Neutrino Physics Course

Lecture XXI

22/6/2021

LMU

Summer 2021

LR theory: neutrino mass

Seesaw mechanism

$$\exists\ \nu_R^i \qquad (i = 1, 2, 3)$$

$$\Downarrow \qquad M_{\nu_R} \gtrsim M_W$$

physical

$$\to M_\nu = - M_D^T \frac{1}{M_N} M_D$$

physical

$$N_{i_L} = C \bar{\nu}_{R_i}^T$$

light (ordinary) neutrino mass matrix

$$\boxed{(\text{def.})\ \nu_{0L}^T\, M_\nu\, \nu_{0L}\ ;\quad \overline{\nu_R}^{\,0} M_D\, \nu_L^0}$$

with $\nu_{0L} = \begin{pmatrix}\nu_e \\ \nu_\mu \\ \nu_\tau\end{pmatrix}_L^0 \leftarrow$ "weak" basis states

(def.) $\nu_{0L} = U_{L\nu}\, \nu_L$

where $\nu_L = \begin{pmatrix}\nu_1 \\ \nu_2 \\ \nu_3\end{pmatrix}_L$

mass eigenstates

$\Downarrow$

$$\nu_{0L}^T M_\nu\, \nu_{0L} = \nu_L^T\, U_{L\nu}^T M_\nu\, U_{L\nu}\, \nu_L$$

$$= \nu_L^T\, m_\nu\, \nu_L$$

$$m_\nu = \mathrm{diag}\,(m_1^\nu, m_2^\nu, m_3^\nu)$$

$$\Leftrightarrow \quad M_\nu = U_{L\nu}^* m_\nu U_{L\nu}^+$$

$$\hookrightarrow \nu_L^+ \underbrace{U_{L\nu}^T U_{L\nu}^*}_{1} m_\nu \underbrace{U_{L\nu}^+ U_{L\nu}}_{1} \nu_L$$

$$= \nu_L^T m_\nu \nu_L \ \checkmark$$

---

analogy

$$\Leftrightarrow \quad f_L^0 = U_L f_L \,,\ f_R^0 = U_L f_R^0$$

$$\Downarrow$$

$$\mathcal{L}_{wu}^{(q)} = \frac{g}{\sqrt{2}} W_\mu^+ \overline{u_L^0} \gamma^\mu d_L^0 =$$

$$= \frac{g}{\sqrt{2}} W_\mu^+ \overline{u_L} \underbrace{U_{uL}^+ U_{dL}}_{V_{CKM}} \gamma^\mu d_L$$

$$V_{CKM} \equiv \underbrace{U_{Lu}^{+} U_{Ld}}_{\text{relative mixing}}$$

$\Updownarrow$ analogy

$$\mathcal{L}_{un}^{(l)} = \frac{g}{\sqrt{2}} W_\mu^+ \overline{\nu_L^0} \gamma^\mu e_L^0 =$$

$$= \frac{g}{\sqrt{2}} W_\mu^+ \overline{\nu_L} \underbrace{U_{L\nu}^{+} U_{Le}}_{V_{PMNS} = U_{L\nu}^{+} U_{Le}} \gamma^\mu e_L$$

basis: $U_{Le} = 1$

so in this basis:

$$\boxed{M_\nu = V_L^* \, m_\nu \, V_L^\dagger}$$

$$\boxed{\nu \equiv \nu_L}$$

$$\Updownarrow$$

$$V_{PMNS} = V_L^\dagger$$

$$M_{\nu_R} = V_R^* \, m_{\nu_R} \, V_R^\dagger$$

$$N_L = C \bar{\nu}_R^T \Rightarrow m_{\nu_R} = m_N$$

$$M_{\nu_R} = M_N^*$$

$$\Downarrow$$

$$\boxed{M_N = V_R \, m_N \, V_R^T}$$

$\boxed{\text{How do you measure } \underline{M}_\nu \text{ and } M_N?}$

① $\underline{M}_\nu = V_L^{*} m_\nu V_L$

$\uparrow$ lepton mixing

$$\mathcal{L}_{cc} = \frac{g}{\sqrt{2}} W_\mu^+ \bar{\nu}_L \gamma^\mu V_L^{\dagger} e_L$$

