Neutrino Physics Course

Lecture XXIV

2/7/2021

LMU

Summer 2021

LR theory: SSB and seesaw

$\rightarrow$ revisited

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or Letter:

$$\boxed{\text{Fundamental of SSB}}$$

$$G_{new} \longrightarrow G_{SM}$$

M_{new}

$$\Big\downarrow \langle \phi_{SM} \rangle = M_W$$

$\shortparallel$

$\langle \phi_{new} \rangle$ $\quad U(1)_{em} \; (\times SU(3)_c)$

$\Downarrow$

$$M_{new} \gg M_W$$

(LHC)

- $M_{new} = \langle \phi_{new} \rangle$

SM = <u>renormalizable</u>

$d=4$ interactions (small, finite # of terms)

$\downarrow$

<u>infinite # predictions</u>

$\Leftrightarrow$ SM does <u>not</u> depend on physics beyond

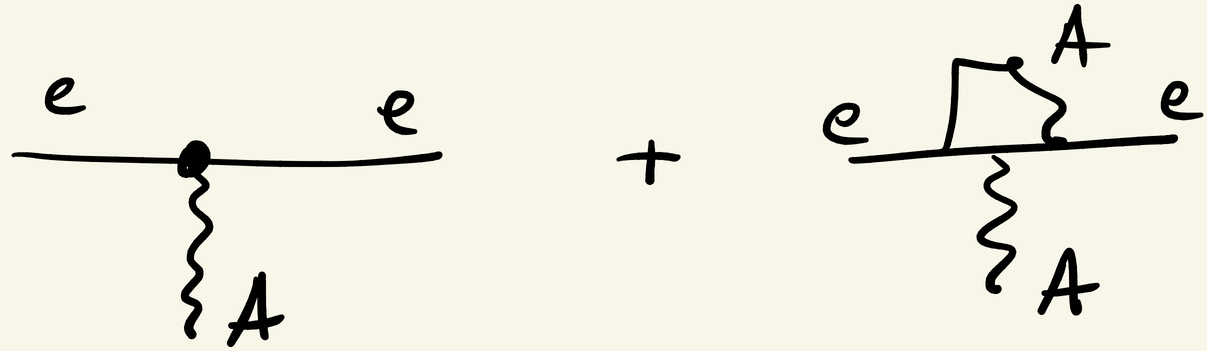
<u>better:</u>

$$\mathcal{A}_{SM} = \mathcal{A}_{SM}\,(\text{direct}) + $$
$$+ O\left(M_W / M_{new}\right)$$

$$\Leftrightarrow \quad M_{new} \to \infty \Rightarrow SM$$

decoupling theorem

<u>recall</u> em charge e:



$$\underline{\underline{e(\mu)}} = e_0 \left(1 + \frac{e_0^2}{16\pi^2} \ln \frac{\Lambda}{\mu} \right)$$

$$\Lambda = \text{cut-off}$$

$$= M_{new}$$

$$\underline{\underline{m_e(\mu)}} = f(\Lambda/\mu)$$

$$\underline{\lambda(\mu)} = \ldots$$

similar

$$\Downarrow$$

$$\boxed{\text{physical amplitudes do \underline{not} depend on } \Lambda \ (\Lambda \to \infty)}$$

$$\Updownarrow$$

$$A_{physical} = A_{SM}\left(1 + O\left(\frac{M_W}{\Lambda}\right)^n\right)$$

$$= A_{SM}\left(1 + O\left(\frac{M_W}{M_{new}}\right)^n\right)$$

Appelquist, Carrazone
'74-'75

LR theory: SSB

$$G_{LR} \xrightarrow[\langle \Delta_R \rangle = M_R]{} G_{SM}$$

$$G_{LR} = SU(2)_L \times \underbrace{SU(2)_R \times U(1)_{B-L}}_{\downarrow \, \langle \Delta_R \rangle \atop U(1)_Y}$$

