# Neutrino Physics Course

## Lecture $\overline{\underline{\mathrm{XXV}}}$

6/7/2021

LMU

Summer 2021

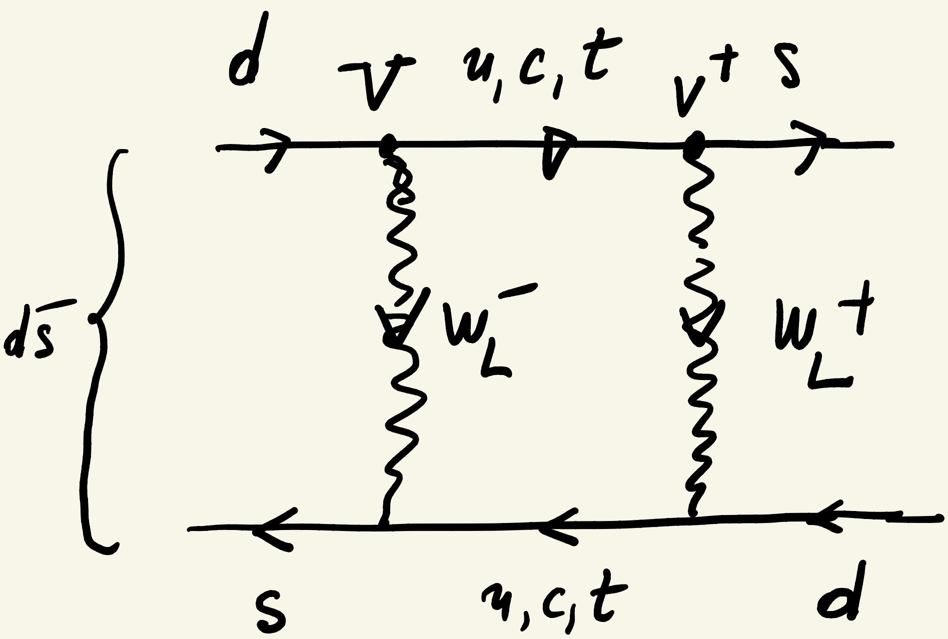
$(\underline{W^{\pm}, Z})$ $\boxed{\text{LEP}}$

Factory $10^{9}$

- <u>Low energy</u> $\boxed{(\text{SM})}$

$$\frac{4G_F}{\sqrt{2}} J_{\mu}^{W} \bar{J}^{\mu}_{W} = \mathcal{H}_{eff}^{(W)}$$

$$J^{\mu}_{W} = \bar{u}_L \gamma^{\mu} d_L + \bar{\nu}_L \gamma^{\mu} e_L$$




(LL)

$\Delta S = 2$

$\bar{d}s$ $\boxed{K^{0} - \bar{K}^{0} \text{ mixing}}$

$$\begin{pmatrix} u \\ d \end{pmatrix}_L^0 \quad \begin{pmatrix} c \\ s \end{pmatrix}_L^0 \quad u_R^0, d_R^0, c_R^0, s_R^0$$

$$J^\mu_w(q) = (\bar{u}_L \, \bar{c}_L) \gamma^\mu V_c \begin{pmatrix} d \\ s \end{pmatrix}_L$$

$$V_c = \begin{pmatrix} \cos\theta_c & \sin\theta_c \\ -\sin\theta_c & \cos\theta_c \end{pmatrix}$$

$$\frac{G_F}{\sqrt{2}} = \frac{g^2}{8 M_{W_L}^2} \Downarrow \quad \boxed{\text{static}}$$

$$H_{eff}(\Delta S = 2) = \{ m_u = 0 \}$$

$$\underbrace{\frac{G_F}{\sqrt{2}}}_{\text{weak}} \underbrace{\left(\frac{\alpha}{4\pi}\right)}_{\text{loop}} \underbrace{\frac{m_c^2}{M_W^2}}_{\text{GIM (mass)}} \underbrace{\sin^2\theta_c \cos^2\theta_c}_{\text{GIM (mixing)}} \underbrace{\bar{s}\gamma_\mu L d \, \bar{s}\gamma^\mu L d}_{d=6}$$

