Neutrino Physics Course

Lecture XX

18/6/2021

LMU

Summer 2021

LR theory : $\not{P}$ spontaneous

Higgs sector

$$G_{LR} = SU(2)_L \times SU(2)_R \times U(1) \times P$$

$$\Delta_L \xleftrightarrow{P} \Delta_R$$

$$\Delta_L \to U_L \, \Delta_L \, U_L^\dagger \, ; \; \Delta_R \to U_R \, \Delta_R \, U_R^\dagger$$

$$(B-L) \; \Delta_{L,R} = 2$$

$$\langle \Delta_R \rangle = \begin{pmatrix} 0 & 0 \\ v_R & 0 \end{pmatrix}, \; \langle \Delta_L \rangle = 0$$

$M_R \propto v_R$

$$SU(2)_R \times U(1)_{B-L} \times P \xrightarrow{M_R} U(1)_Y$$

$$\Delta_{L,R} = \begin{pmatrix} \frac{1}{\sqrt{2}}\delta^{+} & \delta^{++} \\ \delta^{0} & -\delta^{+}\frac{1}{\sqrt{2}} \end{pmatrix}_{L,R}$$

- $SU(2)_L \times U(1)_Y \xrightarrow[\langle \Phi \rangle]{} U(1)_{em}$

$$\Phi \longrightarrow U_L \, \Phi \, U_R^{+}$$

$$\boxed{\Phi \xrightarrow{P} \Phi^{+}}$$

$$\Phi^{+} \longrightarrow U_R \, \Phi^{+} \, U_L^{+}$$

- $\underset{g}{\overset{A_R^{i}}{SU(2)_R}} \times \underset{\bar{g}}{\overset{\bar{B}}{U(1)_{B-L}}} \longrightarrow U(1)_Y$

$$\Downarrow$$

$$B = \sin\theta_R \, A_{3R} + \cos\theta_R \, \bar{B}$$

(Y) $\quad m_B = 0$

$$\boxed{Z_R = \cos\theta_R \, A_{3R} - \sin\theta_R \, \bar{B}}$$

$$m_{Z_R} \gg M_W$$

$$\boxed{\tan\theta_R = \frac{\bar{g}}{g}}$$

$$\boxed{g \equiv g_L = g_R}$$

- $\mathcal{L}_R \Rightarrow$ resulting physics

(i) gauge bosons

$i = 1,2,3$

$$D_\mu \Delta_R = \left(\partial_\mu - i g \hat{T}_{iR} A^i_{\mu R} - i \bar{g} \frac{B-L}{2} B_\mu\right) \Delta_R$$

$$\Delta_R \longrightarrow U_R \Delta_R U_R^{\dagger} =$$

$$= \left(1 + i \theta^i_R T_{iR} + \dots\right) \Delta_R \left(1 - i \theta^i_R T_{iR}\right)$$

$$\left(T_i^R = \frac{\sigma_i}{2} \right)$$

$$= \Delta_R + i \theta^i_R \underbrace{\left[T_{iR}, \Delta_R \right]}_{\Downarrow}$$

$$\hat{T}_{iR} \Delta_R = \left[T_{iR}, \Delta_R \right]$$

$$= \left[\frac{\sigma_i}{2}, \Delta_R \right]$$

$$\Updownarrow \; R$$

$$\left(\hat{T}_{iL}\, \Delta_L = \left[\frac{\sigma_i}{2},\ \Delta_L \right] \right)$$

$$\Downarrow$$

$$D_\mu (\Delta)_R \doteq -i \times$$

$$\frac{g}{2}\left(\begin{bmatrix} A_{3R} & A_{1R} - i A_{2R} \\ A_{1R} + i A_{2R} & - A_{3R} \end{bmatrix}_\mu , \begin{pmatrix} 0 & 0 \\ v_R & 0 \end{pmatrix} \right)$$