$$\Delta_R \longleftrightarrow \Delta_L \quad \therefore$$

$$\langle \Delta_L \rangle = 0, \quad \langle \Delta_R \rangle = M_R$$

$$\varphi_L \longleftrightarrow \varphi_R$$

real, scalar fields

$$(\varphi_L^4 + 2\varphi_L^2\varphi_R^2 + \varphi_R^4)$$

$$V = -\frac{\mu^2}{2}\left(\varphi_L^2+\varphi_R^2\right) + \frac{\lambda}{4}\left(\varphi_L^2+\varphi_R^2\right)^2$$

$$+ \frac{\lambda'}{2}\varphi_L^2\varphi_R^2 \qquad (\lambda > 0)$$

$\lambda' = 0$ $\quad \langle \varphi_L^2 + \varphi_R^2 \rangle = \mu^2/\lambda$

flat direction

$\boxed{\cdot \ \lambda' > 0}$ $\quad \Rightarrow \langle \varphi_L \rangle = 0, \ \langle \varphi_R \rangle \neq 0$

$$m_{\varphi_L}^2 = -\mu^2 + \lambda\langle\varphi_R\rangle^2 + \lambda'\langle\varphi_R^2\rangle$$

$$\simeq \lambda'\langle\varphi_R\rangle^2 \sim M_R^2$$

$$\boxed{\begin{array}{c}\varphi_L \text{ and } \varphi_R \text{ are heavy} \\ m_{\varphi_L, \varphi_R} \sim M_R\end{array}}$$

- $\varphi_L, \varphi_R \longrightarrow \underbrace{\Delta_L, \Delta_R}_{\text{adjoints},\ B-L=2}$

$$\Delta = \begin{pmatrix} \delta^+ & \delta^{++} \\ \delta^0 & -\delta^+ \end{pmatrix}$$

doubly charged

neutral

charged

• turn on ϕ_{SM} !. $\langle \phi_{SM} \rangle = M_W$

LR: $\phi_{SM} \subseteq \Phi$ $(B-L=0)$

$$\boxed{\Phi \to U_L \, \Phi \, U_R^+}$$

just as in the SM

$$V_{SM} = -\frac{\mu_{SM}^2}{2} \phi_{SM}^+ \phi_{SM} + \frac{\lambda}{4} \left(\phi_{SM}^+ \phi_{SM}\right)^2$$

$\langle \phi_{SM} \rangle = v \Rightarrow v^2 = \mu^2/\lambda \sim M_W^2$

$\Downarrow$

$$\langle \Delta_R \rangle = \begin{pmatrix} 0 & 0 \\ v_R & 0 \end{pmatrix} \leftarrow \text{first}$$

$$(v_i \ll v_R)$$

$$\langle \Phi \rangle = \begin{pmatrix} v_1 & 0 \\ 0 & v_2 \end{pmatrix} \leftarrow \text{second}$$

$$\langle \Delta_L \rangle = 0 \qquad \text{Correct !}$$

?

WRONG

$$V = -\frac{\mu_\Delta^2}{2}\left(\mathrm{Tr}\,\Delta_L^\dagger \Delta_L + \mathrm{Tr}\,\Delta_R^\dagger \Delta_R\right)$$

$$+ \frac{\lambda_\Delta}{4}\left[\left(\mathrm{Tr}\,\Delta_L^\dagger \Delta_L\right)^2 + L \leftrightarrow R\right]$$

$$-\frac{\mu_\Phi^2}{2}\,\mathrm{Tr}\,\Phi^\dagger\Phi + \frac{\lambda_\Phi}{4}\left(\mathrm{Tr}\,\Phi^\dagger\Phi\right)^2$$

$$+ \ldots$$

$$+\ \alpha\,\mathrm{Tr}\,\Phi^\dagger\Phi\left(\mathrm{Tr}\,\Delta_L^\dagger\Delta_L + \Delta_R^\dagger\Delta_R\right)$$

$$\boxed{+\ \beta\,\mathrm{Tr}\,\Delta_L^\dagger\,\Phi\,\Delta_R\,\Phi^\dagger}$$

$$\Uparrow$$

$$\Delta_L \to U_L\Delta_L U_L^\dagger, \quad \Phi \to U_L\,\Phi\,U_R^\dagger, \quad \Delta_R \to U_R\Delta_R U_R^\dagger$$

