


Neutrino Physics Course

Lecture VI

17/5/2022

LACU
Spring 2022



Parity and SM Higgs

mechanism

$$V_T = V_0 + a T^2 \Phi^\dagger \Phi$$

$$T \gg M_w (v)$$

only mass parameter

$$a = \lambda + g^2 + \gamma^2 > 0$$

\Downarrow

$$T \gg T_c \simeq 100 \text{ GeV} \simeq 10^{13} \text{ K}$$

$$\langle \phi \rangle_T = 0$$

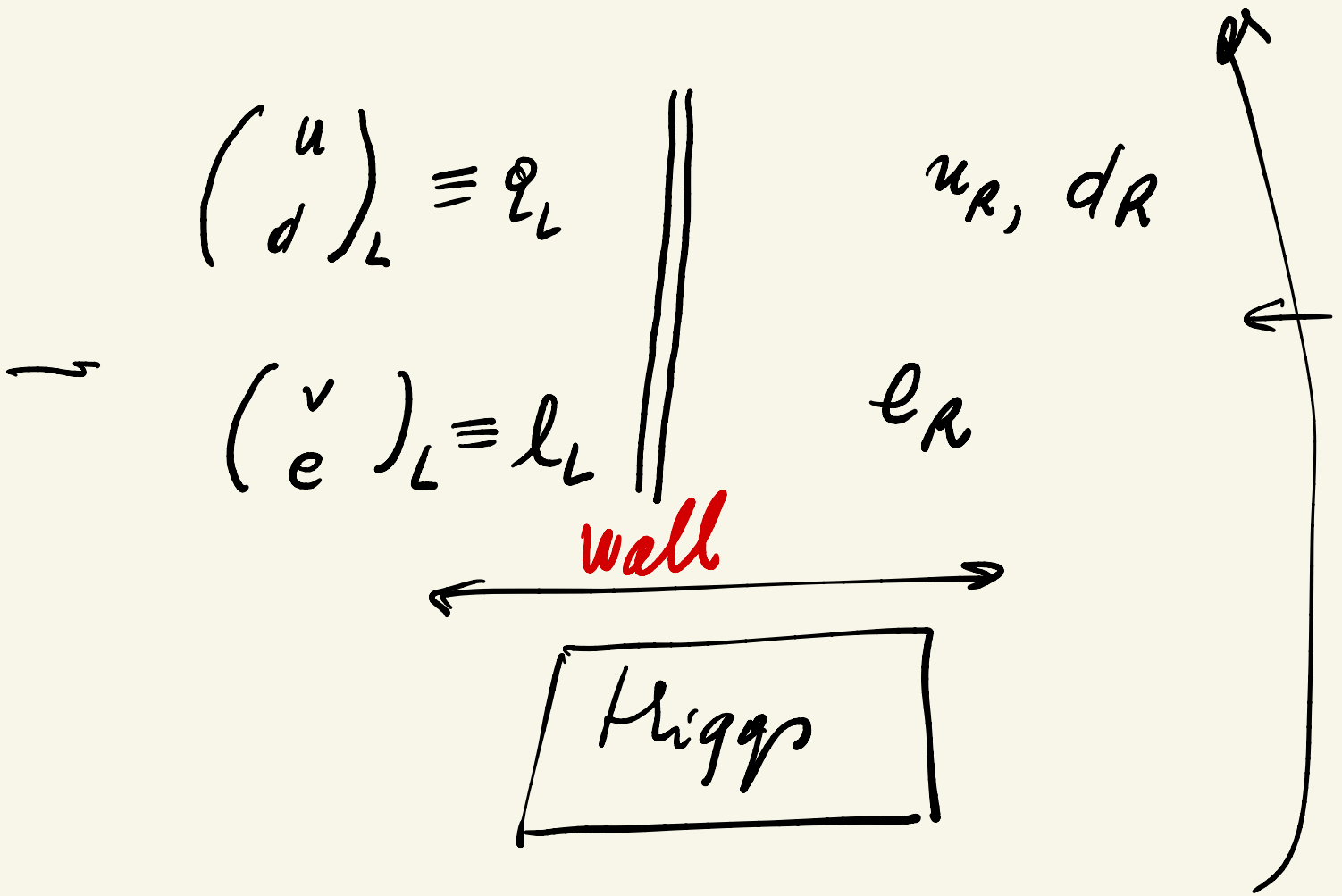
\Rightarrow instead, we probe
Higgs mechanism -
indirectly

$$h : \left[g M_W W^+ W^- + \frac{1}{2} \frac{g}{\cos \theta_W} Z Z_\mu^2 \right. \\ \left. + \frac{g}{2} \frac{m_f}{m_W} \bar{f} f \right]$$

