Neutrino Physics Course

Lecture VII

20/2/2022

LMU
Spring 2022

SM : neutrino mass

$$l_L \equiv \begin{pmatrix} \nu \\ e \end{pmatrix}_L \qquad e_R \, , \quad \cancel{\nu_R}$$

$$\Downarrow$$

$$m_\nu = 0$$

$$\Uparrow$$

$$\mathcal{L}_Y = \bar{l}_L \, y_e \, \Phi \, e_R + h.c.$$

$$\Downarrow \quad \Phi = \begin{pmatrix} 0 \\ \phi_0 \end{pmatrix}$$

$$y_e \, \bar{e}_L \, e_R \, \phi_0$$

$$\phi_0 = \frac{v+h}{\sqrt{2}}$$

$$\text{electron mass} = \underbrace{\bar{e}_L e_R + h.c.}_{\text{Dirac mass term}}$$

$$\Downarrow$$

$$\boxed{m_\nu^D = 0}$$

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neutrino = neutral

$\Downarrow$ (?)

Majorana mass term

$\Downarrow$

$$\boxed{\nu_L^T \, C \, \nu_L = \text{Lorentz invariant} \quad {\color{red}\Delta L = 2}}$$

$\psi \longrightarrow \Lambda \psi$ (Lorentz spinor)

$\Downarrow$

$\psi^c \longrightarrow \Lambda \psi^c$ (—〃—)

$$\therefore \ \psi^c \equiv C \bar{\psi}^T \equiv C \gamma_0 \psi^*$$

$\uparrow$

Dirac charge conjugation

$$\therefore \ C \gamma_\mu C^T = -\gamma_\mu^T$$

$$C^T = -C$$

$\Downarrow$

$$\boxed{C = i\gamma_2\gamma_0} \quad \text{(ours)}$$

$$\gamma_2 = \begin{pmatrix} 0 & \sigma_2 \\ -\sigma_2 & 0 \end{pmatrix}, \quad \gamma^0 = \begin{pmatrix} 0 & 1 \\ 1 & 0 \end{pmatrix}$$

$$\Rightarrow \boxed{C = \begin{pmatrix} i\sigma_2 & 0 \\ 0 & -i\sigma_2 \end{pmatrix}}$$

$$\text{and} \quad \gamma_5 = \begin{pmatrix} I & 0 \\ 0 & -I \end{pmatrix}$$

$$\Downarrow$$

$$\nu_L = \begin{pmatrix} u_L \\ 0 \end{pmatrix}$$

$$\Downarrow$$

$$\boxed{\Delta L = 2}$$

$$\nu_L^T C \nu_L = u_L^T i\sigma_2 u_L \quad \text{(2 comp.)}$$

$\underbrace{\qquad\qquad}_{\text{Inv.}}$ $\qquad$ $\underbrace{\qquad\qquad}_{SU(2)\text{ inv.}}$

recall: $u_L \longrightarrow e^{i \vec{\sigma}/2 \cdot (\vec{\theta} + i \vec{\chi})}$

$\vec{\theta}$: ROT $\qquad$ $\vec{\chi}$: BOOST

· in short:

$$m_\nu^M \left(\nu_L^T C \nu_L + h.c. \right)$$

$$\nu_M = \nu_L + C \bar{\nu}_L^T \quad (4\text{ comp.})$$

$$= \nu_L + C \gamma_0 \nu_L^*$$

$$= \nu_L + i \gamma_2 \nu_L^*$$

$$= \begin{pmatrix} u_L \\ 0 \end{pmatrix} + \begin{pmatrix} 0 & i\sigma_2 \\ -i\sigma_2 & 0 \end{pmatrix} \begin{pmatrix} u_L^* \\ 0 \end{pmatrix}$$

$$\Rightarrow \quad \boxed{\nu_M = \begin{pmatrix} u_L \\ -i\sigma_2 u_L^* \end{pmatrix}} \qquad \boxed{\Delta L = 2}$$

$$u_R \equiv -i\sigma_2 u_L^*$$

$$\Rightarrow \quad \boxed{\bar{\nu}_M \nu_M = \nu_L^T C \nu_L + h.c.} \qquad (1)$$

Majorana mass

$$\boxed{\bullet\ \bar{\nu}_M \gamma^\mu \partial_\mu \nu_M = 2 \bar{\nu}_L \gamma^\mu \partial_\mu \nu_L} \qquad (2)$$

PROVE

$$\mathcal{L}_\nu = i\,\bar{\nu}_L \gamma^\mu \partial_\mu \nu_L - \frac{1}{2} m_\nu^M \nu_L^T C \nu_L$$

$$+ h.c.$$

$$= \frac{1}{2}\left[i\,\bar{\nu}_M \gamma^\mu \partial_\mu \nu_M - m_\nu^M \bar{\nu}_M \nu_M \right]$$

$$\Downarrow$$

$$\not{p}\,\nu_M = m_\nu^M \nu_M$$

$$\Downarrow$$

$$\boxed{p^2 = (m_\nu^M)^2}$$

$$\mathcal{L}_e = i\,\bar{e}\,\gamma^\mu \partial_\mu - m_e\,\bar{e}\,e$$

$$e = e_L + e_R = \begin{pmatrix} u_L^e \\ u_R^e \end{pmatrix}$$

$$\Rightarrow e \to e^{i\alpha} e \Rightarrow$$

$$\partial_\mu j^\mu_{em} = 0$$

Q. if $\nu_L^T C \nu_L =$ inv.

$\Rightarrow$ why don't we write it?

