Neutrino Physics Course

Lecture $\overline{\overline{\text{XXVII}}}$

13/7/2021

LMU
Summer 2021

LR: completing the theory (P)

- Seesaw: (neutrino mass):

final words

P: $M_D = M_D^{+}$

$$\mathcal{M}_\nu = -M_D^T \frac{1}{M_N} M_D + \frac{v_L}{v_R} \mathcal{M}_N^*$$

$$\mathcal{M}_\nu^* = -M_D \frac{1}{M_N^*} M_D^* + \frac{v_L}{v_R} \mathcal{M}_N$$

$$\frac{1}{\sqrt{M_N}} M_\nu^* \frac{1}{\sqrt{M_N}} =$$

$$= -\frac{1}{\sqrt{M_N}} M_D \frac{1}{\sqrt{M_N^*}} \frac{1}{\sqrt{M_N^*}} M_D^* \frac{1}{\sqrt{M_N}} + \frac{v_L}{v_R}$$

$$\Downarrow$$

$$HH^T = S$$

$$S \equiv \frac{v_L}{v_R} - \frac{1}{\sqrt{M_N}} M_\nu^* \frac{1}{\sqrt{M_N}}$$

$$H = \frac{1}{\sqrt{M_N}} M_D \frac{1}{\sqrt{M_N^*}}$$

$\boxed{\text{solution}}$

$$S = O\, s_J\, O^T$$

$$\Rightarrow \quad H = O \sqrt{s_J}\, E\, O^{\dagger}$$

$$\sqrt{s_J}\, E = E^{\dagger} \sqrt{s_J}^{\,*}$$

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s_J = diagonal

- $s_J' \in \mathbb{R}$, $\quad s_J'' = (s, s_0, s^*)$

$\qquad\qquad\qquad\qquad\qquad\qquad s_0 \in \mathbb{R}$

$\Downarrow \qquad\qquad\qquad\qquad\qquad \Downarrow$

$$E_{I} = \mathbb{1} \qquad\qquad E_{II} = \begin{pmatrix} 0 & 0 & 1 \\ 0 & 1 & 0 \\ 1 & 0 & 0 \end{pmatrix}$$

Illustration: ($v_L = 0$)

$$\underline{M}_\nu = -M_D^T \frac{1}{M_N} M_D, \quad M_D^+ = M_D$$

$$\boxed{V_L = V_R} \qquad M_N = V_R\, m_N\, V_R^T$$

$$M_\nu = V_L^*\, m_\nu\, V_L^+$$

$$\boxed{H = ?}$$

$$M_D = \sqrt{m_N}\; H \;\sqrt{m_N^*} \quad (H = H^+)$$

$$M_D = V_R \sqrt{m_N}\; H' \sqrt{m_N}\; V_R^+ \quad (H' = H'^+)$$

$$\Downarrow$$

easy to find $H'(M_D)$

$\boxed{\text{Instead}}$

$$M_\nu = - M_D^T \frac{1}{M_N} M_D$$

$$\|$$

$$V_L^* m_\nu V_L^\dagger$$

$$M_N = V_R m_N V_R^T$$

$$\Downarrow$$

$$\frac{1}{M_N} = V_R^* \frac{1}{m_N} V_R^\dagger$$

Proof:

$$1/M_N \; M_N = V_R^* \frac{1}{m_N} \underbrace{V_R^\dagger V_R}_{1} m_N V_R^T$$

$$= V_R^* V_R^T = 1 \quad \checkmark$$

Q.E.D.