$$\Phi^\dagger \to U_R\,\Phi^\dagger\,U_L^\dagger$$

$$\beta: \to \mathrm{Tr}\;U_L\,\Delta_L^\dagger\,\cancel{U_L^\dagger U_L}\,\Phi\,\cancel{U_R^\dagger U_R}\,\Delta_R\,\cancel{U_R^\dagger U_R}\,\Phi^\dagger\,U_L^\dagger$$

$$= \mathrm{Tr}\;\Delta_L^\dagger\,\Phi\,\Delta_R\,\Phi^\dagger\ \checkmark$$

$$\Downarrow$$

$$\langle \Delta_R \rangle = \begin{pmatrix} 0 & 0 \\ v_R & 0 \end{pmatrix}$$

$$\langle \Phi \rangle = \begin{pmatrix} v_1 & 0 \\ 0 & v_2 \end{pmatrix}$$

$$\Downarrow$$

$$\beta \, \mathrm{Tr} \begin{pmatrix} \delta^- & \delta_0^* \\ \delta^{--} & \delta^- \end{pmatrix}_L \begin{pmatrix} 0 & 0 \\ v_2 v_R & 0 \end{pmatrix} \begin{pmatrix} v_1 & 0 \\ 0 & v_2 \end{pmatrix}$$

$$= \beta \, \mathrm{Tr} \begin{pmatrix} \delta^- & \delta_0^* \\ \delta^{--} & \delta^- \end{pmatrix} \begin{pmatrix} 0 & 0 \\ v_1 v_2 v_R & 0 \end{pmatrix}$$

$$= \beta \, \delta_0^* v_1 v_2 v_R \qquad \begin{matrix} (v \sim M_W) \\ (v \sim v_1, v_2) \end{matrix}$$

$$\Downarrow \qquad \beta_{eff} \, \delta_0^* v^2 v_R$$

$$V_{\Delta_L} = \frac{m_{\Delta_L}^2}{2} \mathrm{Tr}\, \Delta_L^\dagger \Delta_L + \frac{\lambda_\Delta}{4} \left(\mathrm{Tr}\, \Delta_L^\dagger \Delta_L \right)^2$$

$$+ \beta_{eff} \left(\delta_{0L}^* v^2 v_R + h.c. \right)$$

$$\Downarrow$$

$$V_{\delta_L^0} = \frac{m_{\Delta_L}^2}{2} \delta_L^{0*} \delta_L^0 + \frac{\lambda_\Delta}{4} \left(\delta_L^{0*} \delta_L^0 \right)^2$$

$$+ \beta_{eff} \left(\delta_{L0}^* \, v^2 v_R + h.c. \right)$$

Tadpole

$$\Downarrow$$

$$\delta_L^0 \equiv \delta^0$$

$$\frac{\partial V}{\partial \delta_0^*} = \beta_{eff} v^2 v_R + \frac{\overbrace{m_{\Delta_L}^2}^{\simeq M_R^2}}{2} \delta_0$$

$$+ \frac{\lambda_\Delta}{4}(\delta_0^* \delta_0)\delta_0 = 0$$

- $\beta_{eff} = 0 \Rightarrow \langle \delta_0^L \rangle = 0$

- $\beta_{eff} \neq 0 \Rightarrow \beta_{eff} v^2 v_R + \frac{m_{\Delta_L}^2}{2} \delta_0 \simeq 0$

  $\delta_0 = \text{small} \Rightarrow$

$$\Rightarrow \langle \delta_0 \rangle = \langle \delta_L^0 \rangle = v_L \neq 0$$

$$\boxed{v_L = -2\beta_{eff} v^2 v_R / m_{\Delta_L}^2}$$

$v \simeq M_W$, $m_{\Delta_L}^2 \simeq M_R^2 \simeq v_R^2$

$$\boxed{\nu_L \simeq \frac{M_W^2}{M_R}\beta_{eff}}$$

Seesaw

$\boxed{\text{WRONG!}}$

$$\begin{array}{ccc} \nu_L & \multicolumn{2}{c}{\left(\begin{array}{cc} 0 & M_D^T \\ M_D & M_N \end{array}\right)} \\ N_L & & \\ & \nu_L & N_L \end{array}$$