$$+ \bar{g}\ \overline{B_\mu} \begin{pmatrix} 0 & 0 \\ v_R & 0 \end{pmatrix} =$$

$$= \frac{g}{2} v_R \begin{pmatrix} A_{1R} - i A_{2R} & 0 \\ - A_{3R}\, 2 & A_{1R} - i A_{2R} \end{pmatrix}_\mu$$

$$+ \bar{g} v_R \begin{pmatrix} 0 & 0 \\ \bar{B}_\mu & 0 \end{pmatrix}$$

$$\Downarrow$$

$$D_\mu \langle \Delta_R \rangle = -i\, (x)$$

$$v_R \begin{pmatrix} (A_{1R} - i A_{2R})\, g/2 & 0 \\ (-A_{3R}\, g + \bar{g}\, \bar{B}) & \frac{g}{2}\, (A_{1R} - i A_{2R}) \end{pmatrix}_\mu$$

$$\Downarrow$$

$$\mathrm{Tr}\, (D_\mu \langle \Delta_R \rangle)^+ \; \mathrm{Tr}\, (D^\mu \langle \Delta_R \rangle)$$

$$\parallel$$

$$v_R^{\,2}\, \frac{g^2}{\cancel{4}\,2} (A_{1R}^{\,2} + A_{2R}^{\,2}) \cdot \cancel{2} \; +$$

$$+ \frac{1}{2} v_R^{\,2} \left(A_{3R}\, g - \bar{g}\, \bar{B}\right)^2 2$$

$$W_R^{\pm} = \frac{A_{1R} \mp i\, A_{2R}}{\sqrt{2}}$$

$$\Downarrow$$

$$\boxed{M_{W_R} = g\, v_R} \quad \boxed{= m_{A_{1R}} = m_{A_{2R}}}$$

$$Z_R \equiv \frac{g\, A_{3R} - \bar{g}\, \bar{B}}{\sqrt{g^2 + \bar{g}^2}}$$

$$\Rightarrow \boxed{M_{Z_R} = \sqrt{2}\, \sqrt{g^2 + \bar{g}^2}\, v_R}$$

$$\tan \theta_R \equiv \bar{g} / g$$

$$\Downarrow$$

$$\boxed{Z_R = \cos\theta_R \, A_{3R} - \sin\theta_R \, \bar{B}}$$

$$\Downarrow$$

$$\boxed{\frac{M_{Z_R}}{M_{W_R}} = \sqrt{2}\,\frac{1}{\cos\theta_R}}$$

$$\sin\theta_R = \frac{\bar{g}}{\sqrt{g^2+\bar{g}^2}}, \quad \cos\theta_R = \frac{g}{\sqrt{g^2+\bar{g}^2}}$$

- what is θ_R??

$$\tan\theta_W \equiv g'/g$$

$U(1)_Y$

$SU(2)_L$

$$\frac{Y}{2} = T_{3R} + \frac{B-L}{2} \leftarrow$$

$$\Updownarrow$$

$$Q_{em} = T_{3L} + Y/2$$

$$\Downarrow$$

$$\boxed{\begin{gathered} e = g \sin\theta_W = g' \cos\theta_W \\ \frac{1}{e^2} = \frac{1}{g^2} + \frac{1}{g'^2} \Leftarrow \\ (\sin^2\theta_W + \cos^2\theta_W = 1) \end{gathered}}$$

$$\Downarrow$$

LR $\longleftrightarrow$ SM analog

$$\boxed{\frac{1}{g'^2} = \frac{1}{g^2} + \frac{1}{\bar{g}^2}}$$

$$\Rightarrow \quad g' = \frac{g\,\bar{g}}{\sqrt{g^2+\bar{g}^2}}$$

$$\tan\theta_W = g'/g = \frac{\bar{g}}{\sqrt{g^2+\bar{g}^2}} = \sin\theta_R$$

$$\boxed{\tan\theta_W = \sin\theta_R}$$

$$\Downarrow$$

$$\boxed{M_{Z_R} = \frac{\sqrt{2}}{\sqrt{1-\tan^2\theta_W}} \, M_{W_R}}$$

$$\theta_W \simeq 30^\circ \leftrightarrow \sin^2\theta_W = 1/4$$

$$\Downarrow$$

$$M_{Z_R} = \frac{\sqrt{2}}{\sqrt{2/3}} \, M_{W_R} = \sqrt{3} \, M_{W_R}$$

$\boxed{\text{LHC}}$

$$\left.\begin{aligned} M_{W_R} &\gtrsim 5\,\text{TeV} \\ M_{Z_R} &\gtrsim 2\,\text{TeV} \end{aligned}\right\} \text{exp.}$$

