Neutrino Physics Course

Lecture $\overline{\underline{XXVI}}$

9/7/2021

LMU

Summer 2021

Seesaw and LR symmetry:

Case P

LR theory $\rightarrow m_\nu \neq 0$ ($\exists \nu_R$)

$\rightarrow$ $\boxed{\text{seesaw}}$

Crucial:

- $N_L \equiv C\bar{\nu}_R{}^T$

$$N_M = N_L + C\bar{N}_L{}^T \equiv$$

$$\equiv \nu_R + C\bar{\nu}_R{}^T$$

N_M produced through W_R

- $\underline{M}_N = V_R\, m_N\, V_R^T \leftarrow$ probe at LHC

- seesaw

$$\underline{M}_\nu = -M_D^T \frac{1}{M_N} M_D$$

$$\Downarrow$$

$$M_D = f(M_\nu, M_N) \text{ — probe}$$

the origin of neutrino mass

LR symmetry:

(a) C $\quad f_L \to C\bar{f}_R^T \propto f_R^*$

$$\Rightarrow \quad M_D^T = M_D$$

$$\Downarrow$$

$$M_D = i\, M_N \sqrt{1/M_N\, M_\nu}$$

$$\Theta_{\nu N} = \frac{1}{M_N} M_D = i\sqrt{1/M_N\, M_\nu}$$

(b) $\underline{P}$ = parity

$$f_L \longleftrightarrow f_R \qquad \begin{pmatrix} f_L \to U_L f_L \\ f_R \to U_R f_R \end{pmatrix}$$

$$\Downarrow$$

$$Y_\Phi = Y_\Phi^\dagger$$

$$\Phi \to U_L \Phi U_R^\dagger$$

$$\Updownarrow$$

$$\mathcal{L}_Y = \bar{f}_L Y_\Phi \Phi f_R + \bar{f}_R \Phi^\dagger Y_\Phi^\dagger f_L$$

$$\Phi \longrightarrow \Phi^\dagger$$

$$\Downarrow$$

$$\boxed{Y_\Phi = Y_\Phi^\dagger}$$

$$\tilde{\Phi} \equiv \sigma_2 \Phi^* \sigma_2 \Rightarrow \tilde{\Phi} \to U_L \tilde{\Phi} U_R^\dagger$$

$$\Rightarrow \boxed{\tilde{Y}_\Phi = \tilde{Y}_\Phi^\dagger}$$

$$M_D = Y_\Phi \langle \Phi \rangle$$

complex in general

$$\Phi = \begin{pmatrix} \phi_1 & \tilde{\phi}_2 \end{pmatrix}$$

$$\langle \phi_1 \rangle = v_1 \in R$$

$$\langle \phi_2 \rangle = v_2 e^{i\alpha} \in C$$

$$\tan\beta = v_2 / v_1$$

$\epsilon = \tan\beta \sin\alpha$ ← measure of complexity

$\epsilon = 0 \iff \langle\phi\rangle \in R$

$\boxed{\text{\underline{phen.}} \quad \epsilon \ll 1}$

$\boxed{\epsilon = 0 \ \text{\underline{limit}}}$

$$\Rightarrow \boxed{\begin{array}{l} M_D^{+} = M_D \\ M_\ell = M_\ell^{+} \end{array}}$$

SM + seesaw:

$$M_\nu = -M_D^{T} \frac{1}{M_N} M_D$$

$$\Downarrow$$

$$M_D = i\sqrt{M_N}\; O \,\sqrt{M_\nu}$$

$$O^T O = 1, \quad O \in C$$

$$\boxed{M_D = M_D^\dagger \overset{?}{\Longrightarrow} O = \text{fixed}}$$

$$\underbrace{\langle \Delta_L \rangle = 0, \quad \langle \Delta_R \rangle = M_R \gg M_W}_{\text{Maximal } \cancel{P}}$$

$$\not\Downarrow$$

$$\underline{M}_\nu = - \underline{M}_D^T \frac{1}{\underline{M}_N} \underline{M}_D \quad \text{(seesaw)}$$

$$M_D^\dagger = M_D$$

$$\Downarrow$$

$$M_D = f\left(\underline{M}_\nu, \underline{M}_N\right)$$

$\sim SU(2)_L$ triplet

<u>but:</u>

$$\underbrace{\langle \Delta_L \rangle}_{SU(2)_L \text{ triplet}} \propto \frac{\langle \Phi \rangle^2}{\underline{M}_R} \xrightarrow[M_R \to \infty]{} 0$$

