Neutrino Physics Course

Lecture XXII

26/6/2021

LMU

Summer 2021

Seesaw and LR symmetry

(1) $\exists\, \nu_R$

(2) seesaw

predicted

- $\nu_L \longleftrightarrow \nu_R \quad (LR)$
- $M_{\nu_R} \propto \langle \Delta_R \rangle = M_R$

$$\Downarrow$$

seesaw mechanism

$$M_\nu = - M_D^T \frac{1}{M_N} M_D$$

$$\Downarrow$$

$$M_D = i\sqrt{M_N}\, O\, \sqrt{M_\nu}$$

$$O O^T = O^T O = 1$$

<u>general</u>

$$O \in C$$

<u>why?</u> $M_D = $ arbitrary

<u>SM</u>: $M_f = y_f\, v$

$\uparrow$ arbitrary

$\tilde{\Phi} = \sigma_2 \phi^* \sigma_2$

<u>LR</u>:

$$\mathcal{L}_Y = \overline{f_L}\left(Y_\Phi\, \Phi + \tilde{Y}_\Phi\, \tilde{\Phi}\right) f_R$$

$$+ \overline{f_R}\left(Y_\Phi^+\, \Phi^+ + \tilde{Y}_\Phi^+\, \tilde{\Phi}^+\right) f_L$$

$$\boxed{P: \quad f_L \longleftrightarrow f_R, \quad \Phi \longrightarrow \tilde{\Phi}^{+}}$$

$$\Rightarrow \quad Y_\Phi = Y_\Phi^{+}$$

$$\Rightarrow \quad \boxed{y_D = y_D^{+}}$$

$$\boxed{\begin{array}{l} SM: \ CP \text{ is "good"} \\ \qquad \epsilon_{\not{cp}} = 10^{-3} \Rightarrow C \Longleftrightarrow P \end{array}}$$

$$C: \quad \psi \longrightarrow C\bar{\psi}^{T} = C\gamma_0 \psi^{*}$$

$$\Rightarrow \quad \psi_L \longrightarrow C(\bar{\psi}_R)^{T} = (\psi^{c})_L$$

$$\boxed{C: \quad \psi_L \to C\gamma_0 \psi_R^*}$$

Imagine: SO(10) GUT

$$\begin{pmatrix} f \\ f^c \end{pmatrix}_L = \begin{pmatrix} u \\ d \\ u^c \\ d^c \\ e \\ e^c \\ \nu \\ \nu^c \end{pmatrix}_L$$

C = can be gauged

$$G_{LR} = SU(2)_L \times SU(2)_R \times U(1)_{B-L} \times (LR)$$

$$LR = \begin{cases} P \\ C \end{cases}$$

P: $\quad Y_\Phi = Y_\Phi^+ \Rightarrow Y_D = Y_D^+$

$(f_L \longrightarrow f_R)$

C: $\quad (f_L \longrightarrow f_R^*) \Rightarrow y_D = y_D^T$ (?)

$$\mathcal{L}_Y = \bar{f}_L \, Y_\Phi \, \Phi \, f_R + h.c.$$

$$\to \overline{C \bar{f}_R^T} \, Y_\Phi \, \Phi' \, C \bar{f}_L^T + h.c.$$

$$\propto f_R^T \, Y_\Phi \, \Phi' \cdot \bar{f}_L^T =$$

$$= \bar{f}_L^T \, Y_\Phi^T \, \Phi'^T \, f_R$$

coeff = 1

$\boxed{\text{Prove!}}$ $\boxed{***}$

$$\Rightarrow Y_\Phi = Y_\Phi^T, \quad \Phi' = \Phi^T$$

$$\boxed{P: \ \Phi \to \Phi^+, \quad C: \ \Phi \to \Phi^T}$$

$$C: \quad \boxed{Y_D = Y_D^T}$$

$$\Downarrow$$

$$\boxed{\text{LR symmetry}}$$

$$C: \quad f_{L(R)} \to c\gamma_0 f_{R(L)}^*, \quad \Phi \to \Phi^T$$

$$\boxed{Y_D = Y_D^T}$$

$$P: \ f_L \to f_R, \quad \Phi \to \Phi^+, \quad \boxed{Y_D = Y_D^+}$$

$\boxed{\text{Seesaw}}$

$$\underline{M}_\nu = -\underline{M}_D^T \frac{1}{\underline{M}_N} \underline{M}_D$$