weak loop GIM (mass) GIM (mixing) th.ci

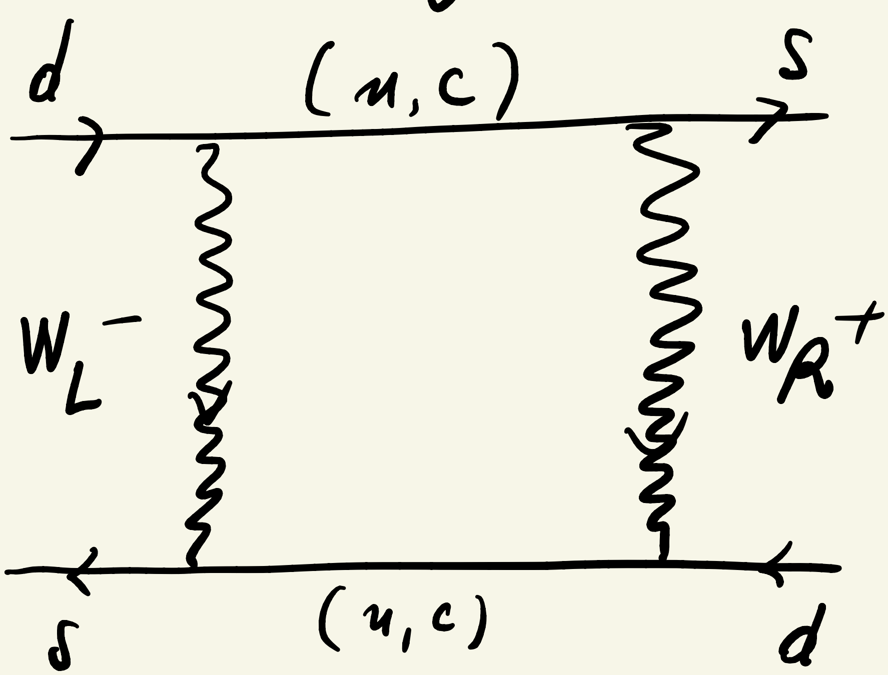
GIM: $m_c = 0 \Leftrightarrow V_c = \mathbb{1}$

• <u>low energy</u> (LR)

$\Delta s = 2 \quad (k - \bar{k})$ | <u>rare process</u>

<u>why?</u> $\dfrac{\delta m_k}{m_k} \simeq 10^{-14}$

$\Downarrow$




(LR)

$$M_{W_R} \gg M_{W_L}$$

Q. Why compute it?

<u>Lazy man's approach:</u>

$$(LR) = (LL)\left(\frac{M_{W_L}}{M_{W_R}}\right)^2 \qquad (+\text{h.c.})$$

$$\Downarrow$$

$$(LR)/(LL) \ll 1$$

Beall, Bander, Soni '1981

Compute LR

$\Downarrow$

$$\boxed{M_{W_R} \gtrsim 2.5\ \mathrm{TeV}}$$

Much better limit

$$\frac{(LR)_{eff}}{(LL)_{eff}} \simeq 10^3 \left(\frac{M_L}{M_R}\right)^2$$

lazy man's mistake:

$$1 \longrightarrow 10^3$$

LL: $\bar{s}\gamma_\mu L d \; \bar{s}\gamma^\mu L d$

$$\langle K^0 | (\bar{s}\gamma^\mu L d)^2 | \bar{K}^0 \rangle = ?$$

$$\downarrow$$

$$= \sum_n \langle K^0 | \bar{s}\gamma_\mu L d | n \rangle \langle n | \bar{s}\gamma^\mu L d | \bar{K}^0 \rangle$$

$$\sum_n |n\rangle\langle n| = 1$$

$$|n\rangle = |0\rangle, |\pi\rangle, |2\pi\rangle \ldots$$

$$\boxed{\text{vacum dominence:}}$$

$$= \langle K^0 | \bar{s}\gamma^\mu L d | 0 \rangle \langle 0 | \bar{s}\gamma_\mu L d | \overline{K^0} \rangle$$

$$+ \boxed{\text{small}}$$

???