$\uparrow$

$SU(2)_L$ singlet

$$\Downarrow$$

$$\mathcal{L}_y^{\Delta} = l_L^T C i\sigma_2 Y_\Delta \Delta_L l_L +$$

$$+ l_R^T C i\sigma_2 Y_\Delta \Delta_R l_R + h.c.$$

$$\Downarrow$$

$$\Rightarrow M_{\nu_R} = Y_\Delta \langle \Delta_R \rangle$$

$$= M_N^*$$

$$N_L = C \overline{\nu_R}^T \propto \nu_R^*$$

$$\Downarrow$$

$$\boxed{M_\nu = \frac{v_L}{v_R} M_N^* - M_D^T \frac{1}{M_N} M_D \qquad M_D^+ = M_D}$$

$$\Downarrow$$

$$M_\nu = \frac{v_L}{v_R} M_N^* - M_D^* \frac{1}{M_N^*} M_D$$

$$\Downarrow$$

$$M_\nu^* = \frac{v_L}{v_R} M_N - M_D \frac{1}{M_N^*} M_D^*$$

$$\Downarrow$$

$$\frac{1}{\sqrt{M_N}} M_\nu^* \frac{1}{\sqrt{M_N}} = \frac{v_L}{v_R} \quad -$$

$$- \underbrace{\frac{1}{\sqrt{M_N}} M_D \frac{1}{\sqrt{M^*}}}_{H} \underbrace{\frac{1}{\sqrt{M_N^*}} M_D^* \frac{1}{\sqrt{M_N}}}_{H^*}$$

$$\therefore \boxed{H^+ = H}$$

Proof:

$$H^+ = \frac{1}{\sqrt{M_N{}^T}}\, M_D{}^+ \, \frac{1}{\sqrt{M_N{}^+}}$$

$$\Downarrow \qquad M_N{}^T = M_N \quad (\text{Majorana})$$

$$H^+ = \frac{1}{\sqrt{M_N}}\, M_D \, \frac{1}{\sqrt{M_N{}^*}} = H \quad \checkmark$$

Q.E.D.

$$\underbrace{\boxed{HH^* = \frac{v_L}{v_R} - \frac{1}{\sqrt{M_N}}\, M_\nu{}^* \frac{1}{\sqrt{M_N}}}}_{\text{symmetric} \equiv S}$$

$$(H^+ = H) \Downarrow \qquad (H^* = H^T)$$

$$\boxed{HH^T = S} \quad (1) \quad (HH^* = S)$$

$$\boxed{S \equiv \frac{V_L}{V_R} \frac{1}{\sqrt{M_N}} M_D^* \frac{1}{\sqrt{M_N}}}$$

$\boxed{H^+ = H}$

Tello, G. S.
'16–'20

$S = HH^*$

$S^* = H^* H$

$$(1) \quad SH = HH^T H = H(H^* H)$$
$$= HS^*$$

2×2 case

Wikipedia

$\sqrt{M}$ = analytic expression

$\Downarrow$

$$\boxed{\begin{gathered} H_{2\times 2} = \sqrt{SS^*}\,\frac{1}{\sqrt{S^*}} \qquad 2\times 2 \\ (HH^T = S, \quad S^T = S) \\ \text{Prove!} \end{gathered}}$$

$\underline{n \times n}$ (3×3)

$$HH^T = S$$

$$S^T = S, \quad H = H^+$$

$$\boxed{\begin{gathered} \left(S \sim M_\nu / M_N\right) \\ \left(H \sim \Theta \sim M_D / M_N\right) \end{gathered}}$$

$$S = V d V^T \qquad V V^+ = V^+ V = 1$$

$\uparrow$ diagonal

$$H = V\sqrt{d}\,V^{+}$$

$$\Rightarrow H^{+} = V\sqrt{d}\;V^{+}$$

$$\Downarrow$$

$$H H^{T} = V\sqrt{d}\,\underbrace{V^{*}\,V^{*}}_{???}\sqrt{d}\,V^{T}$$

fails !

