Neutrino Physics Course

Lecture XVII

8/6/2021

LMU
Spring 2021

Left-Right Symmetric Theory

SM:

$\cancel{P}$ maximal $\Rightarrow$ theory of origin of mass of f

-||- $\Rightarrow$ $m_\nu = 0$

$\Downarrow$

$\boxed{\text{break P spont.}}$

$\boxed{\text{LR theory}}$ $\boxed{1974-1975}$

<u>matter</u>

$$\begin{pmatrix} u \\ d \end{pmatrix}_L \xleftrightarrow{LR = P} \begin{pmatrix} u \\ d \end{pmatrix}_R$$

$$\begin{pmatrix} \nu \\ e \end{pmatrix}_L \xleftrightarrow{LR = P} \begin{pmatrix} \nu \\ e \end{pmatrix}_R$$

$$G_{LR} = ? \quad (\text{minimal})$$

$$G_{LR}^{min} = \underline{SU(2)_L} \times \underset{\underset{T_{3R}}{\uparrow}}{\underline{SU(2)_R}} \Big| \quad ?$$

$$Q_{em} = T_{3L} + T_{3R}$$

$$\rightarrow U_{L,R} = e^{i \vec{\theta}_{L,R} \cdot \vec{T}_{L,R}}$$

$$\vec{T}_{L,R} = \vec{\sigma}/2 \quad (\vec{\theta}_L \neq \vec{\theta}_R)$$

$$\Downarrow$$

$$Q_{em} = \pm 1/2 \qquad L$$

$$\pm 1/2 \qquad R$$

WRONG!

$$\Downarrow$$

$$G_{LR}^{min} = SU(2)_L \times SU(2)_R \times U(1)_{Y'}$$

$$\Downarrow$$

$$Q_{em} = T_{3L} + T_{3R} + \frac{Y'}{2}$$

$$f_L \Rightarrow Y'_L = Y = \begin{cases} 1/3 & q_L \\ -1 & \ell_L \end{cases}$$

$$f_R \Rightarrow Y'_R = Y = \begin{cases} 1/3 & e_R \\ -1 & \ell_R \end{cases}$$

$$\Downarrow$$

$$Y' =$$

$$\boxed{\text{Baryon} = 3 \text{ quarks}}$$

$$B \text{ baryon} = 1 \Rightarrow B_q = 1/3$$

$$L \text{ lepton} = 1$$

$$\Downarrow$$

$$\boxed{Y' = B - L}$$

= global (accidental) sym. in SM

= anomaly-free

$\Rightarrow$ $\boxed{B-L = \text{gauged}}$

$\Downarrow$ LR theory

$$\boxed{\begin{array}{ll} \text{(i)} & Q_{em} = T_{3L} + T_{3R} + \dfrac{B-L}{2} \\ & \\ \text{(ii)} & m_\nu \neq 0 \end{array}}$$

$\downarrow$

Q. How $m_\nu \ll m_e$?

- gauge group
- matter $= (q, \ell)$

$$q_L \equiv \begin{pmatrix} u \\ d \end{pmatrix}_L \qquad \begin{pmatrix} u \\ d \end{pmatrix}_R \equiv q_R$$

$$\bar{q}_L \, M \, q_R$$

$$\underset{SU(2)_L}{q_L \to U_L \, q_L}, \quad \underset{SU(2)_R}{q_R \to U_R \, q_R}$$

$$[T_a, T_b] = i \, \underset{(f_{abc})}{\varepsilon_{abc}} \, T_c$$

SU(2)

$$\Rightarrow T_a = \vec{\sigma}/2$$

$$U_L \equiv e^{i\vec{\theta}_L \cdot \vec{\sigma}/2} \; ; \quad U_R \equiv e^{i\vec{\theta}_R \cdot \vec{\sigma}/2}$$

$$\Downarrow$$

$$\bar{q}_L \underline{M}\, q_R \to \bar{q}_L\, U_L^{\dagger} U_R \underline{M}\, q_R$$