$$V_L = V_R \Rightarrow 1/m_N = V_L^{*} m_N^{-1} V_L^{+}$$

$$\Downarrow$$

$$V_L^{*} m_\nu V_L^{+} = - M_D^{T} V_L^{*} m_N^{-1} V_L^{+} M_D$$

$$M_D = M_D^{+} \Rightarrow M_D = V m_D V^{+}$$

diagonal

$$\Downarrow$$

$$V_L^{*} m_\nu V_L^{+} = - V^{*} m_D \underbrace{V^{T} V_L^{*}}_{1} m_N^{-1} \underbrace{V_L^{+} V}_{1} m_D V^{+}$$

$$\Rightarrow V = V_L \Rightarrow$$

$$V_L^{*} m_\nu V_L^{+} = - V_L^{*} m_D m_N^{-1} m_D V_L^{+}$$

$$\Downarrow$$

$$\boxed{m_D^2 = - m_\nu m_N}$$

$$\Downarrow$$

$$\boxed{M_D = i\, V_L \sqrt{m_\nu m_N}\, V_L^\dagger}$$

$\boxed{\text{PMNS}}$ $\Rightarrow$ $\boxed{O \text{``}=\text{''} 1}$

<u>contrast!</u>

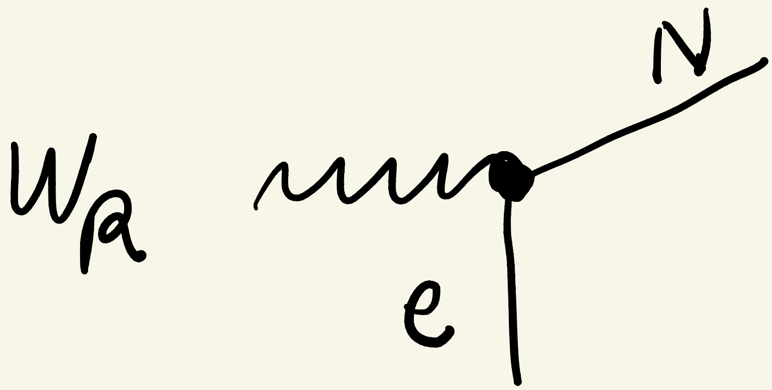
$$M_D = i \sqrt{M_N}\, O \sqrt{M_\nu}$$

$$O^T O = 1, \quad O \in \mathbb{C}$$

$\Downarrow$ claim

$\boxed{LR \Rightarrow \text{"solves" seesaw}}$

$M_N = \boxed{\text{input}}$



$$N \xrightarrow[W_R]{} e_R \, j \, j \quad (M_N)$$

$$N \longrightarrow e_L \, W^+ \quad (M_D) \quad 50\%$$

$$\bar{e}_L \, W^- \quad (M_D) \quad 50\%$$

Completing Majorana "dream"

LR $\rightarrow$ origin of neutrino mass

SM $\rightarrow$ -"- lepton masses (charged)

- W_R $\leftarrow$ crucial role

hadron collider

$\Rightarrow$ quarks enter!

SM:
$$\frac{g}{\sqrt{2}} W^+_{\mu L} \bar{u}_L \gamma^\mu V_{CKM} d_L$$
$$\qquad\qquad\qquad \begin{matrix} \| \\ V_L \end{matrix} \quad + h.c.$$

LR
$$+ \frac{g}{\sqrt{2}} W^+_{\mu R} \bar{u}_R \gamma^\mu V_R d_R \quad + h.c.$$

$$\boxed{V_R = ?}$$

$$\boxed{P \Rightarrow V_L = V_R}$$

but

P is broken (spont.)

Higgs:

(i) $\Delta_L, \quad \Delta_R$ (adjoints)

$$(B-L)\,\Delta = 2\,\Delta$$

$$\langle \Delta_R \rangle = v_R \gg M_W$$

$$\langle \Delta_L \rangle = 0$$

$$\Rightarrow \quad m_\Delta \simeq v_R \quad (M_R)$$

(ii) Φ = bi-doublet

$$\Phi = (\tilde{\phi}_1 \;\; \phi_2)$$

↳ $SU(2)_L$ doublets

$\boxed{\text{SM}}$ $\boxed{\text{Higgs}}$

ϕ - single doublet

$\Downarrow$

$$M_q = Y_q \langle\phi\rangle = Y_q v$$

$$M_W = \frac{g}{2} v$$

diagonal

diagonal

$\Downarrow$

$$h\bar{f}f \leftarrow \frac{g}{2}\frac{m_f}{M_W}$$

2 doublets

$$M_q = Y_q^1 v_1 + Y_q^2 v_2$$

diagonal

$\boxed{\text{NOT diagonal}}$

· Two Higgs doublets in SM

$$\phi_1, \quad \phi_2 \qquad v_i \equiv \langle \phi_i \rangle \in R$$

$$\rightarrow \quad \phi = \frac{v_1 \phi_1 + v_2 \phi_2}{\sqrt{v_1^2 + v_2^2}} \; ; \qquad \langle \phi \rangle = v$$

