Neutrino Physics

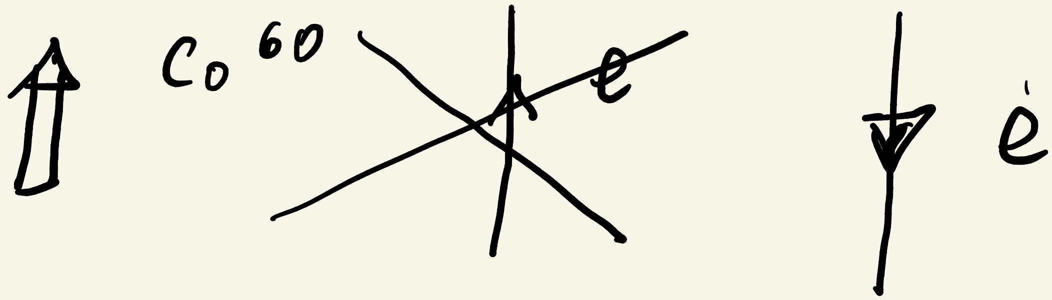
Lecture II

3/5/2022

LHU

Spring 2022

$\boxed{\not{P} = \text{maximal}}$ $\boxed{\text{'1956}}$



$$n \to p + e + \bar{\nu}$$

$$(d \to u + e + \bar{\nu})$$

u u
d

p

d d
u

p

$$\Downarrow$$

1957

$$\boxed{V - A}$$

$\Downarrow$ "V − A was the key"

$$\frac{4 G_F}{\sqrt{2}} J_\mu^{w} \bar{J}^{\mu}_{w}$$

$$\boxed{J^\mu_w = \left(\bar{\nu}_L \gamma_\mu e_L + \bar{u}_L \gamma^\mu d_L\right)}$$

vector + axial vector

$$g_{\mu\nu} = \mathrm{diag}\,(1, -1, -1, -1)$$

$$\{\gamma_\mu, \gamma_\nu\} = 2 g_{\mu\nu}$$

$$\Sigma_{\mu\nu} = \frac{1}{4i}[\gamma_\mu, \gamma_\nu] \quad (1)$$

Lorentz gen

Spinor: $\psi \longrightarrow \Lambda \psi$ (2)

$$\Lambda = \exp\left(i\, \Sigma_{\mu\nu}\, \theta^{\mu\nu}\right)$$

ROT + BOOST

$$\gamma^0 = \begin{pmatrix} 0 & 1 \\ 1 & 0 \end{pmatrix}, \quad \gamma^i = \begin{pmatrix} 0 & \sigma^i \\ -\sigma^i & 0 \end{pmatrix}$$

σ_i = Pauli

$$\gamma^\mu = \begin{pmatrix} 0 & \sigma^\mu_+ \\ \sigma^\mu_- & 0 \end{pmatrix} \quad (3)$$

$$\sigma^\mu_+ = (1; \vec{\sigma})$$

$$\sigma^\mu_- = (1, -\vec{\sigma})$$

$$\{\gamma_5, \gamma_\mu\} = 0 \qquad \gamma_5^2 = 1$$

$$\Rightarrow [\gamma_\mu, \Sigma_{\mu\nu}] = 0$$

$$L = \frac{1+\gamma_5}{2} \qquad R = \frac{1-\gamma_5}{2}$$

$$\psi_L \equiv L\psi \quad , \quad \psi_R \equiv R\psi$$

<u>QED</u> $\boxed{\mathcal{L}_{int} \propto A_\mu \bar{\psi} \gamma^\mu \psi}$

$$A_i \xrightarrow{P} -A_i$$

$$A_0 \xrightarrow{P} A_0 \quad \Rightarrow \quad \boxed{\psi \xrightarrow{P} \gamma^0 \psi}$$

$$\bar{\psi} \gamma^\mu \psi \equiv \psi^\dagger \gamma^0 \gamma^\mu \psi$$

$$\boxed{\gamma_5 = \begin{pmatrix} 1 & 0 \\ 0 & -1 \end{pmatrix}} \qquad \gamma_5 = \underset{t}{\pm i}\, \gamma_0 \gamma^1 \gamma^2 \gamma^3$$