C: $\quad Y_D = Y_D^T$

$$M_D = Y_D \langle \Phi \rangle \Rightarrow \boxed{M_D^T = M_D}$$

$$\Downarrow$$

$$\underline{M}_\nu = -\underline{M}_D \frac{1}{\underline{M}_N} \underline{M}_D \quad (1) / \underline{M}_N \text{(left)}$$

$$\underline{M}_D = ?$$

[$\underline{M}_\nu \leftarrow$ oscillations, $0\nu 2\beta$

input $\left\{ M_N \right.$ $\longleftarrow$ LHC (hadron collider)

$$\frac{1}{M_N} M_\nu = - \frac{1}{M_N} M_D \frac{1}{M_N} M_D$$

$$= - \left(\frac{1}{M_N} M_D \right)^2$$

$$\Downarrow$$

$$\frac{1}{M_N} M_D = i \sqrt{1/M_N \, M_\nu}$$

$$\boxed{M_D = i \, M_N \sqrt{1/M_N \, M_\nu}}$$

pathology: $\sqrt{1}$ = not determined

banning pathologies:

$$M_D = \text{fixed}$$

$$\Rightarrow \boxed{\Theta_{\nu N} = \frac{1}{M_N} M_D = i\sqrt{1/M_N \, M_\nu}}$$

$$|\Theta_{\nu N}^2| = |(-1) \; 1/M_N \, M_\nu|$$

$\Rightarrow$ **Mixing predicted!**

- seesaw ambiguity (SM seesaw)

$$M_D = i\sqrt{M_N} \; O \sqrt{M_\nu}, \quad O = ?$$

• $LR = C \Rightarrow$

$$M_D = i\, M_N \sqrt{\tfrac{1}{M_N} M_\nu}$$

$$\Downarrow$$

$$\cancel{i} \sqrt{M_N}\; O_{LR} \sqrt{M_\nu} = \cancel{i}\, M_N \sqrt{\tfrac{1}{M_N} M_\nu}$$

$$O_{LR} = \frac{1}{\sqrt{M_N}} M_N \sqrt{\tfrac{1}{M_N} M_\nu}\, \frac{1}{\sqrt{M_\nu}}$$

$$\boxed{O_{LR} = \sqrt{M_N} \sqrt{\tfrac{1}{M_N} M_\nu}\, \frac{1}{\sqrt{M_\nu}}}$$

all ambiguity gone!

$$\Uparrow$$

• $A = \sqrt{B} \Leftrightarrow A^2 = B$

$\left(\underline{2 \times 2} \quad \text{explicit } \sqrt{M} \quad (\text{Wikipedia}) \right)$

<u>reminder</u>

$$N_i \to e_j W^+ \quad (e^c W^-)$$

$$\Uparrow$$

given by $\quad \theta_{\nu N} = \frac{1}{M_N} M_D = i\sqrt{1/M_N \, M_\nu}$

measure

$$\Downarrow$$

$$\boxed{\Gamma(N_\nu \to e_j W^+) = \text{predicted}}$$

$$\boxed{\begin{array}{l} \Leftrightarrow \text{ SM charged fermion} \\[2em] m_f \Rightarrow \Gamma(h \to f\bar{f}) = \text{predicted} \end{array}}$$

- <u>Charge conjugation</u> (C)

- $y_\Phi = y_\Phi^T$ <u>symmetric</u>

- $$\mathcal{L}_Y^\Delta = \ell_L^T i\sigma_2 C \Delta_L Y_{\Delta_L} \ell_L +$$
$$+ \ell_R^T i\sigma_2 C \Delta_R Y_{\Delta_R} \ell_R + h.c.$$

$$\left(P: \; f_L \leftrightarrow f_R \Rightarrow Y_{\Delta_L} = Y_{\Delta_R} \right)$$

$C: \; f_L \to \cdots f_R^* \Rightarrow \Delta_L \to \Delta_R^*$

$$\Rightarrow \boxed{Y_{\Delta_L} = Y_{\Delta_R}^*}$$

$$\boxed{\begin{aligned} M_{\nu_R} &= Y_{\Delta_R} \langle \Delta_R \rangle \\ \Rightarrow M_N &= Y_{\Delta_R}^* \langle \Delta_R \rangle \end{aligned}}$$

LH (left-handed) and

RH (right -"-) mixings

$$M_\nu = V_L^* m_\nu V_L^\dagger$$

$$V_L \neq V_R$$

$$M_N = V_R m_N V_R^T$$

$$\left(\quad M_\nu = - M_D^T \frac{1}{M_N} M_D \right.$$

$$\left. \Rightarrow \quad V_L = V_R \right)$$

$$C: \quad f_L \longrightarrow -f_R^*$$

$$\nu_L \longrightarrow -\nu_R^* \quad (N)$$

$$\boxed{\left(\underline{\text{"natural"}}: \quad V_L = V_R^* \quad \text{illustration} \right.}$$