$\downarrow$

$$\rightarrow \quad \phi' = \frac{v_2 \phi_1 - v_1 \phi_2}{\sqrt{v_1^2 + v_2^2}} \; ;$$

$$\langle \phi' \rangle = 0$$

$$\boxed{v^2 \equiv v_1^2 + v_2^2} \qquad \boxed{M_W = \frac{g}{2} v}$$

Q. Are ϕ, ϕ' eigenstates of mass?

A. $\boxed{\text{In general, NOT.}}$

If yes $\Rightarrow$ $\boxed{\phi' \neq \text{Higgs !!!}}$

$m_{\phi'}$ = anything (free)

but

$m_\phi = ?$

eaten by W^+

$$\phi = \begin{pmatrix} \phi^+ = G_W^+ \\ v + h + i G_Z \end{pmatrix}$$

eaten by Z

$$M_W = \frac{g}{2} v$$

$$\xrightarrow[\text{unitary}]{} \begin{pmatrix} 0 \\ v + h \end{pmatrix}$$

Higgs boson

$$\boxed{\begin{array}{c} v = \text{scale of } SU(2) \text{ breaking} \\ (A = \sin\theta \, A_3 + \cos\theta \, B) \end{array}}$$

$m_h \propto$ scale of breaking

$\propto M_W \ (v)$

$$\boxed{m_h = \sqrt{2\lambda}\, v}$$

true, for whatever lies BSM

$\downarrow$

LR theory

$\phi = h$ (usual SM Higgs)

$\phi' = H$ (new doublet)

$\therefore \langle m \rangle = 0$

$$\boxed{\text{Q. } m_H = ?}$$

$LR \longrightarrow SM$

$\langle \Delta_R \rangle = v_R \; (\sim M_{W_R})$

$\Longleftrightarrow \quad m_H \propto v_R$

(i) How come?

(ii) phenomenology?

LR — $\boxed{\text{bi-doublet}}$

$$\mathcal{L}_y^{(LR)} = \bar{f}_L \left(Y_1^f \Phi + Y_2^f \tilde{\Phi} \right) f_R + h.c.$$

$$\rightarrow \boxed{\begin{array}{l} \bar{q}_L \left(Y_1 \Phi + Y_2 \tilde{\Phi} \right) q_R + h.c. \\[1em] q_{L,R} = \begin{pmatrix} u \\ d \end{pmatrix}_{L,R} \end{array}}$$

SM $\quad \phi_{1,2}$ – 2 doublets

$$\mathcal{L}_y^{(SM)} = \bar{q}_L \left(Y_1^d \phi_1 + Y_2^d \phi_2 \right) d_R + h.c.$$

$$+ \bar{q}_L \left(Y_2^u \tilde{\phi}_1 + Y_2^u \tilde{\phi}_2 \right) u_R + h.c.$$

$\longrightarrow$ 4 Yukawas!

SM : ϕ (MSM)

$Y_2 = 0 \Rightarrow Y_d, Y_u$!

