Neutrino Physics Course

Lecture VI

17/5/2022

LMU

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Parity and SM Higgs mechanism

$$V_T = V_0 + a\,T^2\,\Phi^+\Phi$$

$$\left.T \gg M_W\,(v)\right\}$$

only mass parameter

$$a = \lambda + g^2 + y^2 > 0$$

$$\Downarrow$$

$$T \gg T_c \simeq 100\ \text{GeV} \cong 10^{13}\,{}^\circ K$$

$$\langle\phi\rangle_T = 0$$

$\Rightarrow$ instead, we probe

Higgs mechanism –

indirectly

$$h : \left[g M_W W^+ W^- + \frac{1}{2} \frac{g}{\cos\theta_W} M_Z Z_\mu^2 + \frac{g}{2} \frac{m_f}{M_W} \bar{f} f \right]$$

$\Downarrow$

W^+, Z, t, b, τ $\leftarrow$ Higgs origin

Higgs $\longrightarrow$ W, Z masses but f masses too

$$\begin{pmatrix} u \\ d \end{pmatrix}_L \equiv q_L \qquad u_R,\ d_R$$

$$\begin{pmatrix} \nu \\ e \end{pmatrix}_L \equiv l_L \qquad e_R$$

wall

$$\boxed{\text{Higgs}}$$

<u>Single</u> $\Phi \to U\Phi$

$$Y_\Phi = 1$$

$$\mathcal{L}_Y = \bar{q}_L \Phi Y_d d_R + \bar{q}_L i\sigma_2 \Phi^* Y_u u_R$$

$$+ \bar{l}_L \Phi Y_e e_R + h.c.$$

Nucleosynthesis : $T \gtrsim 10\,\mathrm{MeV}$

maximum T probed by cosmology

$$\Gamma(h \to f\bar{f}) \propto \left(\frac{m_f}{M_W}\right)^2 m_h$$

f = 1st gen, 2nd gen, b

$m_h \gg m_f$

$\boxed{\text{Imagine:}}$ $\boxed{P = \text{good symmetry}}$

Glashow '1961

$$\boxed{G_{SM} = SU(2)_L \times U_Y(1)}$$

$$\begin{pmatrix} u \\ d \end{pmatrix}_L \equiv q_L \qquad \begin{pmatrix} u \\ d \end{pmatrix}_R \equiv q_R$$

$$\begin{pmatrix} \nu \\ e \end{pmatrix}_L \equiv \ell_L \qquad \begin{pmatrix} \nu \\ e \end{pmatrix}_R \equiv \ell_R$$

no wall

$$\Downarrow$$

$$\boxed{\exists\ \nu_R}$$

$$\Downarrow$$

$$\boxed{m_\nu \neq 0}$$

$$\mathcal{L}_{mass} = \underbrace{\bar{q}_L M q_R}_{} + h.c.$$

FAILURE

but

$$\boxed{\begin{array}{c} m_u = m_d \\ m_\nu = m_e \end{array}}$$

(all gen.)

- $m_u \simeq 2\text{-}3$ MeV $\qquad m_d \simeq 5$ MeV
- $m_c \simeq 1.5$ GeV $\qquad m_s \simeq 100$ MeV
- $m_t \simeq 175$ GeV $\qquad m_b \simeq 4\text{-}5$ GeV

$$\Downarrow$$

$\Downarrow$

Higgs mechanism

- which Higgs?

$\therefore$

$\boxed{\text{couples to fermions}}$

$\Downarrow$

Adjoint repr. $\quad A$

$$\left.\begin{matrix} q_L \\ l_L \end{matrix}\right\} \longrightarrow U \left.\begin{matrix} q_L \\ l_L \end{matrix}\right\}$$

$$\left.\begin{matrix} q_R \\ l_R \end{matrix}\right\} \longrightarrow U \left.\begin{matrix} q_R \\ l_R \end{matrix}\right\}$$

$$\Downarrow$$

$$\boxed{A \to U A U^\dagger}$$

- $A = A^\dagger \leftarrow$ preserved
- $\mathrm{Tr}\, A \to \mathrm{Tr}\, U A U^\dagger = \mathrm{Tr}\, A$

$$\Downarrow$$

irreducible adjoint:

Hermitian + traceless

$$\Downarrow$$

$$\boxed{A = T_i \varphi_i = \frac{\sigma_i}{2} \varphi_i} \quad i = 1, 2, 3$$

A = 3 components φ_i

$$\cdot\ A \longrightarrow U A U^{\dagger} \qquad U \equiv e^{i\theta_i T_i}$$

$$\longrightarrow (1 + i\theta_i T_i) A (1 - i\theta_i T_i)$$

$+ \, -$

$$= A + i\theta_i [T_i, A]$$

$$\uparrow$$

$$T_i = \frac{\sigma_i}{2}$$

$$\boxed{\hat{T}_i A = \left[\frac{\sigma_i}{2}, A\right]}$$

$$\Downarrow$$

$$A \longrightarrow A + i\theta_j \left[\frac{\sigma_j}{2}, \frac{\sigma_k}{2}\varphi_k\right]$$

$$= A + i(i)\,\theta_j \varphi_k \epsilon_{jki} \frac{\sigma_i}{2}$$

$$= \frac{\sigma_i}{2}\left[\varphi_i - \varepsilon_{ijk}\,\theta_j\,\varphi_k\right]$$

$$\boxed{\varphi_i' = \varphi_i' - \varepsilon_{ijk}\,\theta_j\,\varphi_k}$$

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vector = triplet = column

$$V = \begin{pmatrix} v_1 \\ v_2 \\ v_3 \end{pmatrix}$$

$$V \longrightarrow O\,V$$

$$\boxed{\begin{array}{l} OO^T = O^T O = 1 \\ \det O = 1 \end{array}}$$

$SO(3)$

$$\boxed{SU(2) = SO(3)}$$

$$O = e^{i\theta_i L_i}$$

$$[L_i, L_j] = i\,\varepsilon_{ijk} L_k \quad (*)$$

$$\boxed{\begin{aligned} L_i &= -L_i^*, \\ L_i &= -L_i^T \end{aligned}} \quad L_i^+ = L_i$$

$$\Downarrow$$

$$\boxed{(L_i)_{jk} = -i\,\varepsilon_{ijk}} \quad (*)$$

$$\Downarrow$$

$$V \longrightarrow (1 + i\,\theta_j L_j)\,V$$

$$V_i' = V_i + i\,\theta_j\,(L_j)_{ik}\,V_k$$

$$= V_i + i\theta_j (-i) \varepsilon_{jik} V_k$$

$$= V_i + i(i) \varepsilon_{ijk} \theta_j V_k$$

$$\Rightarrow \quad \boxed{V_i' = V_i - \varepsilon_{ijk} \theta_j V_k}$$

- <u>on the side</u>: $SU(n)$

$$A \to U A U^\dagger$$

$$U = e^{i\theta_i T_i} \leftarrow \underline{\text{fundamental}}$$

$$\varepsilon_{ijk} \longrightarrow f_{ijk} :$$

$$[T_i, T_j] = i f_{ijk} T_k$$

$\underline{SU(n)}$

$$D_\mu = \partial_\mu - ig\, T_i A_\mu^i$$

$$A = T_i A_\mu^i$$

$$A \to \underbrace{U A U^+}_{\substack{\text{global} \\ SU(n)}} + \underbrace{\frac{i}{g}(\partial_\mu U)U^+}_{\text{gauge transf.}}$$

$\Downarrow$

Adjoint

$\Downarrow$

$f = q, l$

- $\mathcal{L}_Y = \bar{f}_L \left(M_f + Y_f \, A \right) f_R + h.c.$

direct mass ($\neq$ SM)

Yukawa ($\sim$ SM)

- SSB $\therefore$ $V = f(A)$

invariant

$$A \to U A U^\dagger$$

<u>Invariants</u> $\quad \mathrm{Tr}\, A = 0$

$$\mathrm{Tr}\, A^2 = \text{inv.}$$

$$(A^2 \to U A^2 U^{+})$$

$$\mathrm{Tr}\, A^3 = ? \quad (=0)$$

$$\text{vector} = \bar{V} = \vec{V}$$

$$\text{inv}\,(\vec{V}) = \vec{V}\cdot\vec{V} = V^2$$

$$\mathrm{Tr}\, A^3 \propto \mathrm{Tr}\, \underbrace{\sigma_i\,\sigma_j\,\sigma_k}_{AS}\ \underbrace{\varphi_i\,\varphi_j\,\varphi_k}_{S}$$

$$= 0$$

$\mathrm{Tr}\, A^4 =$ new? $\Leftarrow$ NOT

$= ?\ (\mathrm{Tr}\, A^2)^2$

$\Updownarrow$

$$\boxed{\begin{aligned} \text{only inv.} &= \mathrm{Tr}\, A^2 \\ &= \vec{v}^{\,2} \end{aligned}}$$