Jordan decomposition

$$S = O\,s_J\,O^{T}$$

$\hookrightarrow$ often diagonal,

(but not always)

$$\boxed{\begin{array}{l}\text{let us discuss} \\ \quad s_J = \text{diagonal}\end{array}}$$

$$S = O \underbrace{s_J}_{\text{diagonal}} O^T = O\, d_J\, O^T$$

$$H H^T = H H^* = S \qquad (H = H^+)$$

$$H = O \sqrt{d_J}\, O^+$$

$$H^+ = O \sqrt{d_J^*}\, O^+ \neq H$$

$$H = O\sqrt{d_J}\,E\,O^+$$

$$\Rightarrow H^+ = O\,E^+\sqrt{d_J^*}\,O^+ = H$$

$$\Rightarrow \boxed{E^+\sqrt{d_J^*} = \sqrt{d_J}\,E}$$

$$\boxed{E = \,?}$$

- $HH^T = S = O\,d_J\,O^T$

$$\shortparallel$$

$HH^* \Rightarrow$

$$\left.\begin{aligned} &\mathrm{Tr}\,HH^* \in R \\ &\mathrm{Tr}(HH^*)^2 \in R \\ &\mathrm{Tr}(HH^*)^3 \in R \end{aligned}\right\} \begin{aligned} &\mathrm{Tr}\,S \in R \\ &\mathrm{Tr}\,S^2 \in R \\ &\mathrm{Tr}\,S^3 \in R \end{aligned}$$

$$\mathrm{Tr}\, S = \mathrm{Tr}\, d_J$$

$\Downarrow$ <u>Prove!</u>

$$\boxed{\begin{aligned} &\mathrm{I})\ d_J^{\mathrm{I}} \overset{\text{diag}}{=} \Lambda(d_1, d_2, d_3) \qquad d_i \in R \\ &\mathrm{II})\ d_J^{\mathrm{II}} \overset{\text{diag}}{=} \Lambda(d, d_0, d^*), \qquad d_0 \in R \end{aligned}}$$

$$E^+ \sqrt{d_J^*} = \sqrt{d_J}\, E \qquad (\text{def. of } E)$$

$$\Downarrow$$

$$\mathrm{I})\ E^{\mathrm{I}} = \mathbb{1} \qquad (d_J^* = d_J)$$

$$E^+ \sqrt{d_J} = \sqrt{d_J}\, E$$

II) $d_J^* = (d^*, d_0, d)$

$$d_J = (d, d_0, d^*)$$

$$\Downarrow$$

$$E^{II} = \begin{pmatrix} 0 & 0 & 1 \\ 0 & 1 & 0 \\ 1 & 0 & 0 \end{pmatrix}$$

$$(E^{II})^+ = E^{II}$$

$$E^{II} \begin{pmatrix} d & & 0 \\ & d^0 & \\ 0 & & d^* \end{pmatrix} E^{II} = \begin{pmatrix} d^* & & 0 \\ & d^0 & \\ 0 & & d \end{pmatrix}$$

$\Rightarrow \boxed{H = H^+}$ (I, II)

$$HH^* = S$$

$$H = O\sqrt{d_J}\, E\, O^*$$

$$H^T = O^* E^T \sqrt{d_J}^T\, O^T$$

$$\Downarrow$$

- $$HH^T = O\sqrt{d_J}\, E\, \underbrace{O^+ O^*}_{1} E^T \sqrt{d_J}^T O^T$$

$$= O\sqrt{d_J}\, E\, E^T \sqrt{d_J}^T O^T$$

- $$HH^* = O\sqrt{d_J}\, E\, \overbrace{O^+ O^*}^{1} \sqrt{d_J}^* E^* O^T$$

$$= O\sqrt{d_J}\, E \sqrt{d_J}^* E^* O^T$$

$$\mathrm{I}: \quad E^{\mathrm{I}} = \mathbb{1}, \quad d_j \in \mathbb{R}$$

$$HH^* = HH^T = O\sqrt{d_J}^{\mathrm{I}}\sqrt{d_J}^{\mathrm{I}}O^T$$

$$= O\,d_J^{\mathrm{I}}\,O^T = S \quad \checkmark$$

$$\mathrm{II}: \quad E^{\mathrm{II}} = \begin{pmatrix} 0 & & 1 \\ & 1 & 0 \\ 1 & 0 & 0 \end{pmatrix}, \quad \left(d_J^T = d_J^*\right)$$