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$(u, c)$ : CP conservation

$$K^0 \xleftrightarrow{CP} \overline{K^0}$$

$$\Rightarrow \quad K^\pm = \frac{K_0 \pm \overline{K^0}}{\sqrt{2}}$$

CP : $K^+$ (1)

$K^-$ (−1)

$\Downarrow$

$$M_{K\bar{K}} \equiv \begin{array}{c} K^0 \\ \bar{K}^0 \end{array} \begin{pmatrix} m_K & \boxed{\delta m_K} \\ \boxed{\delta m_K} & m_K \end{pmatrix} \qquad \boxed{\delta m_K \neq 0}$$

CPT : mass of particle

= mass of anti -"-

$$M_{K\bar{K}} \begin{pmatrix} 1 \\ 1 \end{pmatrix} = (m_K + \delta m_K) \begin{pmatrix} 1 \\ 1 \end{pmatrix}$$

$$M_{K\bar{K}} \begin{pmatrix} 1 \\ -1 \end{pmatrix} = (m_K - \delta m_K) \begin{pmatrix} 1 \\ -1 \end{pmatrix}$$

$$\underset{K, \bar{K}^0}{\begin{pmatrix} 1 \\ 1 \end{pmatrix}} \equiv K_0 + \overline{K_0}$$

$\begin{pmatrix}1\\-1\end{pmatrix} \equiv K_0 - \bar{K}_0$

$$\boxed{K^+ = u\bar{s} \\ \downarrow \\ K^0 \equiv d\bar{s}}$$

$\therefore$ $$\boxed{\Delta m_K = 2\,\delta m_K}$$

$\bar{K}^0 \equiv \bar{d}s$

$\Updownarrow$

$$\delta m_K = \langle K^0 |\, H_{eff}(\Delta s = 2)\, | \bar{K}^0 \rangle$$

$\quad\quad\quad\quad\quad\quad\quad\quad\; \| $

$\quad\quad\quad\quad\quad\quad \cdots \boxed{\bar{s}\gamma_\mu L d\, \bar{s}\gamma^\mu L d}$

$\langle K^0 | = \langle \bar{d}s |$ ; $|\bar{K}^0\rangle = |\bar{d}s\rangle$

$$\boxed{\langle K^0 | \bar{s}\gamma^\mu L d | 0 \rangle = ?}$$

vacuum dominance!

step back: $\boxed{\text{pion decay}}$

$$\pi^- \longrightarrow l\,\bar{\nu}_e$$

$$\left\{\begin{array}{l} \mu\,\bar{\nu}_\mu \\ e\,\bar{\nu}_e \end{array}\right\} \longleftarrow \boxed{\text{negligible}}$$

$$J^\mu_W = \bar{u}_L \gamma^\mu d_L + \bar{\nu}_L \gamma^\mu e_L$$

$\pi^-$ at rest:

$$\vec{J}_{in} = 0 = \vec{S}_{in} + \vec{L}_{in}$$

$e_L$ $\quad$ $(\bar{\nu}_e)_R$

$\Rightarrow S_e^z\,(1/2)$ $\quad$ $\Rightarrow S_{\bar{\nu}}^z\,(1/2)$

$m_e = 0$

$$S^z_{final} = 1$$

$(m_\pi \simeq \overset{\uparrow}{140}\,\text{MeV})$ $\Rightarrow J_z^{\text{final}} = 1$

$(l_z^{\text{final}} = 0)$

$J_z^{\text{in}} = 0$ $\qquad J_z^{\text{final}} = \cancel{0}\ 1$

for electron

$$\Rightarrow \quad \underline{\pi^- \not\to e + \bar{\nu}_e \qquad (m_e = 0)}$$

$$\Rightarrow \boxed{\frac{\Gamma(\pi \to \mu^- \bar{\nu}_\mu)}{\Gamma(\pi^- \to e^- \bar{\nu}_e)} \simeq \left(\frac{m_\mu}{m_e}\right)^2 \simeq 10^4}$$