$$\mathcal{L}_Y^{\Delta} = l_L^T C\, i\sigma_2\, Y_{\Delta_L} \Delta_L\, l_L +$$

$$+ \, l_R^T C i\sigma_2 \, Y_{\Delta_R} \Delta_R \, l_R \; + h.c.$$

gives mass to ν_R

$$M_{\nu_R} = Y_{\Delta_R} \langle \Delta_R \rangle \equiv Y_{\Delta_R} v_R$$

$$\Rightarrow \quad M_N = Y_{\Delta_R}^* v_R \qquad \left(N_L \equiv C \bar{\nu}_R^T \right)$$

$$\boxed{M_\nu = Y_{\Delta_L} v_L - M_D^T \frac{1}{M_N} M_D}$$

<u>seesaw:</u> $\quad M_N \to \infty \implies M_\nu \to 0$

$$(v_R \to \infty)$$

$$v_L \simeq O\left({M_W}^2 / v_R \right) \xrightarrow[v_R \to \infty]{} 0$$

$\boxed{d=5}$ $\boxed{\text{effective}}$ $\boxed{\text{Weinberg '79}}$

$\begin{pmatrix} \nu \\ e \end{pmatrix}_L$ $\quad e_R \quad$ $\boxed{\cancel{B}, \cancel{L}}$

$$\Rightarrow \quad \frac{{\nu_L}^T C \nu_L \; \phi_0 \phi_0}{\Lambda_{\text{neutrino}}}$$

$$\Rightarrow \quad M_\nu \simeq \frac{\langle \phi_0 \rangle^2}{\Lambda_{\text{neutrino}}} \simeq \frac{{M_W}^2}{\Lambda_{\text{neutrino}}}$$

$$\boxed{LR : \Lambda_{\text{neutrino}} = v_R}$$

$$C: \quad f_L \longrightarrow C\bar{f_R}^T = C\gamma_0 f_R^*$$

$$\Rightarrow \quad Y_{\Delta_R} = Y_{\Delta_L}^*$$

$$M_D = M_D^T$$

$$M_N = v_R Y_{\Delta_R}^* = v_R Y_{\Delta_L}$$

(seesaw II) (seesaw I)

$$M_\nu = Y_{\Delta_L} v_L - M_D^T \frac{1}{M_N} M_D$$

$$= \underbrace{\left(\frac{v_L}{v_R}\right)}_{\equiv \epsilon} M_N - M_D \frac{1}{M_N} M_D$$

$$\frac{1}{M_N} M_\nu = \epsilon - \frac{1}{M_N} M_D \frac{1}{M_N} M_D$$

$$\Downarrow$$

$$\left(\frac{1}{M_N} M_D\right)^2 = \left(\epsilon - \frac{1}{M_N} M_\nu\right)$$

$$\boxed{M_D = M_N \sqrt{\epsilon - \frac{1}{M_N} M_\nu}}$$

$$\boxed{M_D = f(M_N, M_\nu)}$$

seesaw = untangled

S.M + neutrino = massive

① $\exists\, \nu_R \longrightarrow$ seesaw (I)

② add triplet $\Delta \to$ seesaw (II)

$\uparrow$ $\boxed{\text{just this}}$

$$Y(l_L) = -l_L$$

$$\mathcal{L}_y^{\Delta} = l_L^T C \, i\sigma_2 \, y_\Delta \, \Delta \, l_L + h.c.$$

$$l = \begin{pmatrix} \nu \\ e \end{pmatrix}_L \qquad \boxed{Y\Delta = 2\Delta}$$

$$V_{\phi,\Delta} = V_\phi + V_\Delta + V_{\phi\Delta}$$

$$V_{\phi\Delta} = \mu \, \phi^T i\sigma_2 \, \Delta^* \phi^T + h.c.$$

$Y\phi = \phi$ nobody saw $\Delta \Rightarrow$

Δ = heavy

$\Downarrow$ $m_\Delta \gtrsim (2-4)10^2 \text{ GeV}$

$$V_\Delta = + \frac{m_\Delta^2}{2} \mathrm{Tr}\,\Delta^\dagger\Delta + \lambda \,---$$