$$E^{\mathrm{II}}\sqrt{d_J^*} = \sqrt{d_J}\,E^{\mathrm{II}}$$

$$\Rightarrow \quad HH^* = O\sqrt{d_J}^{\mathrm{II}}\sqrt{d_J}^{\mathrm{II}}E^{\mathrm{II}}E^{\mathrm{II}*}O^T$$

$$= O\,d_J\,O^T = S$$

$$E^{\mathrm{II}}E^{\mathrm{II}*} = \left(E^{\mathrm{II}}\right)^2 = \mathbb{1}$$

$\boxed{\text{Q.E.D.}}$

- $S_J = d_J =$ diagonal

$$\Rightarrow \quad H = O \sqrt{d_J}\, E\, O^+$$

$$S = O\, d_J\, O^T$$

$$\mathrm{I}: \quad d_J \in R, \quad E^{\mathrm{I}} = \mathbb{1}$$

$$\mathrm{II}: \quad d_J^T = d_J^*, \quad E^{\mathrm{II}} = \begin{pmatrix} 0 & 0 & 1 \\ 0 & 1 & 0 \\ 1 & 0 & 0 \end{pmatrix}$$

<u>completed</u> $\underbrace{\text{for all } S_J \text{ in } 3\times 3}$

$\boxed{\text{Tello, G. S. 2020}}$

$$H = \frac{1}{\sqrt{M_N}}\, M_D\, \frac{1}{\sqrt{M_N^*}}$$

$$S = v_L/v_R - \frac{1}{\sqrt{M_N}}\, M_\nu^*\, \frac{1}{\sqrt{M_N}}$$

$$\Rightarrow \boxed{M_D = \sqrt{M_N}\, H \sqrt{M_N^*}}$$

<u>illustrate</u> $\quad v_L = 0$ (seesaw)

$$\boxed{v_L = v_R} \quad \text{(example)}$$

$$M_\nu \equiv V_L^*\, m_\nu\, V_L^\dagger$$

$$M_N = M_{\nu_R}^* = V_R m_N V_R^T$$

$$S = -\frac{1}{\sqrt{M_N}} M_\nu^* \frac{1}{\sqrt{M_N}}$$

$$= -\sqrt{1/M_N}\, M_\nu^* \sqrt{1/M_N}$$

$$= -\sqrt{V_R^* \frac{1}{m_N} V_R^\dagger}\; V_L m_\nu V_L^T \sqrt{V_R^* \frac{1}{m_N} V_R^\dagger}$$

$$= \sqrt{V_L^* \frac{1}{m_N} V_L^\dagger}\; V_L m_\nu V_L^T \sqrt{V_L^* \frac{1}{m_N} V_L^\dagger}$$

<u>Task:</u>

$$M_D = f(M_N, M_\nu)$$

$$= f(m_\nu, m_N, V_L)$$

$$\sqrt{U M U^{-1}} = U \sqrt{M}\, U^{-1}$$

$\boxed{\text{Task}}$ find explicit (**)

$$M_D = f\,(m_N, m_\nu, V_L)$$

(**)

$\boxed{\text{Summary}}$

$$P = LR \Rightarrow$$

$$M_D = M_D^+$$

$$M_D = \sqrt{M_N}\, H \sqrt{M_N}^*$$

$$HH^T = S^* = V_L/V_R - \frac{1}{\sqrt{M_N}} M_\nu^* \frac{1}{\sqrt{M_N}}$$

· Jordan $S = O \, \Lambda_J \, O^T$

$H = O \sqrt{\Lambda_J} \, E \, O^+ \quad \therefore \sqrt{\Lambda_J} \, E = E^+ \sqrt{\Lambda_J}^*$

$E = f(\Lambda_J)$

find E for $\Lambda_J = d_J$

Tasks

① a) $d_J = d_J^* \Rightarrow E = 1$

b) $d_J^T = d_J^* \Rightarrow E = \text{anti } \mathbb{1}$

② $M_D = f(V_L, m_N, m_\nu)$