$$\neq \bar{q}_L \underline{M}\, q_R$$

$$\Downarrow \quad \boxed{\text{Higgs mechanism}}$$

$$\Rightarrow \mathcal{L}_y = \bar{q}_L\, Y_{\Phi}\, \Phi\, q_R + h.c.$$

$$+ \cdots ?$$

$$G_{SM} \xrightarrow[M_{W_L}]{} U(1)_{em}$$

$$D_\mu = \partial_\mu - i g_L \hat{A}_{L\mu} \cdot \vec{T}_L$$

$$- i g_R \vec{A}_{R\mu} \cdot \vec{T}_R$$

$$- i g_{BL} B_\mu^{(BL)} \frac{B-L}{2}$$

LR

$$g_L \equiv g_R \equiv g$$

$$\Rightarrow \quad W_\mu^\pm \equiv W_{\mu L}^\pm$$

$$\Rightarrow \quad W_{\mu R}^\pm$$

$$\Rightarrow \quad \boxed{M_{W_R} \gg M_{W_L}}$$

$$G_{LR} \longrightarrow G_{SM} \longrightarrow U(1)_{em}$$

$$\underline{M}_{W_R} \qquad \underline{M}_{W_L}$$

new Higgs

(def. of Φ)

$$\mathcal{L}_Y = \bar{q}_L \, Y_\Phi \, \Phi \, q_R + h.c.$$

$$q_{L,R} \to U_{L,R} \, q_{L,R}$$

$$\Rightarrow \boxed{\Phi \to U_L \, \Phi \, U_R^+}$$

$$\boxed{= \text{matrix}}$$

Φ = doublet of $SU(2)_L$

— II — of $SU(2)_R$

bi – doublet

$$Q_{em} = T_{3L} + T_{3R} + \frac{B-L}{2}$$

$$(B-L)\,\Phi = ?$$

$$(B-L)\,q_L = (B-L)\,q_R$$

$$\Rightarrow (B-L)\,\Phi = 0$$

$$\Phi = \begin{pmatrix} a & b \\ c & d \end{pmatrix} \qquad \Downarrow$$

$$U = e^{i\vec{\theta}\cdot\vec{T}}$$

$$\Downarrow$$

$$U^{+} = e^{i\vec{\theta}\,(-\vec{T})}$$

$$\Downarrow$$

$$Q_{em}\,\Phi = T_{3L}\,\Phi - \Phi\,T_{3R}$$

$$= \begin{pmatrix} \frac{1}{2} & 0 \\ 0 & -1/2 \end{pmatrix} \begin{pmatrix} a & b \\ c & d \end{pmatrix} - \begin{pmatrix} a & b \\ c & d \end{pmatrix} \begin{pmatrix} 1/2 & 0 \\ 0 & -1/2 \end{pmatrix}$$

$$= \begin{pmatrix} (1/2 - \frac{1}{2})a & -(-1/2)\,b \\ -\frac{1}{2}\,c & (-\frac{1}{2} + \frac{1}{2})\,d \end{pmatrix}$$

$$= \begin{pmatrix} 0 \cdot a & (+1)\,b \\ (-1)\,c & 0 \cdot d \end{pmatrix}$$

<u>SM dublet</u> $\quad \phi = \begin{pmatrix} \varphi^{+} \\ \varphi^{0} \end{pmatrix}$

$$\begin{pmatrix} b \\ d \end{pmatrix} = \phi$$

$\begin{pmatrix} a \\ c \end{pmatrix} = ?$

$\phi \longrightarrow U \Phi$

$\phi^{+} \longrightarrow \Phi^{+} U^{+}$

$\cancel{E = ma^2}$

$\cancel{E = mb^2}$

$\phi^{T} \longrightarrow \Phi^{T} U^{T}$

$E = mc^2!$

$\phi^{*} \longrightarrow U^{*} \phi^{*}$

$$\tilde{\phi} \equiv i\sigma_2 \, \phi^{*} \longrightarrow U \tilde{\phi}$$

$$\Longleftrightarrow \quad \underbrace{\phi^{T} \, i\sigma_2 \, \phi}_{\parallel} = \text{invariant}$$

$$\epsilon = \begin{pmatrix} 0 & 1 \\ -1 & 0 \end{pmatrix}$$

$$\phi = \begin{pmatrix} \uparrow \\ \downarrow \end{pmatrix} \qquad (\uparrow\downarrow - \downarrow\uparrow)$$

$$\Downarrow$$

$$\boxed{\Phi = \begin{pmatrix} \tilde{\phi} & \phi \end{pmatrix}} \qquad \Phi \to U_L \Phi U_R^\dagger$$