$$L = \begin{pmatrix} 1 & 0 \\ 0 & 0 \end{pmatrix}, \quad R = \begin{pmatrix} 0 & 0 \\ 0 & 1 \end{pmatrix}$$

$$\psi = \begin{pmatrix} u \\ v \end{pmatrix} = \begin{pmatrix} u_L \\ u_R \end{pmatrix}$$

$$\psi_L = L\psi = \begin{pmatrix} u \\ 0 \end{pmatrix}$$

$$\psi_R = R\psi = \begin{pmatrix} 0 \\ v \end{pmatrix}$$

$\Downarrow$

From (1) – (3):

$$u_{L,R} \to e^{i \vec{\sigma}/2 \cdot (\vec{\theta} \pm i \vec{\varphi})}$$

$$\boxed{\begin{aligned} \theta_{0i} &= \varphi_i \\ \theta_{ij} &= \varepsilon_{ijk}\,\theta_k \end{aligned}}$$

ROT (Euler) BOOST

P: $\psi \to \gamma^0 \psi$

$$\begin{pmatrix} u_L \\ u_R \end{pmatrix} \to \begin{pmatrix} 0 & 1 \\ 1 & 0 \end{pmatrix} \begin{pmatrix} u_L \\ u_R \end{pmatrix} = \begin{pmatrix} u_R \\ u_L \end{pmatrix}$$

$$\boxed{u_L \longleftrightarrow u_R} \Leftarrow \text{true}$$

$\Updownarrow$

<u>often:</u> $\psi_L \longrightarrow \psi_R$ symbolic

W bozon @ CERN

$\uparrow$

at rest

$m_e \simeq$ MeV

$\Uparrow \qquad W^- \longrightarrow e + \bar{\nu}$

$M_W \simeq 80$ GeV $\quad (m_p \simeq$ GeV$)$

$\Rightarrow m_e = m_\nu = 0 \quad (\ll E_e, E_\nu)$

$\boxed{m = 0}$ Dirac:

$$\boxed{p^\mu \gamma_\mu \psi = m \psi}$$

$m = 0 \qquad \Downarrow$

$$p^\mu \gamma_\mu = 0 \Rightarrow$$

$$\begin{pmatrix} 0 & E - \vec{p}\cdot\vec{\sigma} \\ E + \vec{p}\cdot\vec{\sigma} & 0 \end{pmatrix} \begin{pmatrix} u_L \\ u_R \end{pmatrix} = 0$$

$$\Rightarrow \quad (E \pm \vec{p}\cdot\vec{\sigma})\, u_{L,R} = 0$$

$$|\vec{p}| = E \qquad\qquad \vec{p} \equiv |\vec{p}|\, \hat{p}$$

$$\Rightarrow \quad \frac{\vec{\sigma}}{2}\cdot\hat{p}\, u_{L,R} = \mp \frac{1}{2}\, u_{L,R}$$

$$\vec{S} = \frac{\vec{\sigma}}{2}$$

$$\Downarrow$$

$$\boxed{\underbrace{\vec{S}\cdot\hat{p}}_{\text{helicity}}\, u_{L,R} = \mp\frac{1}{2}\, u_{L,R}}$$

$$\boxed{h u_L = -\frac{1}{2} u_L \qquad h u_R = +1/2}$$

$$\Downarrow$$

$S_z^W = 1$ $\Uparrow$ $\qquad \cancel{e_L}\ \ S_z^e = -\frac{1}{2}\ \Downarrow \qquad e_L\ \ S_z^e = \frac{1}{2}\ \Downarrow$

W^-

$$J_z^{in} = J_z^{f}$$

$l_z = 0$ along z-axis

$$\Downarrow$$

$$\boxed{S_z^{in} = S_z^{f}}$$

$\cancel{P}$ int. (maximal $\cancel{P}$)

$$\sim \left(\bar{\nu}_L e_R + \dots \right)$$

$$\cancel{\| \quad \bar{\nu} \frac{1-\gamma_5}{2} e}$$

$\downarrow$ united

$$\text{weak} \sim \underbrace{\bar{J}_{\mu L} J^{\mu}_L}$$

'50s

$$\boxed{\text{triumph of QED as a gauge theory}}$$

$$\mathcal{L}_D = i\,\bar{\Psi}\,\gamma^\mu \partial_\mu \Psi - m\,\bar{\Psi}\Psi$$