$$\boxed{V_R^+ V_L^* = 1}$$

$$\cdot \; M_D = i\,(M_N)\sqrt{1/M_N \, M_\nu}$$

$$= i\,(V_R m_N V_R^T)\sqrt{V_R^* \frac{1}{m_N} V_R^+ V_L^* m_\nu V_L^+}$$

$$\Uparrow$$

$$M_N \frac{1}{M_N} = 1$$

$$\Downarrow$$

$$V_R m_N \underbrace{V_R^T V_R^*}_{1} \frac{1}{m_N} V_R^+ =$$

$$= V_R V_R^+ = 1 \checkmark$$

$$\Downarrow$$

$$M_N = V_R m_N V_R^T \Rightarrow \frac{1}{M_N} = V_R^* \frac{1}{m_N} V_R^+$$

$$\Downarrow$$

$$M_D = i \left(V_R m_N V_R^T \right) \sqrt{V_L \frac{1}{m_N} m_\nu V_L^+}$$

$\boxed{\text{Theorem}}$ $V \sqrt{M} V^+ = \sqrt{V M V^+}$

iff $V^\dagger V = 1 = V V^\dagger$

<u>Proof:</u>

$$\left(V\sqrt{M}\,V^\dagger\right)^2 = V\sqrt{M}\,\overbrace{V^\dagger V}^{1}\sqrt{M}\,V^\dagger$$

$$= V M V^\dagger$$

$$\left(\sqrt{V M V^\dagger}\right)^2 = V M V^\dagger \checkmark$$

Q.E.D.

$$\Rightarrow \sqrt{V_L \frac{1}{m_N} m_\nu V_L^\dagger} = V_L \sqrt{\frac{1}{m_N} m_\nu}\, V_L^\dagger$$

$$(V_R = V_L^*)$$

$$\Downarrow$$

$$M_D = i\, V_L^{*}\, m_N \underbrace{V_L^{+} V_L}_{1} \sqrt{1/m_N\, m_\nu}\; V_L^{+}$$

$$\boxed{M_D = i\, V_L^{*} \sqrt{m_N\, m_\nu}\; V_L^{+}}$$

illustration : $V_R = V_L^{*}$

$(C = 1000)$

$$\boxed{V V^{+} = 1 \Rightarrow |V_{ij}| \leq 1}$$

(a) $M_D \sim \sqrt{m_N m_\nu}$ natural value

(b) $M_D^{T} = i\, V_L^{*} \sqrt{m_N m_\nu}\; V_L^{+} = M_D$ ✓

(c) $\Theta_{\nu N} = \frac{1}{M_N} M_D = i\, V_L \sqrt{1/m_N}\, m_\nu\; V_L^{+}$

$$\boxed{\theta_{\nu N}^2 \simeq m_\nu / m_N}$$

· N is produced by W_R

$\longrightarrow$ decays via $\begin{cases} W_R \\ W_L \ (\theta_{\nu N}) \end{cases}$

· $O = ?$ $\quad C :$ $V_R = V_L^*$

$$O_C = \sqrt{M_N} \sqrt{1/M_N \, M_\nu} \sqrt{1/M_\nu} =$$

$$\sqrt{V_R m_N V_R^T} \sqrt{V_R^* \tfrac{1}{m_N} \underbrace{V_R^+ V_L^*}_{1} m_\nu V_L^+} \sqrt{V_L m_\nu V_L^T}$$

$$= \sqrt{V_R m_N V_R^T} \sqrt{V_L \tfrac{1}{m_N} m_\nu V_L^+} \sqrt{V_L m_\nu^{-1} V_L^T}$$

$$= \sqrt{V_L^* m_N V_L^+}\; V_L \sqrt{\tfrac{1}{m_N} m_\nu}\; V_L^+ \sqrt{V_L m_\nu^{-1} V_L^T}$$

$$= \sqrt{\underbrace{V_L V_L^+}_{1} V_L^* m_N V_L^+}\; V_L \sqrt{\tfrac{1}{m_N} m_\nu}\; V_L^+ \sqrt{V_L m_\nu^{-1} V_L^T \underbrace{V_L V_L^+}_{1}}$$

$$= V_L \sqrt{V_L^* V_L^* m_N \underbrace{V_L^+ V_L}_{1}} \sqrt{\tfrac{1}{m_N} m_\nu^{-1}}\; V_L^+ V_L \sqrt{m_\nu^{-1} V_L^T V_L}\; V_L^+$$

$$= V_L \sqrt{V_L^+ V_L^* m_N} \sqrt{\tfrac{1}{m_N} m_\nu} \sqrt{m_\nu^{-1} V_L^T V_L}\; V_L^+$$

$$\cong \sqrt{m_N} \sqrt{\tfrac{1}{m_N} m_\nu} \sqrt{m_\nu^{-1}} \simeq O(1)$$

$$\boxed{V_L^T V_L = ?}$$

Comment: if $V_L \in R \Rightarrow V_L V_L^T = 1$

$\Downarrow$

$$O_c = V_L \sqrt{m_N \, 1/m_N \, m_\nu \, 1/m_\nu} \, V_L^T = \mathbb{1}$$

$\boxed{2 \times 2}$ $$\sqrt{\mathbb{1}} = \{\sigma_1, \sigma_3, \sigma_2\}$$

$$\sqrt{\mathbb{1}} = M = \begin{pmatrix} a & c \\ d & b \end{pmatrix}$$

$$M^2 = \text{- ---} = \mathbb{1} \Rightarrow \text{relations}$$