$$V \stackrel{?}{=} f(? \; \Phi^\dagger \Phi \; ?)$$

$$\Phi^\dagger \Phi \to U_R \Phi^\dagger U_L^\dagger U_L \Phi U_R^\dagger$$

$$= U_R \Phi^\dagger \Phi U_R^\dagger$$

$$\mathrm{Tr}\, \Phi^\dagger \Phi \to \mathrm{Tr}\, \Phi^\dagger \Phi$$

$$\Downarrow$$

$$\boxed{V = f(\mathrm{Tr}\, \Phi^\dagger \Phi)}$$

$$\mathrm{Tr}\,\Phi^{+}\Phi \;\propto\; \phi^{+}\phi \quad !!$$

coeff. = ?

$$\phi = \begin{pmatrix} R_1 + i R_2 \\ R_3 + i R_4 \end{pmatrix} = \begin{pmatrix} \varphi^{+} \\ \varphi^{0} \end{pmatrix}$$

$$\Rightarrow \phi^{+}\phi = \sum_{i=1}^{4} R_i^2 \Rightarrow$$

$$\boxed{G(V) = SO(4)}$$

$$SO(4) = SU(2)_L \times SU(2)_R$$

$\# = 6$ $\qquad\qquad$ $\# = 6$

$r = 2$ $\qquad\qquad$ $r = 2$

$$\Phi = \begin{pmatrix} \tilde{\phi} & \phi \end{pmatrix} \qquad \phi = \begin{pmatrix} \varphi^+ \\ \varphi^0 \end{pmatrix}$$

$$\tilde{\phi} \equiv i\sigma_2 \phi^* = \begin{pmatrix} 0 & 1 \\ -1 & 0 \end{pmatrix} \begin{pmatrix} \varphi^- \\ \varphi^{0*} \end{pmatrix} = \begin{pmatrix} \varphi_0^* \\ -\varphi^- \end{pmatrix}$$

$$= \begin{pmatrix} \varphi_0^* & \varphi^+ \\ -\varphi^- & \varphi^0 \end{pmatrix}$$

$$\uparrow \qquad \uparrow$$

$SU(2)_L$ doublets

$$\boxed{\Phi \to U_L \, \Phi \, U_R^\dagger}$$

$$\langle \Phi \rangle = \begin{pmatrix} v & 0 \\ 0 & v \end{pmatrix} \qquad (v \in \mathbb{R})$$

$$\mathcal{L}_Y = \bar{q}_L \, Y_\Phi \, \Phi \, q_R =$$

#

$$\ni (\bar{u}\,\bar{d})_L \, y_\Phi v \begin{pmatrix} 1 & 0 \\ 0 & 1 \end{pmatrix} \begin{pmatrix} u \\ d \end{pmatrix}_R$$

$$= y_\Phi v \left(\bar{u}_L u_R + \bar{d}_L d_R \right)$$

$$\Downarrow$$

$m_u = m_d$ Disaster !

not good

$$\boxed{\text{What to do?}}$$

$$\Phi = \left(\tilde{\phi} \quad \phi \right)$$

doublets

$$\Phi \to U_L \, \Phi \, U_R^{\dagger}$$

- Y_ϕ = number
- one vev $\equiv v$ in $\Phi = (\tilde{\phi} \;\; \phi)$

$\uparrow$

$m_u \neq m_d$

$$\Phi = \begin{pmatrix} \text{doublet} & \text{doublet} \\ (0,-1) & (+,0) \end{pmatrix}$$

charges

from the group theory structure

$$\Rightarrow \Phi = \begin{pmatrix} \tilde{\phi}_1 & \phi_2 \end{pmatrix} \quad \underline{\underline{\text{a must!}}}$$

(complex) L_i - dublet

$$\phi_2 \to U_L \, \phi_2$$

$$\phi_1 \to U_L \, \phi_1$$

$$\left(\Rightarrow \tilde{\phi}_1 \to U_L \, \tilde{\phi}_1 \right)$$

$$\Phi = \begin{pmatrix} \varphi_1^{0*} & \varphi_2^{+} \\ -\varphi_1^{-} & \varphi_2^{0*} \end{pmatrix} \qquad \Phi \to U_L \, \Phi \, U_R^{+} \;\leftarrow SU(2)_R$$

$$\Rightarrow \boxed{\langle \Phi \rangle = \begin{pmatrix} v_1^{*} & 0 \\ 0 & v_2 \end{pmatrix}}$$

$$\Downarrow$$

$$\boxed{\begin{aligned} m_u &= Y_\Phi v_1^* \\ &\neq \\ m_d &= Y_\Phi v_2 \end{aligned}}$$