<u>2gen</u> $\quad V_L = \begin{pmatrix} \cos\theta_\ell & \sin\theta_\ell \\ -\sin\theta_\ell & \cos\theta_\ell \end{pmatrix}$

$$= \frac{g}{\sqrt{2}} W_\mu^+ \left(\bar{\nu}_{Le}^{0} \gamma_\mu e_L^{0} + \nu_{\mu L}^{0} \gamma^\mu \mu_L^{0} \right)$$

$$= \frac{g}{\sqrt{2}} W_\mu^+ \left(\bar{\nu}_{eL} \gamma^\mu e_L + \bar{\nu}_{\mu L} \gamma^\mu \mu_L \right)$$

$$e_L = e_L^0, \ ---$$

(diagonal charged leptons)

$$= \frac{g}{\sqrt{2}} W_\mu^+ \Big(\bar{\nu}_{1L} \gamma^\mu \cos\theta \, e_L - \bar{\nu}_{2L} \gamma^\mu \sin\theta_e \, e_L$$

$$+ \bar{\nu}_{1L} \gamma^\mu \sin\theta \, \mu_L + \bar{\nu}_{2L} \gamma^\mu \cos\theta_e \, \mu_L \Big)$$

$$\theta_e = \theta_{12}^{(\nu)} \simeq 30^\circ \leftarrow \text{solar}$$

$$\left(\theta_{23}^{(\nu)} \simeq 45^\circ, \quad \theta_{13}^{(\nu)} \simeq 10^\circ \right)$$

atmospheric

long baseline

$$\nu_e = \nu_1 \cos\theta + \nu_2 \sin\theta$$

$$---$$

$\nu_e \longrightarrow \nu_\mu$ (sun)

$\nu_\mu \to \nu_\tau$ (atm)

$$P(\nu_e \to \nu_\mu) = \sin^2 2\theta \, \sin^2 \frac{\Delta m^2 L}{4E}$$

$$\Rightarrow \quad \Delta m^2_A \simeq 10^{-3}\, eV^2, \quad \theta_A \simeq 45^0$$

$$\Delta m^2_\odot \simeq 10^{-5}\, eV^2, \quad \theta_\odot \simeq 30^0$$

$$m_\nu = ?$$

KATRIN: β decay $\Rightarrow m_\nu \leq 1\, eV$

$$\left.\begin{array}{l} \text{GERDA} \\ \text{MAJORANA} \\ \text{NEMO, EXO...} \end{array}\right\} : 0\nu 2\beta \Rightarrow m_\nu \leq 1\, eV$$

$M_\nu \Leftarrow$ being probed!

- $M_N = ?$

seesaw : $\exists N$ in SM

$$\theta_{\nu N} \simeq \frac{m_D}{m_N}$$

$$\sigma(N) \propto |\theta_{\nu N}^2| = \left(\frac{m_D}{m_N}\right)^2 = \frac{m_\nu}{m_N}$$

zero!

- $M_\nu = - M_D^T \frac{1}{M_N} M_D \qquad (*)$

$\Downarrow$

$$M_D = i\sqrt{M_N}\, O \sqrt{M_\nu} \quad (**)$$

$$O O^T = O^T O = 1 \,, \quad O \in \mathbb{C}$$

$M_D = Y_D v$, $\quad M_u = Y_u v$ - - -

SM: $h\, y_f\, \bar{f} f$ $\qquad f$ = charged fermion

$$y_f = \frac{g}{2} \frac{m_f}{M_W}$$

$$\text{mass}(f) \longrightarrow y(f)$$

analog

$\Longleftarrow \quad m(f) \longrightarrow M_\nu, M_N$

$M_{N_1}, M_N \longrightarrow M_D$ fails

in seesaw

$\boxed{\text{Why LR theory?}}$

(i) $\not{P}$ spont.