$\Downarrow$

$\boxed{\text{LR must be equally predictive!}}$

$$\tilde{\phi} = (i\sigma_2)\,\phi^*(-i\sigma_2)$$

$$= -\sigma_2\,\phi^*\,\sigma_2$$

$\boxed{\text{Tello, PhD thesis}}$

Y_1, Y_2

$\hookrightarrow M_u, M_d$

$\Downarrow$

all interactions are fixed

$\langle h \rangle = v, \ \langle H \rangle = 0$

$\Downarrow$

$$\mathcal{L}_y(h) = \mathcal{L}_y^{SM}(h)$$

$$\mathcal{L}_y(H) = \alpha(\cdot +) H^0 \bar{d}_L V_L^+ \left(\frac{m_u}{M_W}\right) V_R d_R + h.c.$$

(derive)

$$M_u = V_{uL} m_u V_{uR}^+$$

$$H^0 \bar{d}_L^0 M_u d_R^0 / M_W$$

$$V_L \equiv U_{uL}^+ U_{dL}, \ V_R = U_{uR}^+ U_{dR}$$

$$\langle \phi \rangle = \begin{pmatrix} v_1 & 0 \\ 0 & v_2 e^{ia} \end{pmatrix}$$

$$\underbrace{v_2 e^{ia}}_{a \neq 0 \text{ in general}}$$

$$(\tan\beta \equiv v_2/v_1 \; ; \; a)$$

$$v_2 < v_1 \qquad \boxed{v^2 \equiv v_1^2 + v_2^2}$$

$$v_2 = 0 \Rightarrow v_1 - \text{real (SM)}$$

$$\boxed{M_W = \frac{g}{2} v}$$

$\downarrow$ precise

$$\boxed{\frac{g}{\cos\beta} H_0 \, \bar{d}_L \, V_L^{+} \frac{m_u}{M_W} V_R \, \phi_R \text{ th.c.}}$$

$$\boxed{\text{DERIVE}}$$

$P: \quad Y_1 = Y_1^+, \quad Y_2 = Y_2^+$

$$\Downarrow$$

$$\boxed{\begin{array}{c} M_d - M_d^+ = i\,\epsilon\, M_u \quad (*) \\ \\ \epsilon \equiv \tan 2\beta \sin a \end{array}}$$

$\epsilon = 0 \iff a = 0$ or $\tan\beta = 0$ $(v_2 = 0)$

$$\Rightarrow \boxed{\begin{array}{c} M_d = M_d^+ \\ M_u = M_u^+ \end{array}}$$

ϵ = measure of P breaking

in Φ sector

$$\boxed{M_\ell = M_q^{+} \Rightarrow V_R = V_L}$$

$$\boxed{(*) \Rightarrow \varepsilon \leq m_b / m_t \simeq 10^{-2}}$$

$$\Downarrow$$

$$\boxed{V_R - V_L \simeq O(\varepsilon)}$$

<u>Holy Grail</u>

Tello

$$V_{R_{ij}} = V_{L_{ij}} - i\,\epsilon \frac{(V_L)_{ik} (V_L^{+} m_u V_L)_{kj}}{m_{d_k} + m_{d_j}} + \cdots$$

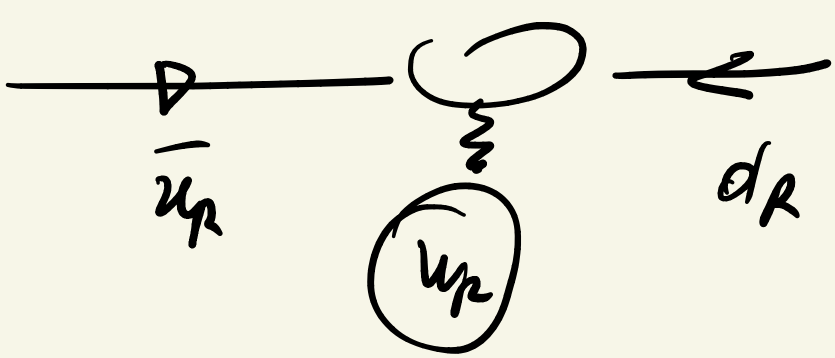
$$\boxed{\theta_R^{(i)} \simeq \theta_L^{(i)}} \quad \text{mixings}$$

$$\theta_R^{(12)} \equiv \theta_R^{(c)} \simeq \theta_L^{(12)} \quad (\equiv \theta_L^{c})$$

$$- \underset{\sim\, 10^{-2}}{\epsilon}\; \underset{\sim\, 10^{-2}}{\theta_{23}^{L}}\; \underset{\sim\, 10^{-3}}{\theta_{13}^{L}}\; m_t / m_s \quad (\sim 10^{3})$$

$$\lesssim 10^{-3} \; !$$

$$(V_R \simeq V_L)$$



LR works (miracle):