- <u>$SU(2)_R$ doublet</u>

$$\psi = \begin{pmatrix} \psi^+ \\ \psi^0 \end{pmatrix}$$

$$\psi \longrightarrow U_R \psi$$

$$\psi^\dagger \longrightarrow \psi^\dagger U_R^\dagger$$

$$\Rightarrow \psi^\dagger = \underline{\underline{\begin{pmatrix} \psi^- & \psi_0^* \end{pmatrix}}}$$

$$\Downarrow$$

$$\Phi = \begin{array}{c} L \\ \downarrow \end{array}\begin{pmatrix} \phi_1^{0*} & \phi_2^{+} \\ -\phi_1^{-} & \phi_2^{0} \end{pmatrix}$$

$$\xrightarrow[R]{}$$

$\boxed{\text{bi - doublet}}$

$\rightarrow$ $\boxed{\text{add generations}}$

$\Rightarrow$ Y_Φ = matrix in generation space

$$\Rightarrow \quad \underline{M}_u = Y_\Phi v_1^*$$

$$\underline{M}_d = Y_\Phi v_2$$

$$\Downarrow$$

$$M_u \propto M_d \Rightarrow U^u_{L,R} = U^d_{L,R}$$

$$\Rightarrow \begin{cases} m_u \propto m_d & \text{wrong!} \\ V_{CKM} = U_{Lu}^{+} U_{Ld} = 1 & \text{wrong!} \end{cases}$$

<u>what to do?</u>

$$\left.\begin{aligned} \mathcal{L}_{SM} &= (\bar{u}\,\bar{d})_L\, Y_d\, \phi\, d_R + \\ &+ (\bar{u}\,\bar{d})_L\, Y_u\, \underbrace{i\sigma_2 \phi^*}_{\tilde{\phi}}\, u_R \end{aligned}\right\}$$

$\phi \in C$

$$\Longrightarrow \boxed{Y_u \neq Y_d}$$

Φ = complex bi-doublet =

= 2 different $SU(2)_L$ doublets

$$\Phi \longrightarrow U_L \, \Phi \, U_R^{+}$$

LR

$$\tilde{\Phi} = i\sigma_2 \Phi^* (-i\sigma_2)$$

SM

$$\phi \rightarrow U \phi$$

$$\Rightarrow \tilde{\phi} = i\sigma_2 \phi^*$$

$\Downarrow$

$$\mathcal{L}_Y = \bar{q}_L\, Y\, \Phi\, q_R + \bar{q}_L\, \tilde{Y}\, \tilde{\Phi}\, q_R + h.c.$$

$$\tilde{\Phi} = \begin{pmatrix} 0 & -i \\ i & 0 \end{pmatrix} \begin{pmatrix} a & b \\ c & d \end{pmatrix}^* \begin{pmatrix} 0 & -i \\ i & 0 \end{pmatrix}$$

$$\Phi = (\tilde{\phi}_1 \ \ \phi_2) \overset{?}{\Rightarrow} \tilde{\Phi} = (\tilde{\phi}_2 \ \ \phi_1)$$

*** check ***

$$\langle \Phi \rangle = \begin{pmatrix} v_1^* & 0 \\ 0 & v_2 \end{pmatrix} \Rightarrow$$

$$\langle \tilde{\Phi} \rangle = \begin{pmatrix} v_2^* & 0 \\ 0 & v_1 \end{pmatrix}$$

$$\Downarrow$$

$$\mathcal{L}_Y \to \bar{\ell}_L \left(y \langle \Phi \rangle + \tilde{y} \langle \tilde{\Phi} \rangle \right) \ell_R$$

$$\Rightarrow \boxed{\begin{aligned} M_u &= v_1^* y + v_2^* \tilde{y} \\ M_d &= v_2 y + v_1 \tilde{y} \\ M_u &\neq M_d \end{aligned}}$$