore \quad Y' \ell_L = (-1)\, \ell_L$

$Y' q_L = \frac{1}{3} q_L \quad \Rightarrow \quad \boxed{Y' = B - L}$

$B q = \frac{1}{3} q, \quad L q = 0$

$B \ell = 0, \quad L \ell = \ell$

$$L \longleftrightarrow R$$

$$\Rightarrow \boxed{\bar{Y} f_R = (B-L) f_R}$$

$$\Downarrow$$

$$\boxed{\bar{Y} = B - L} \quad \boxed{\text{on } L \text{ and } R}$$

exactly the global symmetry of SM

$$\Downarrow$$

gauge symmetry in LR

$$\Downarrow$$

$$\boxed{Q = T_{3L} + T_{3R} + \frac{B-L}{2}}$$

$$\mathcal{L}_Y = \bar{f}_L \, y \, \Phi \, f_R + h.c.$$

Q. $(B-L)\,\Phi = \,?$

$$\boxed{\text{A. } (B-L)\,\Phi = 0}$$

$\Downarrow$

$$Q\,\Phi = T_{3L}\,\Phi - \Phi\,T_{3R}$$

$$= \left(\frac{\sigma_3}{2}\,\Phi - \Phi\,\frac{\sigma_3}{2}\right)$$

$$Q \equiv Q_{em}$$

$$\Phi = \begin{pmatrix} a & b \\ c & d \end{pmatrix}$$

$$Q\, a = 0, \quad Q\, d = 0$$

$$Q\, c = -c, \quad Q\, b = +1$$

$$\Downarrow$$

$$\boxed{\Phi = \begin{pmatrix} \varphi^0 & \varphi^+ \\ \varphi^- & -\varphi_0^* \end{pmatrix}} \quad (\text{minimal})$$

S_M Higgs

$$\phi_G = \begin{pmatrix} \varphi^+ \\ \varphi^0 \end{pmatrix} \overset{\sim}{\longleftrightarrow} \boxed{\begin{pmatrix} \varphi^0 \\ \varphi^- \end{pmatrix}} = \phi_D$$

$$\Downarrow$$

$$\tilde{\phi}_G = i\sigma_2 \phi^* = \begin{pmatrix} 0 & 1 \\ -1 & 0 \end{pmatrix} \begin{pmatrix} \varphi^- \\ \phi_0^* \end{pmatrix}$$

$$= \begin{pmatrix} \phi_0^* \\ -\varphi^- \end{pmatrix}$$

$$\tilde{\phi}_D \equiv i\sigma_2 \, \phi_D^* = \begin{pmatrix} \varphi^+ \\ -\varphi_0^* \end{pmatrix}$$

$$\Rightarrow \quad \Phi^+ = \begin{pmatrix} \varphi_0^* & \varphi^+ \\ \varphi^- & -\varphi^0 \end{pmatrix}$$

$$\Phi = \begin{pmatrix} \varphi^0 & \varphi^+ \\ \varphi^- & -\varphi_0^* \end{pmatrix} \leftarrow$$

$$\boxed{\begin{aligned} \mathrm{Tr}\, \Phi^+ \Phi &= \left(|\varphi_0|^2 + \varphi^+ \varphi^- \right) 2 \\ &= 2\, \phi^+ \phi \end{aligned}}$$

$$\Downarrow$$

$\phi^\dagger \phi$ = SM invariant

$$V_{SM} = f(\phi^\dagger \phi) = f(\mathrm{Tr}\, \Phi^\dagger \Phi)$$

$$\Phi \to \underbrace{U_L \, \Phi \, U_R^\dagger}_{\boxed{SU(2)_L \times SU(2)_R}}$$

$$\parallel$$

?

$$\phi = \begin{pmatrix} \phi^0 \\ \phi^- \end{pmatrix} = \begin{pmatrix} R_1 + i R_2 \\ R_3 + i R_4 \end{pmatrix}$$

$$i = 1, \ldots, 4 : R_i \in \mathbb{R}$$

$$\phi^+\phi = \underbrace{R_1^2 + R_2^2 + R_3^2 + R_4^2}_{SO(4)}$$

$$\Downarrow$$

$$\boxed{SU(2) \times SU(2) = SO(4)}$$

$r = 1 + 1 = 2$ $\qquad\qquad$ $r = 2$

\# gen. $= 3 + 3 = 6$ $\qquad\qquad$ $\frac{4 \cdot 3}{2} = 6$

$$\mathcal{L}_Y = \bar{f}_L \, Y \, \Phi \, f_R + h.c.$$

$$\Big\downarrow \langle \Phi \rangle$$

masses

<u>but:</u> $\langle \Phi \rangle = \begin{pmatrix} v & 0 \\ 0 & -v \end{pmatrix}$

