Biophysics of Macromolecules SS 2021
PROBLEM SET 2

① Protein synthesis

— Bacteria divide every 20 min ⟹ Need to double amount of protein!

— Bacteria have mass of $\approx 1pg$; (⟶ see Problem Set 1)

— Assume 20% of the cell is protein

⟹ $0.2 \cdot 1pg = 0.2 \cdot 10^{-12}$ g of protein

or roughly $0.2 \cdot 10^{-12} \cdot 6 \cdot 10^{23} \approx 10^{11}$ Da of protein every 20 min;

— Amino acid have on average $\underline{100 \; Da}$.

⟹ Need to synthesize 10^9 amino acids in 20 min

— There are $2 \cdot 10^4$ ribosomes :

$$\Rightarrow Rate = \frac{10^9 \; amino \; acids}{2 \cdot 10^4 \cdot 20 min \cdot 60 \frac{s}{min}} \approx 40 \; \frac{amino \; acids}{s}$$

$$\underline{for \; each \; ribosome.}$$

② Two-state protein folding

a) $K_{eq} = \dfrac{[N]}{[U]}$

$$\dfrac{1 - f_u}{f_u} = \dfrac{1 - \dfrac{[U]}{[U]+[N]}}{\dfrac{[U]}{[U]+[N]}} = \dfrac{[U]+[N]-[U]}{[U]}$$

$$= \dfrac{[N]}{[U]} = K_{eq}$$

b) $\Delta G_f = -RT \ln(K_{eq})$

$$\Rightarrow K_{eq} = e^{-\frac{\Delta G_f}{RT}} = \dfrac{1 - f_u}{f_u}$$

$$\Rightarrow f_u \left(1 + e^{-\frac{\Delta G_f}{RT}} \right) = 1$$

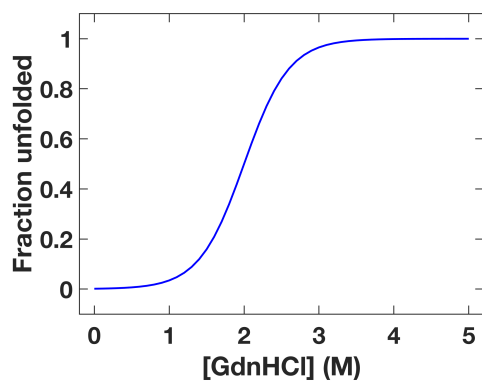$$\Rightarrow f_u = \dfrac{1}{1 + e^{-\frac{\Delta G_f}{RT}}}$$

You get the same result if you
consider the two state system, w/
$G_{unfolded} = 0$ and $\Delta G_f = G_{folded} - G_{unfolded}$

The fraction unfolded is equal to
the probability of being unfolded

$$f_u = P_u = \frac{e^{-\frac{G_u}{RT}}}{Z}$$

where $Z = \sum_{i=states} e^{-G_i/RT}$ the
partition
function.

c) See matlab script for the plotting routine.

$$d) \quad f_u = \cfrac{1}{1 + e^{-\frac{\left(-4\,\frac{kcal}{mol}\right)}{0.6\;kcal/mol}}} \simeq 0.0013$$

$$\simeq 10^{-3}$$