$\vec{v} = v_z \hat{z}$ $\qquad$ $v = \begin{pmatrix} 0 \\ 0 \\ v_3 \end{pmatrix}$

only one component

$A \to U A U^{+} =$ diagonal

$(A = A^{+})$

$$\Rightarrow \quad A \longrightarrow \begin{pmatrix} A_3 & 0 \\ 0 & -A_3 \end{pmatrix}$$

only A_3

$\Longleftrightarrow$ 1 invariant

<u>SU(5)</u> $\qquad F = 5$ dim

$$F \to U F$$

$$\boxed{\text{how many invariants ?}}$$

$$F \to \begin{pmatrix} 0 \\ 0 \\ 0 \\ 0 \\ f \end{pmatrix} \Longrightarrow 1 \text{ inv.}$$

(F^+F)

$$\boxed{V = -\mu^2 \, \mathrm{Tr}\, A^2 + \lambda (\mathrm{Tr}\, A^2)^2}$$

$$\Downarrow$$

$$\mathrm{Tr}(A^2) \neq 0$$

$\cancel{+\lambda' \mathrm{Tr}^4}$

$\mathrm{Tr}\, A^4 \propto (\mathrm{Tr}\, A^2)^2$

$$\Downarrow$$

$$\boxed{\langle A \rangle \to \begin{pmatrix} v & 0 \\ 0 & -v \end{pmatrix}}$$

$$\Downarrow$$

$$\mathcal{L}_Y = \bar{f}_L (M_f + Y_f A) f_R$$

$$\downarrow$$

$$M = M_f + y_f \langle A \rangle$$

$$\Downarrow$$

$$\boxed{\begin{aligned} M_u &= M_f + y_f v \\ M_d &= M_f - y_f v \end{aligned}}$$

$$\Downarrow$$

$$m_b \ll m_t \Rightarrow m_b \simeq 0$$

$$m_b = m_f - y_f v \simeq 0$$

$$m_t \simeq m_f + y_f v$$

$$\Rightarrow y_f \neq m_f$$

- $\boxed{\text{NO connection between } m_f \text{ and } Y_f}$

- connection between m_f and M_W, M_Z ?

$$\langle A \rangle = \begin{pmatrix} v & 0 \\ 0 & -v \end{pmatrix}$$

$$\Downarrow$$

$$\boxed{\begin{aligned} M_W &= g v \\ M_Z &= 0 \\ M_A &= 0 \end{aligned}}$$

$$D_\mu A = (\partial_\mu - ig\hat{T}_i A_\mu^i) A$$

$$\hat{T}_i A = \left[\frac{\sigma_i}{2}, A\right]$$

$$\mathrm{Tr}\,(D_\mu\langle A\rangle)(D^\mu\langle A\rangle) =$$

$$\boxed{M_Z = 0}$$

$$\Downarrow$$

$\boxed{\text{more things}}$

$$\Downarrow$$

- no connection between
  M_W, M_Z
- no connection with m_f
- no connection between masses and couplings

SM $\quad \Phi$ = doublet

$$\langle \Phi \rangle = \begin{pmatrix} 0 \\ v \end{pmatrix}$$

$$D_\mu \Phi = (\partial_\mu - i g T_i A_\mu^i)\Phi$$

$$\boxed{m_{A_i} \neq 0 \iff T_i \langle \Phi \rangle \neq 0}$$

$$\Uparrow$$

$$D_\mu \langle \Phi \rangle = -ig\, T_i A_\mu^i \langle \Phi \rangle$$

if $$\boxed{T_i \langle \Phi \rangle = 0 \Rightarrow m_{A_i} = 0}$$

- $T_i \langle \Phi \rangle \neq 0 \qquad i = 1, 2, 3$

  $T_i = \sigma_i/2$

  $Y \langle \Phi \rangle \neq 0$

  but: $\underbrace{\left(T_3 + \frac{1}{2} Y\right)}\langle \Phi \rangle = 0$

Q_{em}

$$\boxed{Q_{em}\langle\Phi\rangle = 0 \iff m_A = 0}$$

$$\Downarrow$$

$$\boxed{\text{photon is massless in SM}}$$

$SU(3)$: adjoint

$A \to U A U^\dagger \qquad (A = U^\dagger)$

\# of invariant $\longrightarrow$

Simplest form of A

$$A \longrightarrow \begin{pmatrix} a & & \\ & b & \\ & & -(a+b) \end{pmatrix}$$

$$\Rightarrow \underbrace{\mathrm{Tr}\,A^2, \quad \mathrm{Tr}\,A^3}$$

Independent invariants