$$\boxed{\text{theory: } \quad M_{Z_R} \gtrsim 8\,\text{TeV}}$$

$$\boxed{\begin{aligned} Z_R &= \cos\theta_R \, A_{3R} - \sin\theta_R \, \bar{B} \\ &= \sqrt{1-\tan^2\theta_W}\, A_{3R} - \tan\theta_W \, \bar{B} \end{aligned}}$$

Scales: can they be predicted?

SM

$$M_W = \frac{g}{2} v$$

$$m_h^2 = 2\lambda v^2$$

$$\frac{\lambda}{4} (\Phi^\dagger \Phi)^2$$

$$\lambda = ?$$

$$m_h^2 = 2\lambda \frac{4 M_W^2}{g^2} = \frac{8 \lambda M_W^2}{g^2}$$

$$\lambda = ? \Rightarrow m_h = ?$$

$$e = g \sin\theta_W$$

measure

$$\frac{G_F}{\sqrt{2}} = \frac{g^2}{8 M_W^2} = \frac{e^2}{8 (\underline{M_W \sin\theta_W})^2}$$

$$\Leftrightarrow \quad \frac{g}{\sqrt{2}} \bar{u}_L \gamma^\mu d_L W_\mu^+ + h.c.$$

measure G_F, θ_W !

$$M_W \sin\theta_W = 40 \text{ GeV}$$

$\boxed{\text{need } Z}$

$$\frac{g}{\cos\theta_W} Z_\mu \bar{f} \gamma^\mu \left[T_3 - Q \sin^2\theta_W \right] f$$

$$\boxed{\Rightarrow \quad \theta_W \simeq 30^\circ}$$

Higgs

$$m_h^2 = 2\lambda v^2 = \frac{8\lambda M_W^2}{g^2}$$

$$+ \frac{\lambda}{4} h^4$$

$+$ 

$$g^2 \frac{m_f^2}{M_W^2\, m_h^2} = \frac{y^2}{m_h^2} \longleftrightarrow \frac{g^2}{M_W^2}$$

high precision low E physics

Prediction of Higgs SM!

① $\lambda = \frac{g^2}{8} \frac{m_h^2}{M_W^2}$

② $G_H = g^2 \frac{m_f^2}{M_W^2\, m_h^2}$

(LOWER LIMIT)

① $M_{W_R} \gtrsim 2.5 - 3$ TeV

low E physics

$$\frac{4 G_F}{\sqrt{2}} J_\mu^{w} J^\mu_w$$

$$J^\mu_w = \bar{u}_L \gamma^\mu d_L + \bar{\nu}_L \gamma^\mu e_L$$

beaten by high E LHC !

② $\boxed{\text{UPPER LIMIT}}$?

SM: $m_\nu = 0$

exp. $m_\nu \neq 0$

LR : $m_\nu \neq 0 \Rightarrow M_R \leq 10^{14}$ GeV (?)

$\boxed{\text{NOT exciting}}$

Central result (to emerge)

<u>$0\nu 2\beta$</u> $\Delta L = 2$

- if observed
- and if $e = e_R$

$\Rightarrow \boxed{M_{W_R} \leq 10 \text{ TeV}}$

$\boxed{\text{LHC: } W_R \to 6\text{-}8 \text{ TeV}}$

STAY TUNED!