Peskin, many others

$\Downarrow$

pion decay = proof of weak int. being dual

$\boxed{\text{WRONG!}}$

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$\boxed{\text{compute the rate}}$

$$\mathcal{H}_{eff}^{(L)} = \frac{4G_F}{\sqrt{2}}\, \bar{u}_L \gamma^\mu d_L \; \bar{e}_L \gamma^\mu \nu_L$$

$$= \frac{G_F}{\sqrt{2}}\, \bar{u} \gamma^\mu (1+\gamma_5) d \; \bar{e} \gamma_\mu (1+\gamma_5) d$$

let's <u>not</u> assume chirality

Imagine

$$\mathcal{H}_{eff} = \frac{G_F}{\sqrt{2}} \bar{u} \gamma^\mu (a+b\gamma_5) d \, \bar{e} \gamma^\mu (a'+b'\gamma_5)\nu$$

$$\pi^- = u\bar{d}$$

$$\langle 0 | \mathcal{H}_{eff} | \pi^- \rangle =$$

$$= \frac{G_F}{\sqrt{2}} \langle 0 | \bar{u} \gamma^\mu (a+b\gamma_5) d | u\bar{d} \rangle \times$$

$$\times \bar{e} \gamma^\mu (a'+b'\gamma_5)\nu$$

$$\boxed{\langle 0 | \bar{u} \gamma^\mu (a+b\gamma_5) d | \underset{\text{pseudo-scalar}}{\pi^-} \rangle = ?}$$

$$\langle 0|\bar{u}\gamma^\mu d|\pi^-\rangle = 0$$

$$\langle 0|\bar{u}\gamma^\mu\gamma_5 d|\pi^-\rangle \equiv \sqrt{2}\, f_\pi\, p_\mu$$

(Lorentz)

pion decay constant

$$A = a\sqrt{2}\, f_\pi\, p_\mu\, \bar{e}\gamma^\mu (a' + b'\gamma_5)\nu$$

$$p_\mu = p_\mu^e + p_\mu^{\bar{\nu}}$$

$$p_\mu\, \bar{e}\gamma^\mu (a' + b'\gamma_5)\nu$$

$$= m_e\, \bar{e}(a' + b'\gamma_5)\nu$$

$(m_\nu = 0)$

$$p_\mu \gamma^\mu \psi = m \psi$$

$$A(\pi^- \to l \bar{\nu}) \propto m_e$$

for any axial int.

$$A(\pi^- \to l \bar{\nu}) \propto G_F \, f_\pi \, m_\mu \; \bar{e}\,(a' + b'\gamma_5)\,\nu$$

$$\Downarrow$$

$$\Gamma(\pi^- \to l \bar{\nu}) \cong G_F^2 \, f_\pi^2 \, m_\mu^2 \, \frac{m_\pi}{8\pi}$$

$f_\pi = ?$ $\qquad$ $m_\mu \cong 10^{-1}$ GeV $\simeq m_\pi$

$$\simeq 10^{-10} f_\pi^2 \, 10^{-2} \cdot 10^{-2} \, \text{GeV}$$

$$\tau_{\pi^-} \simeq 10^{14} f_\pi^{-2} \, \text{GeV}^{-1} \simeq 10^{14} f_\pi^{-2} \, 10^{-14} \, \text{cm}$$