\Downarrow

W^+, Z, t, b, τ & Higgs
origin

Higgs \rightarrow W, Z masses
but f masses too



single $\Phi \rightarrow U \Phi$

$\gamma \Phi = 1$ \Uparrow

$$\mathcal{L}_Y = \bar{\ell}_L \Phi \gamma_d d_R + \bar{\ell}_L i \gamma_5 \Phi^* \gamma_u u_R + \bar{\ell}_L \Phi \gamma_e e_R + \text{h.c.}$$

Nucleosynthesis : $T \gtrsim 10 \text{ MeV}$

maximum T probed
by cosmology

$$\Gamma(h \rightarrow f\bar{f}) \propto \left(\frac{m_f}{M_W}\right)^2 m_h$$

$f = 1\text{st gen}, 2\text{nd gen.}$

b

$m_h \gg m_f$

Imeyve:

$P = \text{good}$
symmetry

Glashow '1961

$$G_{SM} = SU(2)_L \times U(1)_Y$$

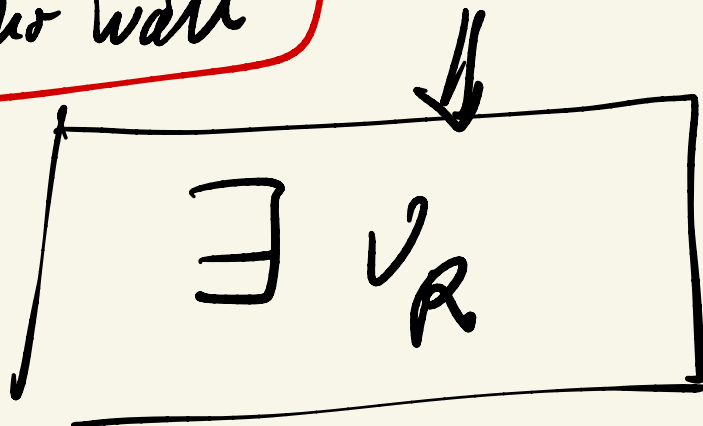
$$\begin{pmatrix} u \\ d \end{pmatrix}_L \equiv q_L$$

$$\begin{pmatrix} u \\ d \end{pmatrix}_R \equiv q_R$$

$$\begin{pmatrix} \nu \\ e \end{pmatrix}_L \equiv l_L$$

$$\begin{pmatrix} \nu \\ e \end{pmatrix}_R \equiv l_R$$

no wall





$$m_\nu \neq 0$$

$$\mathcal{L}_{\text{mass}} = \bar{q}_L M q_R + \text{h.c.}$$



but

FAILURE

$$\begin{array}{l} m_u = m_d \\ m_s = m_c \end{array}$$

(all qu.)

• $m_u \simeq 2-3 \text{ MeV}$

$m_d \simeq 5 \text{ MeV}$

• $m_c \simeq 1.5 \text{ GeV}$

$m_s \simeq 100 \text{ MeV}$

• $m_t \simeq 175 \text{ GeV}$

$m_b \simeq 4-5 \text{ GeV}$



Higgs mechanism

• which Higgs?