$$\Downarrow$$

$$i\gamma^\mu \partial_\mu \Psi = m\Psi$$

$$\Psi \longrightarrow e^{i\alpha Q_{em}}\Psi \qquad U(1)$$

$$Q_{em}\Psi = q_{em}\,\Psi$$

$$\uparrow$$

$e: -1, \quad u: 2/3, \quad d: -1/3$

$$\Downarrow$$

$$\partial^\mu j_\mu = 0 \qquad j_\mu = \bar{\Psi}\,\gamma_\mu\, Q_{em}\,\Psi$$

(Noether)

$$\alpha \longrightarrow \alpha(x)$$

$$\partial_\mu \longrightarrow D_\mu \quad \therefore$$

$$(D_\mu \psi) \longrightarrow e^{i\alpha(x)} D_\mu \psi$$

$$\Downarrow$$

$$D_\mu = \partial_\mu - i e Q_{em} A_\mu$$

$$\Rightarrow \boxed{A_\mu \longrightarrow A_\mu + \frac{1}{e} \partial_\mu \alpha(x)}$$

$$\Uparrow$$

$$\mathcal{L}_D \longrightarrow i \bar{\psi} \gamma^\mu D_\mu \psi - m \bar{\psi}\psi$$

$$= \mathcal{L}_D + e \bar{\psi} \gamma^\mu Q_{em} \psi A_\mu$$

$$e\, j_\mu^{em} A^\mu$$

$$\alpha_{em} = \frac{e^2}{4\pi} \simeq 1/100 \qquad (e \simeq 1/3)$$

Step back

$W \Uparrow \; S_z^W = 1$

$W^- \to e + \bar{\nu}$?

helicity

$$(\psi^c) \equiv C \bar{\psi}^T = C \gamma_0 \psi^*$$

Charge conjugation

$$\psi \to e^{i\alpha Q_{em}} \psi$$

$$\psi^* \to e^{-i\alpha Q_{em}} \psi = e^{i\alpha(-Q_{em})} \psi$$

$$\begin{pmatrix} Q_{em}\psi = q\psi \\ Q_{em}\psi^* = -q\psi \end{pmatrix}$$

$$\psi^c \to 1\,\psi^c$$

$$\Rightarrow \boxed{\begin{array}{c} C\gamma_\mu C^T = -\gamma_\mu^T \\ C^T = -C = -C^* \end{array}}$$

$$C = i\gamma_2\gamma_0$$

$$\Downarrow$$

$$\psi^c(\bar{\psi}) = i\gamma_2 \psi^*$$

$$\Downarrow$$

$$\psi^c = \begin{pmatrix} 0 & i\sigma_2 \\ -i\sigma_2 & 0 \end{pmatrix} \begin{pmatrix} u_L \\ u_R \end{pmatrix}^*$$

$$= \begin{pmatrix} i\sigma_2 u_R^* \\ -i\sigma_2 u_L^* \end{pmatrix}$$

$$\Downarrow$$

$$\underbrace{\begin{pmatrix} u_L \\ 0 \end{pmatrix}}_{\psi_L} \xrightarrow{C} \underbrace{\begin{pmatrix} 0 \\ -i\sigma_2 u_L^* \end{pmatrix}}_{(\psi^c)_R}$$

$$\Rightarrow (\bar{\nu})_R \qquad \boxed{C\bar{\nu}_L^T = (C\bar{\nu}^T)_R}$$

$$\uparrow (\bar{\nu})_R \quad \Downarrow$$

$$\left.\begin{array}{l} \Uparrow \; S_z^{\bar{\nu}} = +1/2 \\[2em] \downarrow e_L \quad \Uparrow \; S_z^{e} = +1/2 \end{array}\right\} \; \begin{array}{c} S_z^{f} = +1 \\ \Downarrow \\ J_z^{f} = +1 \end{array}$$

$$\boxed{\text{Gauge principle}}$$

em : $\quad j^\mu_{em} A^\mu$

weak: $\quad j^\mu_W W_\mu^+ + \bar{j}^\mu_W W_\mu^-$

$$j^\mu_W = \bar{\nu}_L \gamma^\mu e_L + \bar{u}_L \gamma^\mu d_L$$

minimal gauge group (ew)