Construction of LR

① $G_{LR}^{min} = SU(2)_L \times SU(2)_R \times U(1)_{B-L}$

② $\begin{pmatrix} u \\ d \end{pmatrix}_L \quad \begin{pmatrix} \nu \\ e \end{pmatrix}_L \Longleftrightarrow \begin{pmatrix} \nu \\ e \end{pmatrix}_R \quad \begin{pmatrix} u \\ d \end{pmatrix}_R$

$$\nu_L \longleftrightarrow \nu_R$$

$$\Downarrow$$

$$m_\nu \neq 0$$

③ Higgs mechanism

$$\therefore \quad LR \xrightarrow[M_R]{} SM$$

only SM states $\quad m = 0$

$$\Leftrightarrow \quad m_{\nu_R} \propto M_R$$

$$\Downarrow \quad \Delta_{L,R}$$

$$\mathcal{L}_Y^\Delta = l_L^T i\sigma_2 C \Delta_L Y_\Delta l_L$$

$$\Updownarrow P \qquad \qquad \| $$

$$+ l_R^T i\sigma_2 C \Delta_R Y_\Delta l_R \quad + h.c.$$

$$\boxed{\langle \Delta_R \rangle \neq 0} \quad \Downarrow$$

$$= \left(\nu_R^T e_R^T \right) \begin{pmatrix} 0 & 1 \\ -1 & 0 \end{pmatrix} C Y_\Delta \begin{pmatrix} \delta_R^+/\sqrt{2} & \delta_R^{++} \\ \delta_R^0 & -\delta_R^+/\sqrt{2} \end{pmatrix} \begin{pmatrix} \nu_R \\ e_R \end{pmatrix}$$

$$+ R \to L$$

$$= \begin{pmatrix} \nu_R^T & e_R^T \end{pmatrix} C\, y_\Delta \begin{pmatrix} \delta_R^0 & -\delta_R^+/\sqrt{2} \\ \delta_R^+/\sqrt{2} & \delta_R^{++} \end{pmatrix} \begin{pmatrix} \nu_R \\ e_R \end{pmatrix} + h.c.$$

$$= \nu_R^T C\, y_\Delta \nu_R\, \delta_R^0 - \nu_R^T C\, y_\Delta\, e_R\, \delta_R^+/\sqrt{2}$$

$$+ e_R^T C\, y_\Delta\, \nu_R\, \delta_R^+/\sqrt{2} + e_R^T C\, y_\Delta\, e_R\, \delta_R^{++}$$

$$+ h.c.$$

$$= \nu_R^T C\, y_\Delta \nu_R\, \delta_R^0 + e_R^T C\, y_\Delta\, e_R\, \delta_R^{++}$$

$$+ e_R^T C \left(y_\Delta - y_\Delta^T \right) \bar{\nu}_R\, \bar{\delta}_R^+ + h.c.$$

$\boxed{\text{does } \delta_R^+ \text{ exist?}}$

$\boxed{SM}$ $\quad \Phi = \begin{pmatrix} \varphi^+ \\ \varphi^0 \end{pmatrix} = \begin{pmatrix} \varphi^+ \\ v + h + iG_Z \end{pmatrix}$

does φ^+ exist in SM?

<u>NO!!!</u> $\quad W^\pm$ eats $\varphi^\pm$

Z eats G_Z

$\boxed{LR}$ $\quad \delta_R^+$ exists ???

$W_R^\pm$ eats $\delta_R^\pm$

$\Downarrow$

new Higgs

$$\mathcal{L}_Y^{(\Delta_R)} = \nu_R^T C\, Y_\Delta\, \nu_R\, (v_R + h_R \cancel{+ i\, G_R})$$

eaten by Z_R

$$+\, e_R^T C\, Y_\Delta\, e_R\, \delta_R^{++}$$

$$\Rightarrow \boxed{M_{\nu_R} = \underbrace{Y_\Delta}_{\text{matrix (generations)}} v_R \qquad (1)}$$

$$\Rightarrow \boxed{e_R^T C\, \frac{M_{\nu_R}}{v_R}\, e_R\, \delta_R^{++} \qquad (2)}$$