∴

coupled to fermions



Adjoint rep. A

$$\left. \begin{matrix} \psi_L \\ \psi_L \end{matrix} \right\} \rightarrow U \left. \begin{matrix} \psi_L \\ \psi_L \end{matrix} \right\}$$



$$\left. \begin{matrix} \psi_R \\ \psi_R \end{matrix} \right\} \rightarrow U \left. \begin{matrix} \psi_R \\ \psi_R \end{matrix} \right\}$$

$$A \rightarrow UAU^\dagger$$

- $A = A^\dagger \leftarrow$ preserved
- $\text{Tr} A \rightarrow \text{Tr} UAU^\dagger = \text{Tr} A$

irreducible adjoint:

Hermitian + traceless

$$A = T_i \varphi_i = \frac{\sigma_i}{2} \varphi_i \quad i=1,2,3$$

$A = 3$ components φ_i

$$\cdot A \rightarrow UAU^+ \quad U \equiv e^{i\theta_i T_i}$$

$$\rightarrow (1 + i\theta_i T_i) A (1 - i\theta_i T_i)$$

$$= A + i\theta_i [T_i, A]$$

$$\uparrow T_i = \frac{\sigma_i}{2}$$

$$\hat{T}_i A = \left[\frac{\sigma_i}{2}, A \right]$$

↓

$$A \rightarrow A + i\theta_j \left[\frac{\sigma_j}{2}, \frac{\sigma_u}{2} \varphi_u \right]$$

$$= A + i(i)\theta_j \varphi_u \in j\omega_i \frac{\sigma_i}{2}$$

$$= \frac{\sigma_i}{2} [\varphi_i - \varepsilon_{ijk} \theta_j \varphi_k]$$

$$\varphi_i' = \varphi_i - \varepsilon_{ijk} \theta_j \varphi_k$$

vector = triplet = column

$$V = \begin{pmatrix} v_1 \\ v_2 \\ v_3 \end{pmatrix} \quad \therefore$$

$$V \rightarrow O V$$

$SO(3)$

$$\begin{aligned} O O^T &= O^T O = 1 \\ \det O &= 1 \end{aligned}$$

$$\boxed{SU(2) = SO(3)}$$



$$0 = e^{i\theta_i L_i}$$

$$[L_i, L_j] = i \epsilon_{ijk} L_k \quad (*)$$

$$\boxed{L_i = -L_i^*, \quad L_i^\dagger = L_i}$$
$$L_i = -L_i^T$$



$$\boxed{(L_i)_{jk} = -i \epsilon_{ijk}} \quad (*)$$



$$V \rightarrow (1 + i \theta_j L_j) V$$

$$V_2' = V_2 + i \theta_j (L_j)_{ik} V_k$$

$$= V_i + i \theta_j (-i) \epsilon_{jia} V_a$$

$$= V_i + i(i) \epsilon_{ija} \theta_j V_a$$

$$\Rightarrow \boxed{V_i' = V_i - \epsilon_{ija} \theta_j V_a}$$

• on the side: $SU(n)$

$$A \rightarrow U A U^\dagger$$

$$U = e^{i \theta_i T_i} \leftarrow \underline{\text{fundamental}}$$

$$\epsilon_{ija} \rightarrow f_{ija} \dots$$

$$[T_i, T_j] = i f_{ij} T_k$$

SU(n)

$$D_\mu = \partial_\mu - ig T_i A_\mu^i$$

$$A = T_i A_\mu^i$$

$$A \rightarrow \underbrace{U A U^\dagger}_{\substack{\text{global} \\ \text{SU}(n)}} + \underbrace{\frac{i}{g} (\partial_\mu U) U^\dagger}_{\substack{\text{gauge} \\ \text{transf.}}}$$

Adjoint



$$f = \varrho, \ell$$

$$\bullet \mathcal{L}_Y = \bar{f}_L (M_f + Y_f A) f_R + h.c.$$

\uparrow
 direct mass
 ($\neq SM$)

\uparrow
 Yukawa
 ($\sim SM$)

$$\bullet SSB \quad \therefore \quad V = f(A)$$

\nearrow
 invariant

$$A \rightarrow UAU^\dagger$$

invariants

$$Tr A = 0$$

$$Tr A^2 = \text{inv.}$$

$$(A^2 \rightarrow U A^2 U^\dagger)$$

$$T_r A^3 = ? \quad (= 0)$$

$$\text{vector} = \vec{V} = \vec{V}$$

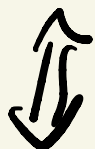
$$\text{inv}(\vec{V}) = \vec{V} \cdot \vec{V} = V^2$$

$$T_r A^3 \propto T_r \underbrace{\sigma_i \sigma_j \sigma_k}_{A_S} \underbrace{\psi_i \psi_j \psi_k}_S$$