(ii) $\exists\ \nu_R\ (N) \leftarrow$ a must

(iii) N physical $\Leftrightarrow$ can be produced at LHC

• reminder: discovery of $W\ (W_L)$
+ P violation

$$\frac{d\Gamma}{d\Omega}\left(W^- \to e^- \bar{\nu}\right) \propto$$

$$\propto (1 \pm \cos\theta)^2$$

$p \quad + \quad \bar{p}$

$(d) \qquad\quad (\bar{u})$

$d_L \qquad\qquad (\bar{u})_R$

$\Leftarrow \qquad\qquad \Leftarrow$

$\downarrow$

W^- at rest $\quad S_z^W = \pm 1$

$$\left(M_{\nu_R} = y_\Delta \langle \Delta_R \rangle \right)$$

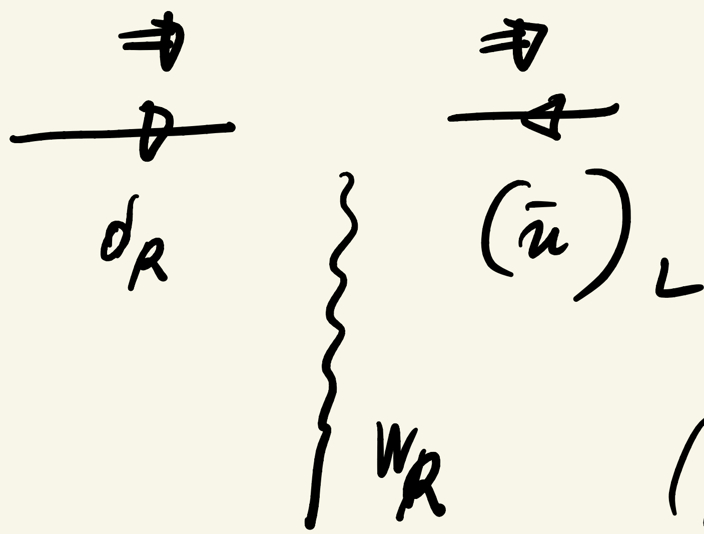
$\Downarrow$

discovery of W_R ?

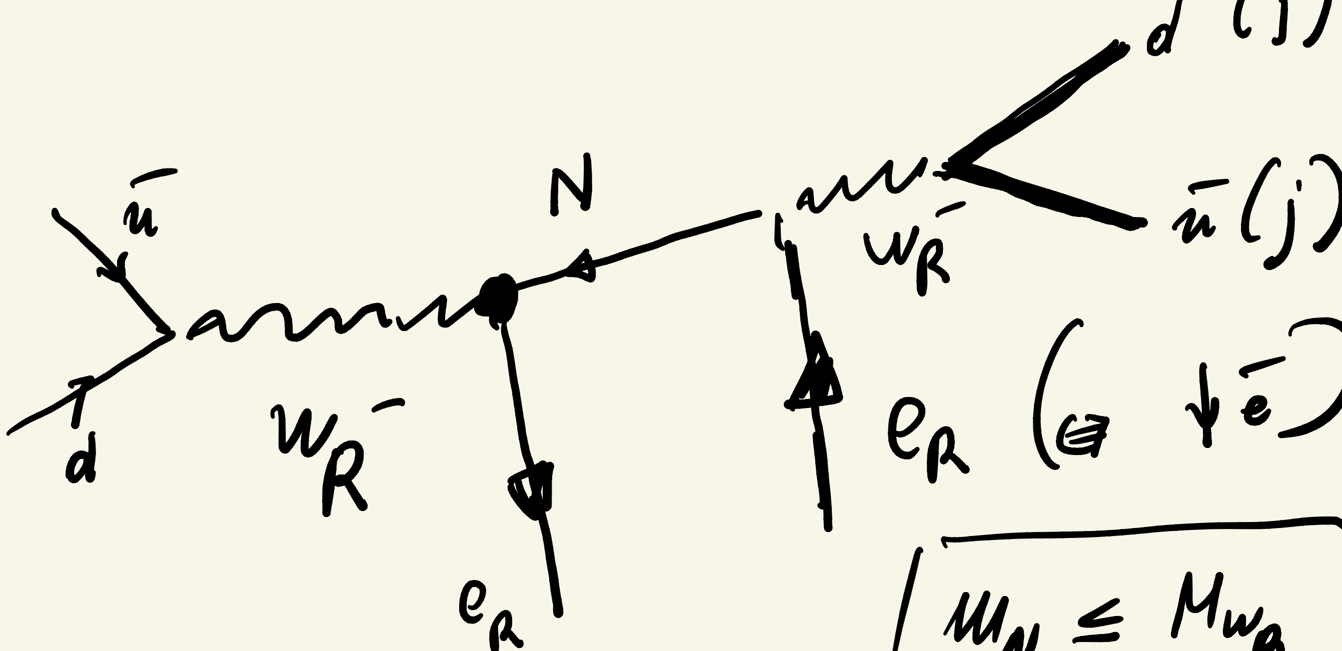
$\boxed{M_{\nu_R} \gtrsim M_W}$

$p \quad + \quad p$

$(d) \qquad\qquad (\bar{u})$



(polarized)