$\ell = e, \mu, \tau$ (generations)

$$\nu_L \Rightarrow \quad N_L \equiv C \bar{\nu}_R^{\,T} = C\gamma_0 \, \nu_R^*$$
$$= i\gamma_2 \, \nu_R^*$$
$$= \begin{pmatrix} 0 & i\sigma_2 \\ -i\sigma_2 & 0 \end{pmatrix} \nu_R^*$$

$$\Rightarrow \boxed{M_N = M_{\nu_R}^*}$$

$$\Downarrow$$

$$\boxed{e_R^T \, C \, \frac{M_N^*}{v_R} \, e_R \, \delta_R^{++}}$$

$$\Downarrow$$

• e = diagonal

$$\boxed{M_N = V_R \, m_N \, V_R^T}$$

diagonal

$$M_N^T = M_N$$

leptonic mixing in W_R

$$\mathcal{L}_{W_L} = \frac{g}{\sqrt{2}} \left[\bar{u}_L^0 \gamma^\mu d_L^0 + \bar{\nu}_L^0 \gamma^\mu e_L^0 \right] W_{\mu L}^+$$

$$+ \frac{g}{\sqrt{2}} \left[\bar{u}_R^0 \gamma^\mu d_R^0 + \bar{\nu}_R^0 \gamma^\mu e_R^0 \right] W_{\mu R}^+$$

$$\nu_R^0 = U_{\nu_R}\, \nu_R$$

$$e_R^0 = U_{e_R}\, e_R$$

$$\Rightarrow \frac{g}{\sqrt{2}}\, \overline{\nu_R}\, \underbrace{U_{\nu_R}^{\dagger}\, U_{e_R}}_{=1}\, e_R\, W_{\mu R}^{+}$$

$$= \frac{g}{\sqrt{2}}\, \overline{\nu_R}\, U_{\nu_R}^{\dagger}\, e_R\, W_{\mu R}^{+}$$

$$M_N = V_R\, m_N\, V_R^T$$

$$M_{\nu_R} = V_R^*\, m_N\, V_R^{\dagger}$$

$$\boxed{U_{\nu_R} = V_R^*}$$

find $N \Rightarrow M_N$

m_N, mixings V_N

- $$e_R^T \frac{M_N^*}{v_R} e_R \; \delta_R^{++} + e_R^+ \frac{M_N}{v_R} e_R^* \; \delta_R^{--}$$

$$\delta_R^{--} \longrightarrow e_R^i e_R^j \begin{pmatrix} e_R e_R \\ e_R \mu_R \\ \mu_R \tau_R \\ \vdots \end{pmatrix}$$

M_N^{ij}

$\boxed{\Delta L = 2}$

$\boxed{\text{To be studied!}}$

$\boxed{\text{Truth} = \text{in production of } N}$

$\Updownarrow$

$\boxed{\Delta L = 2, \quad \text{Majorana theory}}$

SM

$$V_{CKM} = U_{uL}^{+} \, U_{d_L}$$

exp

$$\Longrightarrow (a) \; U_{d_L} = 1$$

$$V_{CKM} = U_{uL}^{+}$$

(b) $U_{uL} = 1 \Rightarrow V_{CKM} = U_{dL}$

$$\Rightarrow \quad V_{PMNS}^{+} \equiv \underbrace{U_{\nu L}^{+} \, U_{eL}}$$

$$U_{eL} = 1 \Rightarrow V_{P} = U_{\nu L}$$

$$\boxed{\begin{gathered} M_{\nu} = U_{\nu L} \, m_{\nu} \, U_{\nu L}^{T} \\ \Uparrow \\ M_{\nu}^{T} = M_{\nu} \end{gathered}}$$

$$M_\nu = V_{PMNS}\, m_\nu\, V_{PMNS}^T \quad ?$$

$$\longrightarrow \left(V_{PMNS}^*\, m_\nu\, V_{PMNS}^\dagger \right)$$