$$\simeq f_\pi^{-2} \, 10^{-10} \, \text{sec} \simeq 10^{-8} \, \text{sec}$$

$$\Rightarrow \boxed{f_\pi \simeq 100 \text{ MeV}}$$

<u>Strong int.</u> $\Rightarrow$ SU(2) isospin

$\rightarrow$ SU(3) symmetry

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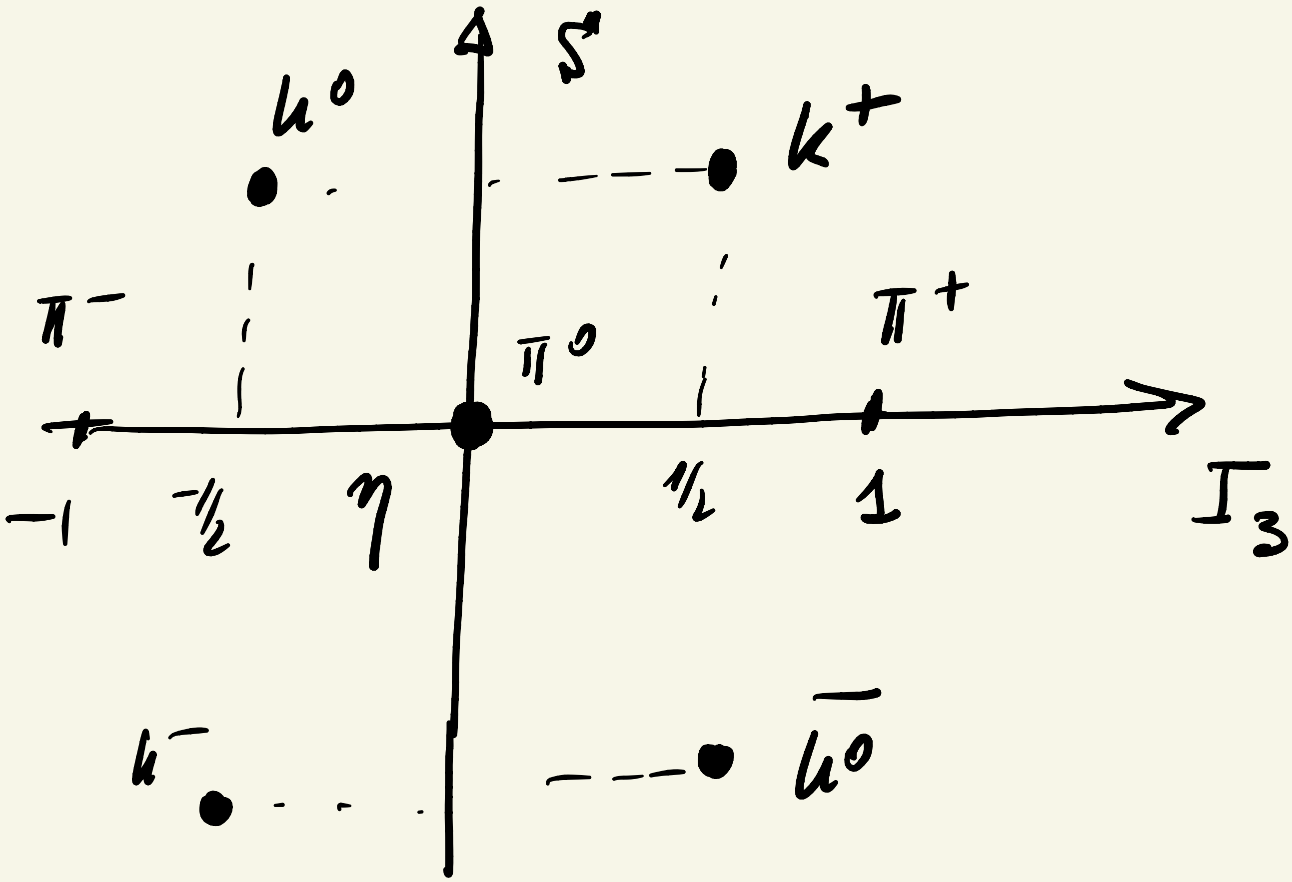



$$\boxed{SU(3) \Rightarrow f_K \simeq f_\pi}$$

$$\langle K^0 | (\bar{s}\gamma^\mu L d)^2 | \bar{K}^0 \rangle =$$

$$= \sum_n \langle K^0 | \bar{s}\gamma^\mu L d | n \rangle \langle n | \bar{s}\gamma^\mu L d | K^0 \rangle$$