$\Downarrow$

$$\mathcal{L}_{mass} = \begin{pmatrix} \bar{u}_L & \bar{d}_L \end{pmatrix} y \begin{pmatrix} v & 0 \\ 0 & -v \end{pmatrix} \begin{pmatrix} u_R \\ d_R \end{pmatrix} + h.c.$$

$$= \left(\bar{u}_L u_R - \bar{d}_L d_R \right) y v + h.c.$$

$\Downarrow$

$$\boxed{m_u = -m_d}$$ $\boxed{\text{BAD}}$

$\Downarrow$

$$\Phi = \begin{pmatrix} \varphi_1^0 & \varphi_2^+ \\ \varphi_1^- & -\varphi_2^{0*} \end{pmatrix}$$

$$= \begin{pmatrix} \phi_1 & \tilde{\phi}_2 \end{pmatrix}_D$$

$$= \begin{pmatrix} \tilde{\phi}_2 & \phi_2 \end{pmatrix}_G$$

$$\Downarrow$$

$$\mathcal{L}_Y = \bar{f}_L \, Y \, \Phi \, f_R + h.c.$$

$$\langle \Phi \rangle = \begin{pmatrix} v_1 & 0 \\ 0 & -v_2 \end{pmatrix}$$

$$\Downarrow$$

$$\boxed{\begin{array}{c} \underline{M}_u = Y v_1 \, ; \quad M_d = -Y v_2 \\ \text{BAD} \quad M_u \propto M_d \end{array}} \qquad (1)$$

<u>SM</u> $\underline{M}_u = Y_u v \, ; \quad M_d = Y_d v$

good

- From (1) $\Rightarrow$ $\boxed{M_u \propto M_d}$

$m_u \simeq 3$ MeV $\qquad m_d \simeq 5$ MeV

$m_c \simeq 1.5$ GeV $\qquad m_s \simeq 100$ MeV

$m_t \simeq 175$ GeV $\qquad m_b \simeq 5$ GeV

$\oplus \quad V_{CKM} = U_{uL}^{\dagger} U_{dL} \neq 1\,!$

$$M_q \to U_{Lq} M_q U_{Rq}^{\dagger} = m_q$$

$$\boxed{\begin{array}{l} \underline{\text{but:}} \quad \text{if } \quad M_u \propto M_d \\ \qquad\qquad\qquad \Downarrow \\ \qquad\qquad U_u = U_d \Rightarrow V_{CKM} = 1 \end{array}}$$

SM

$$Y_d \neq Y_u$$

$$\mathcal{L}_Y^e = \bar{q}_L Y_d \phi_G d_R + $$

$$+ \bar{q}_L Y_u \underbrace{i\sigma_2 \phi^*}_{\tilde{\phi}_G} u_R + h.c.$$

$\uparrow$

because : · if $e \Rightarrow \bar{e}$

· if $\phi \Rightarrow \tilde{\phi}$

$\Downarrow$

$$\boxed{\text{if } \bar{\Phi} \Rightarrow \tilde{\bar{\Phi}} \; !}$$

· $\phi \to U\phi$, $\tilde{\phi} \to U\tilde{\phi}$

$\Downarrow$

SM $\qquad \tilde{\phi} = i\sigma_2 \phi^*$

$\Downarrow$

$$\boxed{\begin{gathered}\tilde{\Phi} = \sigma_2 \Phi^* \sigma_2 \; \therefore \\ \Phi \to U_L \Phi U_R \Longrightarrow \\ \tilde{\Phi} \to U_L \tilde{\Phi} U_R \\ \text{PROVE!}\end{gathered}}$$

$\Downarrow$

$$\mathcal{L}_Y = \bar{f}_L \left(Y \Phi f_R + \tilde{Y} \tilde{\Phi} f_R \right) + h.c.$$

LRSM: $\quad Y, \tilde{Y}$

SM: $\quad Y_u, Y_d$

Summary:

- $\phi \subseteq \Phi \quad (\Phi \to U_L \Phi U_R^\dagger)$

  same Yukawa predictions

- $Q = T_{3L} + T_{3R} + \frac{B-L}{2}$

but:

$$\boxed{G_{LR} \xrightarrow[\langle \text{new Higgs} \rangle]{} G_{SM}}$$

$$SU(2)_L \times SU(2)_R \times U(1)_{B-L} \xrightarrow{\langle \text{new Higgs} \rangle} SU(2)_L \times U(1)_Y$$

$$SU(2)_R \times U(1)_{B-L}$$

$\downarrow$ (new higgs)

$$U(1)_Y$$

$$\Downarrow$$

$$\boxed{\Delta_L \longleftrightarrow \Delta_R}$$

$\therefore$

$$\langle \Delta_L \rangle = 0, \quad \langle \Delta_R \rangle = v_R \neq 0$$

$$\boxed{M_R \gg M_L \ (M_W)}$$

. at M_R ignore Φ

- $SU(2)_L \times U(1)_Y \longrightarrow U(1)_{em}$

$$\langle \Phi \rangle$$

$$\Phi = \begin{pmatrix} \phi_1 & \tilde{\phi}_2 \end{pmatrix}$$

$$\langle \phi_i \rangle = v_i$$

$$\Downarrow$$

$$\boxed{M_{W_L}^2 = \frac{g^2}{4}\left(v_1^2 + v_2^2\right)}$$

$\underline{\cancel{CP}\ ?}$

$$V_{CKM} \in \mathbb{C} \Rightarrow \cancel{CP}$$

$$\|$$

$$U_{Lu}^{+} U_{Ld} \leftarrow M \rightarrow \text{diag.}$$

$$M_f = y_f v$$

Source of CP