$G_{min} = ?$

$\Downarrow$

$$\boxed{G_{min} = SU(2)}$$

D = doublet $\qquad D \to UD$

$U^+U = UU^+ = 1$

$\det U = 1$

$U = e^{iH}$ $\qquad H = H^+$

$\mathrm{Tr}\, H = 0$

$\Downarrow$

$$H = \sum \theta_i T_i \qquad \theta_i = \text{Euler}$$

$$i = 1, 2, 3$$

$$[T_i, T_j] = i\,\varepsilon_{ijk} T_k$$

$$\Rightarrow \boxed{T_i = \frac{1}{2}\sigma_i} \qquad D = \begin{pmatrix} u \\ d \end{pmatrix}$$

$$\mathcal{L}_D(SU(2)) = i\bar{D}\gamma^\mu \partial_\mu D - m\bar{D}D$$

$$D \to e^{i\theta_i T_i} D$$

$$\longrightarrow \theta_i = \theta_i(x) \quad \text{gauging}$$

$$\Rightarrow \partial_\mu = D_\mu \quad \therefore \quad D_\mu D \to e^{i\vec{T}\cdot\vec{\theta}} D_\mu D$$

$$\boxed{D_\mu = \partial_\mu - i g T^i A_\mu^i}$$

Prove:

$$A_\mu^i \to \underbrace{\varepsilon^{ijk}\theta^j A_\mu^k}_{} + \frac{1}{g}\partial_\mu \theta^i$$

gauge

Yang, Mills '54

Shaw '54

$$D_\mu = \partial_\mu - ig\frac{\sigma_i}{2}A_\mu^{\,i} \to$$

$$-i\frac{g}{2}\begin{pmatrix} A_3 & A_1 - iA_2 \\ A_1 + iA_2 & -A_3 \end{pmatrix}_\mu$$

$$\Downarrow$$

$$\to i\bar{D}\gamma^\mu D_\mu D = \frac{g}{2}\bar{D}\gamma^\mu \begin{pmatrix} & \\ & \end{pmatrix}_\mu D$$

$$= \frac{g}{2}\begin{pmatrix}\bar{u} & \bar{d}\end{pmatrix}\gamma^\mu \begin{pmatrix} A_3 u + (A_1 - iA_2)d \\ (A_1 + iA_2)u - A_3 d \end{pmatrix}_\mu$$

$$\Downarrow$$

$$A_{3\mu}: \frac{g}{2}\left(\bar{u}\gamma^\mu u - \bar{d}\gamma^\mu d\right)$$

$$(A_1 - i A_2)_\mu \frac{g}{2} \underbrace{\bar{u} \gamma^\mu d} + h.c.$$

$$\Rightarrow \quad \frac{A_1 - i A_2}{\sqrt{2}}{}_\mu = W_\mu^+$$

$$\frac{A_1 + i A_2}{\sqrt{2}}{}_\mu = W_\mu^-$$

$$\rightarrow \boxed{\frac{g}{\sqrt{2}} \bar{u} \gamma^\mu d \, W_\mu^+ + h.c.}$$

<u>but:</u> weak current = LH

$$\Downarrow$$

$$D = \begin{pmatrix} u_L \\ d_L \end{pmatrix}$$

$SU(2) \Rightarrow$

$$Q_{em} = \sum c_i T_i \Rightarrow$$

neutral

$$\boxed{Q_{em} = T_3}$$

$$\begin{pmatrix} u \\ d \end{pmatrix} \begin{matrix} \rightarrow 2/3 \\ \rightarrow -1/3 \end{matrix} \qquad \begin{pmatrix} \nu \\ e \end{pmatrix} \begin{matrix} \rightarrow 0 \\ \rightarrow -1 \end{matrix}$$

$$A_\mu \big(\bar{\psi}\gamma^\mu\psi = \underbrace{\bar{\psi}_L \gamma^\mu \psi_L + \bar{\psi}_R \gamma^\mu \psi_R\big)}_{}$$

$\underline{P}$ (LR symmetric)

Glashow

SU(2) U(1)

$$Q_{em} = T_3 + \frac{Y}{2}$$

arbitrary

$$\bullet \quad \bullet \quad \Rightarrow \quad Y = \text{- - -}$$