$|0\rangle \longrightarrow$ dominates

$\boxed{\text{assumption}}$

$(\cos\theta_c \simeq 1)$

$$\delta m_K = \langle K^0 | \mathcal{H}_{eff}^{(\Delta S=2)} | \bar{K}^0\rangle =$$

$$= \frac{G_F}{\sqrt{2}} \frac{\alpha}{4\pi} \left(\frac{m_c}{M_W}\right)^2 \sin^2\theta_c \, \langle K^0 | (\bar{s}\gamma^\mu L d)^2 | \bar{K}^0\rangle$$

$$= \frac{G_F}{\sqrt{2}} \frac{\alpha}{4\pi} \quad - \text{''} - \quad \langle K^0 | \bar{s}\gamma_\mu L d | 0\rangle \langle 0 | \bar{s}\gamma^\mu L d | \bar{K}^0\rangle$$

vacuum dominance

$$f_{\pi^-} = f_{\pi^0} \equiv f_\pi \quad (SU(2))$$

$$SU(3) \quad f_{K^-} = f_{K^0} \equiv f_K \quad (SU(2))$$

$$\delta m_u = \frac{G_F}{\sqrt{2}} \frac{\alpha}{4\pi} \sin^2\theta_c \frac{m_c^2}{M_W^2} f_\pi^2 \, m_K$$

dimensional

$m_u \simeq m_d \simeq \text{MeV}$

$m_s \simeq 100 \ \text{MeV}$

$$\boxed{\frac{\delta m_u}{m_u} = \frac{G_F}{\sqrt{2}} \frac{\alpha}{4\pi} \sin^2\theta_c \frac{m_c^2}{M_W^2} f_\pi^2}$$

$G_F \simeq 10^{-5}\,\text{GeV}^{-2}$, $f_\pi \simeq 100\,\text{MeV} \simeq 10^{-1}\,\text{GeV}$

$\sin^2\theta_c \simeq 1/25$, $\frac{\alpha}{4\pi} \simeq 10^{-3}$

$\Downarrow$

$$\frac{\delta m_n}{m_n} \simeq 10^{-8} \cdot 10^{-2} \cdot 10^{-1} \left(\frac{m_c}{M_W}\right)^2 \simeq 10^{-14}$$

Gaillard, Lee '74

$\boxed{\text{exp}}$

$$\Rightarrow \left(m_c/M_W\right)^2 \simeq 10^{-3}$$

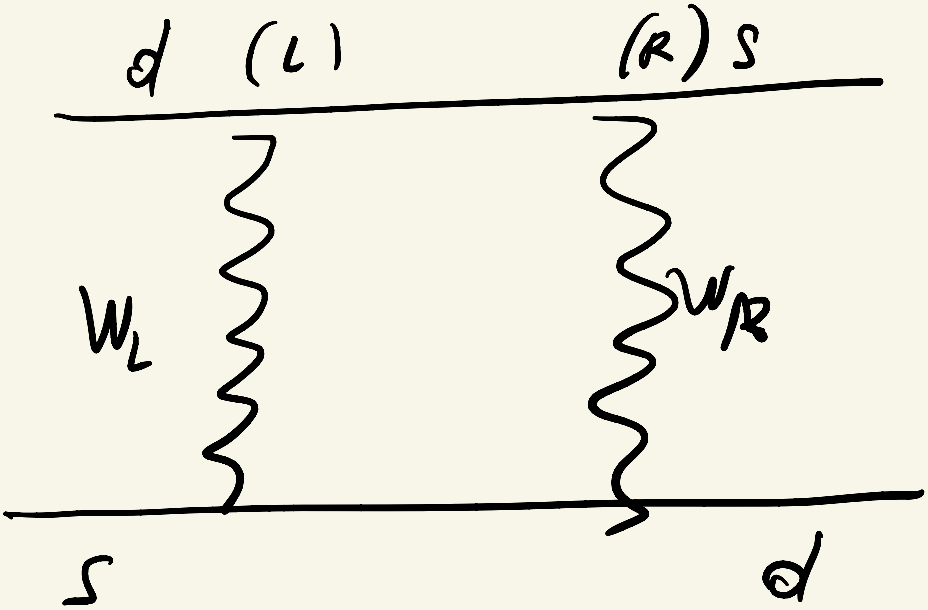
$$\Rightarrow \boxed{m_c \simeq 1.5\ \text{GeV}}$$