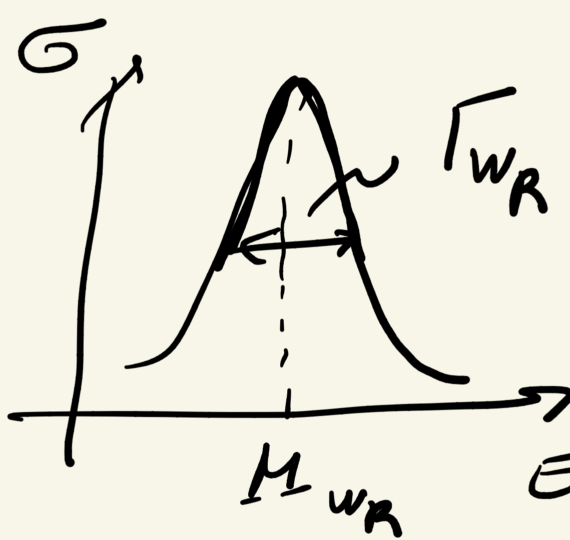
$\boxed{m_N \le M_{W_R}}$

$$\boxed{\begin{aligned} N_M &= N_L + C\bar{N}_L^T \\ &\qquad N_L \equiv C\bar{\nu}_R^T \Rightarrow C\bar{N}_L^T = \nu_R \\ &= \nu_R + C\bar{\nu}_R^T \end{aligned}}$$

$$\boxed{W_R \text{ discovery} \Leftrightarrow N \text{ discovery}}$$

$M_{W_R} \gtrsim 5\,\text{TeV}$ (not yet seen @ LHC)

$$\boxed{M_{W_R} = ?}$$



$$\frac{1}{p^2 - M_{W_R}^2 + i M_{W_R} \Gamma_R}$$

$$\sigma \propto \frac{1}{(p^2 - M_{W_R}^2)^2 + M_R^2 \Gamma_R^2}$$

$$\Gamma_{W_R} \equiv \Gamma_R \ll M_{W_R} \equiv M_R$$

$$M_{W_R} = E_{W_R}\,(\text{rest}) =$$

$$= E_e + E_{\bar{e}} + E_j + E_j$$

$$\vec{p}_{W_R} = 0 = \vec{p}_e + \vec{p}_{\bar{e}} + \vec{p}_j + \vec{p}_j$$

(decay of W_R)

(decay of N)

$$E_N = E_{e^-} + E_j + E_j$$

$$\vec{p}_N = \vec{p}_{e^-} + \vec{p}_j + \vec{p}_j$$

N = Majorana

$$= N_L + C\bar{N}_L^T \; (\equiv \nu_R)$$

$$C\bar{\nu}_R^T \qquad (N_L) \qquad (N_R)$$

$$= C\bar{\nu}_R^T + \nu_R$$

$$L \xleftrightarrow[(P)]{} R$$

$$W^+_{\mu L}\bar{\nu}_L \gamma^\mu e_L + W^+_{\mu R}\bar{\nu}_R \gamma^\mu e_R$$

$$= W^{+}_{\mu L}\bar{\nu}_L\gamma^\mu e_L + W^{+}_{\mu R}\,\overline{N_R}\,\gamma^\mu e_R$$

$$\overline{N_R}\,\gamma^\mu e_R = \overline{e^c_L}\,\gamma^\mu (N^c)_L$$

e enters (e_R)

positron goes out $(e^c)_L$

$$\boxed{\text{Majorana! } \quad (N^c)_L \equiv N_L}$$

$\Downarrow$

$$W^{+}_{\mu R}\,\overline{N_R}\,\gamma^\mu e_R = \overline{e^c_L}\,\gamma^\mu N_L\, W^{+}_{\mu R}$$

$+ h.c.$

$+ \; h.c.$

$\overline{N_L}\,\gamma^\mu e^c_L\, W^{-}_{\mu R}$

$$\mathcal{L}_{cc}^{(R)} = \left[\bar{e}_R \gamma^\mu N_R W_{\mu R}^- + \overline{e^c_L} \gamma^\mu N_L W_{\mu R}^+\right] \frac{g}{\sqrt{2}}$$