$$\phi \to \langle\phi\rangle = \begin{pmatrix} 0 \\ v \end{pmatrix}$$

$$V_{\phi\Delta} = \mu \,(0 \; v) \begin{pmatrix} 0 & 1 \\ -1 & 0 \end{pmatrix} \begin{pmatrix} 0 & 0 \\ \delta_0^* & 0 \end{pmatrix} \begin{pmatrix} 0 \\ v \end{pmatrix}$$

$$+ h.c.$$

$$= \mu \,(0 \; v) \begin{pmatrix} 0 & 1 \\ -1 & 0 \end{pmatrix} \begin{pmatrix} 0 \\ \delta_0^* v \end{pmatrix} + h.c.$$

$$= \mu v^2 \delta_0^* + h.c.$$

$$\frac{\partial V}{\partial \delta_0^*} = \mu v^2 + m_\Delta^2/2 \; \delta_0 + \text{———} = 0$$

$$\langle\delta_0\rangle \neq 0$$

$$\boxed{\langle\delta_0\rangle \simeq \mu \frac{v^2}{m_\Delta^2}}$$

$\langle \delta_0 \rangle \equiv v_\Delta$

$$\underline{M}_\nu = Y_\Delta \langle \delta_0 \rangle$$

small $\Leftrightarrow$ μ – small, m_Δ – large

$\boxed{\text{seesaw (1) vs seesaw (2)}}$

$\underline{M}_\nu = -M_D^T \frac{1}{M_N} M_D$	$\underline{M}_\nu = Y_\Delta v_\Delta$
• $M_D = ?$	• $Y_\Delta = ?$
• cannot produce N	• produce Δ – weak int.

$$\longrightarrow \quad \Delta = \begin{pmatrix} \delta^+ & \delta^{++} \\ \delta^0 & -\delta^+ \end{pmatrix}$$

$$\delta^{++}, \ \delta^+ \leftarrow \ \gamma, \ Z$$

$$\mathcal{L}_y^\Delta = l_L^T C \, i\sigma_2 \, y_\Delta \, \Delta \, l_L =$$

$$= \left(\nu_L^T \ e_L^T \right) C \begin{pmatrix} 0 & 1 \\ -1 & 0 \end{pmatrix} \begin{pmatrix} \delta^+ & \delta^{++} \\ \delta^0 & -\delta^+ \end{pmatrix} y_\Delta \begin{pmatrix} \nu \\ e \end{pmatrix}_L + h.c.$$

$$= \left(\nu_L^T \ e_L^T \right) C \begin{pmatrix} \delta^0 & -\delta^+ \\ -\delta^+ & -\delta^{++} \end{pmatrix} y_\Delta \begin{pmatrix} \nu \\ e \end{pmatrix}_L + h.c.$$

$$= \nu_L^T C \, y_\Delta \nu_L - \nu_L^T C \, \delta^+ y_\Delta e_L$$

$$- e_L^T C \, \delta^+ y_\Delta \nu_L - e_L^T C \, y_\Delta e_L \, \delta^{++} + h.c.$$

$$Y_\Delta = \text{matrix}$$

$$\begin{pmatrix} \nu \\ e \end{pmatrix}_L = \begin{pmatrix} \nu \\ e \end{pmatrix}_L^i$$

$$\Rightarrow \quad M_\nu = Y_\Delta v_\Delta$$

$$\delta^{++} : \quad e_L^{i\,T} C \frac{M_\nu^{ij}}{v_\Delta} e_L^j \, \delta^{++}$$

$$\boxed{M_\nu = V_L^* \, m_N \, V_L^\dagger}$$

leptonic mixing

$$\delta^{--} \longrightarrow ee \quad (M_\nu)^{11}$$

$$\delta^{--} \longrightarrow e\mu \quad (M_\nu)^{12}$$

$\boxed{\text{type I}}$

$$\delta^{--} \longrightarrow \tau\tau \quad (M_\nu)^{33}$$

$$\Downarrow$$

Γ predict branching ratios

Schwetz, ... 2000