$\Downarrow$

great success at GIM (SM)

+ vacuum dominance

# LR $K-\bar{K}$ diagram




$$\frac{LR}{LL} = e \left(\frac{M_L}{M_R}\right)^2$$

$$\mathcal{H}_{eff}^{(\Delta S=2)}(LR) = \frac{G_F}{\sqrt{2}} \frac{\alpha}{4\pi} \sin^2\theta_c \left(\frac{m_c}{M_W}\right)^2 \left(\frac{M_L}{M_R}\right)^2$$

$$\times (8)\ \ln \frac{m_c^2}{M_W^2} (\bar{s} L d)(\bar{s} R d)$$

numerics

$(8)$ explain later (think*)

can be derived

- numerical: $8 \times 8$

- $\langle K^0 | \bar{s} L d\, \bar{s} R d | \bar{K}^0 \rangle =$

$= \{$ vacuum dominance $\}$

$$= \langle K^0 | \bar{s} L d | 0 \rangle \langle 0 | \bar{s} R d | \bar{K}^0 \rangle$$

$$= \frac{1}{4} \langle K^0 | \bar{s} \gamma_5 d | 0 \rangle \langle 0 | \bar{s} \gamma_5 d | \bar{K}^0 \rangle (-1)$$

$$\frac{\langle LR \rangle}{\langle LL \rangle} \simeq \frac{\langle K^0 | \bar{s}\gamma_5 d | 0 \rangle \langle 0 | \bar{s}\gamma_5 d | \bar{K}^0 \rangle}{\underbrace{\langle K^0 | \bar{s}\gamma_5\gamma_\mu d | 0 \rangle}_{p_\mu f_K} \langle 0 | \bar{s}\gamma_5\gamma^\mu d | \bar{K}^0 \rangle}$$

$$p_\mu \bar{s}\gamma_5\gamma_\mu d = m_s \bar{s}\gamma_5 d$$

$(m_d = m_u = 0)$ $(\sim \text{MeV})$

$m_s \simeq 100$ MeV)

$$\Downarrow$$

$$\underbrace{\bar{s}\gamma_5 d}_{(LR)} = \left(\frac{p_\mu}{m_s}\right) \underbrace{\bar{s}\gamma_\mu\gamma_5 d}_{(LL)}$$

$\Downarrow$ vacuum dominance

$$\langle 0 | \bar{s}\gamma_\mu(1\pm\gamma_5)d|\bar{K}^0\rangle =$$

$$= \langle 0|\bar{s}\gamma_\mu\gamma_5 d|\bar{K}^0\rangle(\pm)$$

$$\langle 0|\bar{s}(1\pm\gamma_5)d|\bar{K}^0\rangle = \langle 0|\bar{s}\gamma_5 d|\bar{K}^0\rangle (\pm)$$

$$\Downarrow$$

$$\frac{\langle LR\rangle}{\langle LL\rangle} = \frac{p_\mu p^\mu}{m_s^2} = \frac{m_K^2}{m_s^2} \simeq 25$$

(vacuum dominance)

$m_K \simeq 500$ MeV

$m_s \simeq 100$ MeV

$$\Downarrow$$

$$\frac{\langle LR \rangle_{eff}}{\langle LL \rangle_{eff}} \simeq \underbrace{8 \cdot 8 \cdot 25}_{10^3} \left(\frac{M_L}{M_R}\right)^2$$

$$\Rightarrow \boxed{M_{W_R} \gtrsim (2.5 - 3)\,\text{TeV}}$$

$$\boxed{\text{'80s}}$$

obsolete !

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$$\boxed{\text{LHC}} \qquad \boxed{M_{W_R} \gtrsim 5\,\text{TeV}}$$