$$= 0$$

$$\text{Tr } A^4 = \text{new?} \Leftarrow \text{NOT}$$

$$= ? (\text{Tr } A^2)^2$$



$$\begin{aligned} \text{only i.v.} &= \text{Tr } A^2 \\ &= \bar{V}^2 \end{aligned}$$

$$\bar{V} = V_2 \hat{z} \quad V = \begin{pmatrix} 0 \\ 0 \\ v_3 \end{pmatrix}$$

only one component

$$A \rightarrow U A U^+ = \text{diagonal} \\ (A = A^+)$$

$$\Rightarrow A \rightarrow \begin{pmatrix} A_3 & 0 \\ 0 & -A_3 \end{pmatrix}$$

only A_3

$\Leftrightarrow 1$ invariant

SU(5)

$F = 5$ dim

$$F \rightarrow U F$$

how many invariants?

$$F \rightarrow \begin{pmatrix} 0 \\ 0 \\ 0 \\ 0 \\ 0 \end{pmatrix} \Rightarrow 1 \text{ inv.}$$

$$(F^+ F)$$

$$V = -\mu^2 \text{Tr} A^2 + \lambda (\text{Tr} A^2)^2$$

\Downarrow

$$\text{Tr}(A^2) \neq 0$$

$$\begin{array}{l} + \lambda' \text{Tr}^4 \\ \hline \text{Tr} A^4 + \lambda (\text{Tr} A^2)^2 \end{array}$$

\Downarrow

$$\langle A \rangle \rightarrow \begin{pmatrix} v & 0 \\ 0 & -v \end{pmatrix}$$

\Downarrow

$$\mathcal{L}_Y = \bar{f}_L (M_f + Y_f A) f_R$$

\downarrow

$$M = M_f + Y_f \langle A \rangle$$



$$\begin{aligned} M_u &= M_f + Y_f u \\ M_d &= M_f - Y_f u \end{aligned}$$



$$M_b \ll M_t \Rightarrow M_b \approx 0$$

$$\begin{aligned} M_b &= M_f - Y_f u \approx 0 \\ M_t &\approx M_f + Y_f u \end{aligned}$$

$$\Rightarrow Y_f \neq M_f$$

NO connection between
 ψ_f and γ_f

connection between ψ_f and

M_w, M_z ?

$$\langle A \rangle = \begin{pmatrix} u & 0 \\ 0 & -u \end{pmatrix}$$

\Downarrow

$$M_w = g v$$

$$M_z = 0$$

$$M_A = 0$$

$$D_\mu A = (\partial_\mu - ig \hat{T}_i A_\mu^i) A$$

$$\hat{T}_i A = \left[\frac{\sigma_i}{2}, A \right]$$

$$T_\nu (D_\mu A) (D^\mu A) =$$

$M_z = 0$

move figs

- no connection between H_u, H_d
 - no connection with u_f
 - no connection between u and s and couplings
-

(SM)

$\Phi = \text{doublet}$

$$\langle \Phi \rangle = \begin{pmatrix} 0 \\ v \end{pmatrix}$$

$$D_\mu \Phi = (\partial_\mu - ig T_i A_\mu^i) \Phi$$

$$\mu_{A_i} \neq 0 \Leftrightarrow T_i \langle \Phi \rangle \neq 0$$



$$D_\mu \langle \Phi \rangle = -i \rho T_i A_\mu^i \langle \Phi \rangle$$

$$\text{if } T_i \langle \Phi \rangle = 0 \Rightarrow \mu_{A_i} = 0$$

$$\bullet T_i \langle \Phi \rangle \neq 0 \quad i = 1, 2, 3$$

$$T_i = \sigma_{i/2}$$

$$\gamma \langle \Phi \rangle \neq 0$$

$$\text{but: } (T_3 + \frac{1}{2} \gamma) \langle \Phi \rangle = 0$$



Q_{em}

$$Q_{em} \langle \Phi \rangle = 0$$

$$\Leftrightarrow m_A = 0$$



photon is massless
in SM

$SU(3)$: adjoint

$$A \rightarrow U A U^\dagger \quad (A = U^\dagger)$$

of invariants \longrightarrow