A. SU(2) : $l = \begin{pmatrix} \nu \\ e \end{pmatrix}_L$

$\Rightarrow$ $\boxed{\text{SU(2) forbids such a term}}$

$$\nu_L^T \; C \; \nu_L \quad \Leftarrow \quad SU(2)$$

$$T_3: \frac{1}{2} + \frac{1}{2} = 1 \neq 0$$

$$\boxed{\text{NOT inv.}}$$

- <u>electron</u> : $\overline{e_L}\, e_R\, \phi^0$

$$T_3: +\frac{1}{2} + 0 + (-1/2) = 0$$

- neutrino

$$\nu_L^T \; C \; \nu_L \; \phi^0$$

$$T_3: \frac{1}{2} + \frac{1}{2} + (-1/2) = +1/2$$

Weinberg 1979

$$\nu_L^T C \nu_L \; \phi^0 \phi^0 \left(\frac{1}{\Lambda}\right)$$

T_3: $\frac{1}{2} + \frac{1}{2} - \frac{1}{2} - \frac{1}{2} = 0$

d = 5 Weinberg operator

$\Downarrow$ (effective theory)

$$\boxed{\frac{1}{2} \frac{\nu_L^T C \nu_L \, (h+v)(h+v)}{\Lambda} \qquad \boxed{\Lambda \gtrsim 10 \; M_W}}$$

$\Downarrow$

(i) $m_\nu^{M} = \frac{1}{2} v^2 / \Lambda \qquad M_W = g/2 \, v$

$$\sim M_W^2 / \Lambda \leq 1\, eV$$

$$\boxed{\Lambda \geq \frac{10^4\, GeV^2}{10^{-9}\, GeV} = 10^{13}\, GeV}$$

(ii) $\mathcal{L}_Y = \dfrac{\nu_L^T C \nu_L h v}{\Lambda}$

$$y_\nu \simeq \frac{v}{\Lambda} = \frac{v^2}{v\Lambda} \simeq \frac{m_\nu}{v} \simeq M_W$$

$$\Rightarrow y_\nu \simeq \frac{m_\nu}{M_W} \leq \frac{10^{-9}}{100} = 10^{-11}$$

$$\Gamma(h \to \nu\nu) \propto y_\nu^2 \, m_h$$

$$B(h \to \nu\nu) \simeq y_\nu^2 \simeq 10^{-22}$$

$$\boxed{\text{NO PHYSICS}}$$

$$\underbrace{(\nu \ \nu)}_{T_3 = 1} \qquad \underbrace{\phi^0 \ \phi^0}_{T_3 = -1}$$

what if: another $\phi^0 = \phi^{0\prime}$

$$\Downarrow$$

$(\nu\nu)\ \phi^0\phi^{0\prime},\quad \phi^{0\prime}\phi^{0\prime}$

$\Rightarrow$ modifying SM

Weinberg!

if SM good (up to $\gtrsim 10$ TeV)

$\Rightarrow$ only $\nu\nu\,\dfrac{\phi^0\phi^0}{\Lambda}$

$\Downarrow$ a must

$$\nu \longrightarrow \ell = \begin{pmatrix}\nu \\ e\end{pmatrix}_L$$

$$\phi^0 \longrightarrow \phi$$ (SU(2) dublet)

$$\left(\phi = \begin{pmatrix}\phi^0 \\ \phi^+\end{pmatrix}\right)$$

$$\Downarrow$$

$$\nu_L^T C \nu_L \, \phi_0 \phi_0 \longrightarrow$$

$$\rightarrow \boxed{\cancel{\ell_L^T C i\sigma_2 \ell_L \quad \Phi^T i\sigma_2 \Phi}}$$

(0) ???