$$N \to e_R + W_R^+ \quad \Big| \quad N \to (e^c)_L + W_R^-$$

$$\boxed{\Gamma(N \to e + W_R^{+*}) = \Gamma(N \to e^c + W_R^{-*})}$$

$*$ = off-shell

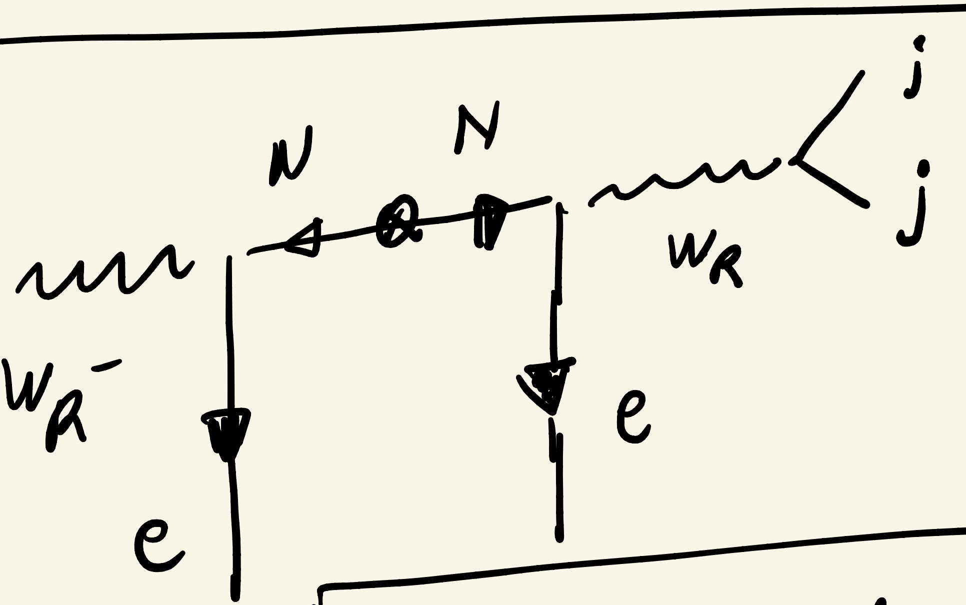
$\Updownarrow$

$$\boxed{\Gamma(N \to e + u + \bar{d}) = \Gamma(N \to \bar{e} + \bar{u} + d)}$$

$$\Downarrow \qquad\qquad \Downarrow$$

$$100\, e \quad = \quad 100\, \bar{e}$$

iff $N =$ Majorana



$\boxed{\Delta L = 2 \text{ direct at LHC}}$

(iv) $\Delta L = 2$ direct

(v) "Majorantity" probed

(vi) $M_N = V_R \, m_N \, V_R^T$

$\uparrow$ direct

$\swarrow$ probed at LHC

$\Updownarrow$

$$\left[\begin{array}{ll} \text{final state:} & e\,e \\ & e\,\mu \quad (\mu e) \\ & e\tau \quad (\tau e) \\ & \mu\tau \\ & \mu\mu \\ & \tau\tau \end{array} \right]$$

$$\Rightarrow \quad M_D = i\sqrt{M_N}\; O \;\sqrt{M_\nu}$$

??

$$\theta_{\nu N} \simeq \frac{1}{M_N} M_D$$

↑ меньше

$$M_D \Longrightarrow \theta_{\nu N}$$

$$\Updownarrow$$

$$W^{+}_{\mu L}\, \overline{N_L}\, \gamma^\mu\, \theta_{\nu N}\, e_L \frac{g}{\sqrt{2}}$$

$$\Rightarrow \quad N \to e + W_L^{+}$$

$$\to e^c + W_L^{-}$$

$\Downarrow$

$$\boxed{\Gamma(N \to e + W_L^+) = \Gamma(N \to \bar{e} + W_L^-)}$$

$\Uparrow$

given by M_D!

(vii) M_D = predicted

$\Leftrightarrow$ $\mathcal{O}$ = fixed

STAY TUNED

$$\mathcal{L}_y^{\Delta} = l_L^T i\sigma_2 \Delta_L C Y_\Delta l_L +$$

$$\underset{\mathcal{P}}{\parallel}$$

$$+ l_R^T i\sigma_2 \Delta_R C Y_\Delta l_R + h.c.$$

$$\mathcal{L}_\Phi = \bar{l}_R \left(Y_\Phi \Phi + \tilde{Y}_\Phi \tilde{\Phi} \right) l_L$$

$$+ \bar{l}_L \left(Y_\Phi^+ \Phi^+ + \tilde{Y}_\Phi^+ \tilde{\Phi}^+ \right) l_R$$

$$l_L \longleftrightarrow l_R \qquad \left(\tilde{\Phi} = \sigma_2 \Phi^* \sigma_2 \right)$$

$$\Phi \longleftrightarrow \Phi^+$$

$$\tilde{\Phi} \longleftrightarrow \tilde{\Phi}^+$$

$$\Rightarrow \boxed{Y_\Phi = Y_\Phi^+}$$

$$\tilde{\varphi}\,\bar{\Phi} = \tilde{\varphi}\,\bar{\Phi}^{+}$$

$$\Rightarrow \boxed{\varphi_D \stackrel{P}{=} \varphi_D^{+}}$$

$$\Downarrow$$

$$\Theta = \text{fixed !!}$$

$$\Leftrightarrow \quad \underset{\text{predicted}}{M_D = f(M_N, M_U)}$$