SM: $m_t \simeq 2 M_W$

$\theta_{13} \simeq 10^{-3}$

$\Downarrow$

$$\boxed{V_R \simeq V_L}$$

$\Downarrow$

$$\frac{g}{\cos 2\beta\, M_W} H_0\, \bar{d}_L V_L^{\dagger} \underbrace{\frac{m_u}{M_W}}_{\text{diagonal}} V_L\, d_R + h.c.$$

$\boxed{\text{2 generations}}$ $\quad V_L \equiv V_c = \begin{pmatrix} c & s \\ -s & c \end{pmatrix}$

Cabibbo

$$\bar{s}_L d_R \; : \; \frac{g}{\cos\beta}\left(V_L^{\dagger}\frac{m_u}{m_W}V_L\right)_{21}$$

$$\propto \left(V^{T}_{L\,21}\, m_u\, V_{L\,11} + V^{T}_{L\,22}\, m_c\, V_{L\,21}\right)$$

$$= c\,s\, m_u + c(-s)\, m_c$$

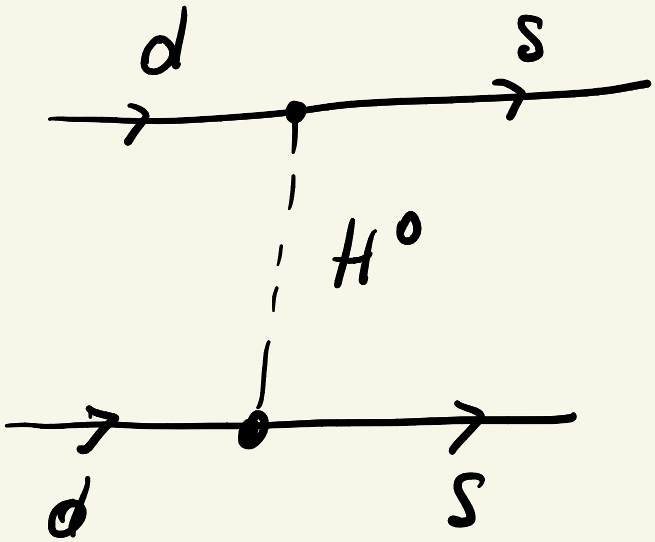
$$= -(m_c - m_u)\cos\theta_c \sin\theta_c$$

Flavor violation: (i) $\theta_c \neq 0$

(ii) $m_c - m_u \neq 0$

$$\Downarrow$$

$K - \bar{K}$ mixing $(\Delta S = 2)$



$$\boxed{\sin^2\theta_c \simeq 1/25}$$

$$\mathcal{H}_{eff}^{(\Delta S=2)}(H_0) = \frac{g^2}{\cos^2\beta}\,\frac{m_c^2\sin^2\theta_c\cos^2\theta_c}{M_H^2\,M_W^2}$$

$$\lesssim \mathcal{H}_{eff}^{(\Delta S=2)}(SM) \simeq G_F\,\frac{\alpha}{4\pi}\,\frac{m_c^2 \;\text{-- '' --}}{M_W^2}$$

$$\Downarrow$$

$$\frac{1}{m_H} \lesssim G_F\,\frac{\alpha}{4\pi} \qquad (\text{exp.})$$

$$\simeq 10^{-5}\cdot 10^{-3}\ \mathrm{GeV}^{-2}$$

$$\Rightarrow \boxed{\begin{array}{c} m_H \gtrsim 10^4 \text{ GeV} \\ \hline M_{W_R} \gtrsim 5 \text{ TeV} \end{array}} \quad (\simeq 10 \text{ TeV})$$

Q. How can we achieve that?

$\Longleftrightarrow$ Where is m_H coming from?