$$\Phi \to U \Phi$$

$$\Phi^T \to \Phi^T U^T$$

but $U^T U \neq 1$

$$\Phi^T i\sigma_2 \Phi = (\phi^+ \ \phi^0) \begin{pmatrix} 0 & 1 \\ -1 & 0 \end{pmatrix} \begin{pmatrix}\phi^+ \\ \phi^0\end{pmatrix}$$

$$= \phi^+\phi^0 - \phi^0\phi^+ = 0$$

$$\Phi_{vev} = \begin{pmatrix} 0 \\ \phi^0 \end{pmatrix}$$

$$\Rightarrow \boxed{\underbrace{\left(l_L^T\, i\sigma_2\, \Phi \right)}_{SU(2)\times U(1)\ \text{inv.}} C \underbrace{\left(\Phi^T\, i\sigma_2\, l_L \right)}_{SU(2)\times U(1)}} \quad (1)$$

$$= \left(\nu_L^T e_L^T \right) \begin{pmatrix} 0 & 1 \\ -1 & 0 \end{pmatrix} \begin{pmatrix} 0 \\ \phi^0 \end{pmatrix} \quad - - - -$$

$$= \underbrace{\left(\nu_L^T e_L^T \right) \begin{pmatrix} \phi^0 \\ 0 \end{pmatrix}}\quad - - -$$

$$\boxed{\left(\nu_L^T \phi^0 \right) C \left(\phi^0 \nu_L \right)}$$

$$= \left(\nu_L^T C \nu_L \right) \phi^0 \phi^0\ {}_{w}$$

Analysis of $d=5$

$l_L^T i\sigma_2 \Phi \sim$ fermion singlet (SM)

$\rightarrow$ $\exists$ singlet fermion to SM

$N = N_L$

Stay tuned!

(2) triplet x triplet ?

$$l_L^T i\sigma_2 \vec{\sigma} C l_L \cdot \underbrace{\Phi^T i\sigma_2 \vec{\sigma} \Phi}$$

$$\underbrace{\overset{\#}{O} \qquad\qquad\qquad \overset{\#}{O}}_{\text{works !!}}$$

$$\Phi_{un} = \begin{pmatrix} 0 \\ \phi^0 \end{pmatrix}$$

$$\vec{\sigma} = (\sigma_1, \sigma_2, \sigma_3)$$

$$l_L^T i\sigma_2 \vec{\sigma} C l_L \quad \Phi^T i\sigma_2 \vec{\sigma} \Phi =$$

$$= l_L^T i\sigma_2 \sigma_3 C l_L \quad \Phi^T i\sigma_2 \sigma_3 \Phi + \cdots$$

$$\propto (l_L^T \sigma_1 C l_L)(\Phi^T \sigma_1 \Phi) + \cdots$$

$$\Downarrow$$

$$\boxed{\text{Prove (2) reduces to } \quad \nu_L^T C \nu_L \, \phi^0 \phi^0}$$

$$\Longleftrightarrow \quad \boxed{(1) \simeq (2)}$$

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(3) $$\boxed{\underbrace{\left(l_L^T i\sigma_2 \vec{\sigma} \, \Phi\right) C \left(\Phi^T i\sigma_2 \vec{\sigma} \, l_L\right)}_{\text{works}} \quad \text{Prove } (3) \sim (2) \sim (1)}$$

go back to (0)

(0) $\underbrace{\ell_L^T i\sigma_2 C \ell_L}_{\Downarrow \atop 0}$ $\quad \Phi^T i\sigma_2 \Phi$

$$\Phi^T i\sigma_2 \Phi = 0$$

$$\ell_L^T i\sigma_2 C \ell_L = 0 \qquad (C^T = -C)$$

$$\{\ell, \ell\} = 0$$

$\ell \quad \ell \quad \Phi \quad \Phi$

$Y: -1 \; -1 \; +1 \; +1 = 0$

$$\left.\begin{array}{l} Y(\ell) = -1 \\ Y(\Phi) = +1 \end{array}\right.$$

$\Downarrow$

$\partial \partial \phi^0 \phi^0$

$Y(\nu) = -1$

$Y(\phi^0) = +1$

$$[T_a, Y] = 0$$

$$\Updownarrow$$

$$SU(2) \times U(1)$$

$$\Downarrow$$

in a multiplet Y_i = equal

$q \equiv \begin{pmatrix} u \\ d \end{pmatrix} \Rightarrow Y_u = Y_d = Y_q$

$l \equiv \begin{pmatrix} \nu \\ e \end{pmatrix} \Rightarrow Y_\nu = Y_e = Y_l$

$$\Phi^{+}\Phi \longrightarrow \phi_0^{*}\phi^{0}$$

but: $\nu\nu\ \phi^{0}\phi^{0}$

T_3: $\frac{1}{2}\ \frac{1}{2}\ -\frac{1}{2}\ -\frac{1}{2} \quad = 0$

Y: $-1\ -1\ +1\ +1 \quad = 0$

if $\nu\nu\ \phi^{0}\phi^{0*}$

T_3 $+\frac{1}{2}+\frac{1}{2}\ -\frac{1}{2}+\frac{1}{2} = 1$

Y: $-1\ -1\ +1\ -1 \quad = -2$