Neutrino Physics Course

Lecture XVI

4/6/2021

Neutrino mass and $\boxed{\text{parity}}$

<u>$m_\nu = 0$?</u> $\qquad G_{SM} = SU(2)_L \times U(1)$

$$\begin{pmatrix} \nu \\ e \end{pmatrix}_L \qquad e_R \,, \; \cancel{\nu_R}$$

$\uparrow$ singlet

$$\Downarrow$$

- $\cancel{m_D \, \bar{\nu}_L \, \nu_R}$
- $\cancel{m_\nu^M \, \nu_L^T C \nu_L}$ $\Leftarrow$ gauge sym.

However, <u>charged fermions</u>

$\Longleftrightarrow$ P violation

$$m_e, \quad m_u, \quad m_d \ll M_W$$

$$m_f \; (Q \neq 0) \leq M_W$$

$$\left.\begin{matrix} \begin{pmatrix} \nu \\ e \end{pmatrix}_L & e_R \\ \begin{pmatrix} u \\ d \end{pmatrix}_L & u_R, \; d_R \end{matrix}\right\} \Longrightarrow m_f = 0$$

$$\Downarrow$$

quarks

$$\mathcal{L}_Y = (\bar{u}\,\bar{d})_L^i \, Y_d^{ij} \, \Phi \, d_R^j + (\bar{u}\,\bar{d})_L^i \, Y_u^{ij} \, i\sigma_2 \Phi^* u_R^j + h.c.$$

$$\Downarrow$$

$$M_{u,d} = y_{u,d}\, v$$

$$= U_{L\,u,d}^{+} \underbrace{m_{u,d}}_{\text{diagonal}} U_{R\,u,d}$$

$$\Downarrow$$

$(A_\mu, \underline{\underline{Z_\mu}}) \to$ diagonal

$$\boxed{\text{\underline{no} flavor violation in neutral currents (tree)!}}$$

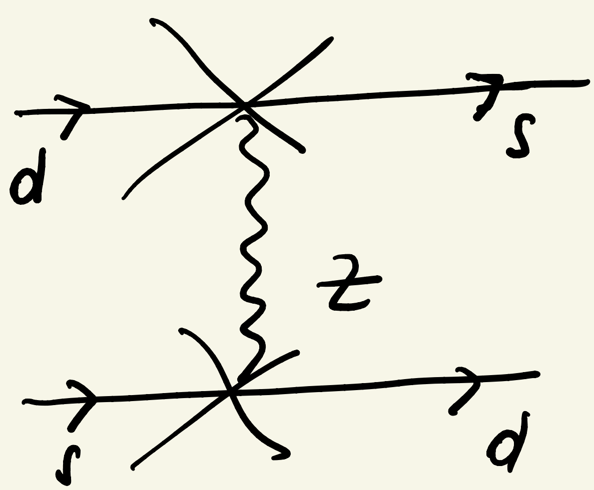
$$\Rightarrow \quad h \frac{m_f}{M_W} \frac{g}{2} \bar{f} f \leftarrow \text{diagonal}$$

· Why flavor violation?

$$\mathcal{L}_{wn} = \frac{g}{\sqrt{2}} W^+_\mu \left(\bar{u}\ \bar{c}\ \bar{t}\right)_L \gamma^\mu V_q \begin{pmatrix} d \\ s \\ b \end{pmatrix}_L$$

$$V_q \equiv U_{Lu}^+ U_{Ld} \qquad\qquad V_q \equiv V_{CKM} \neq 1$$

<u>loops</u>

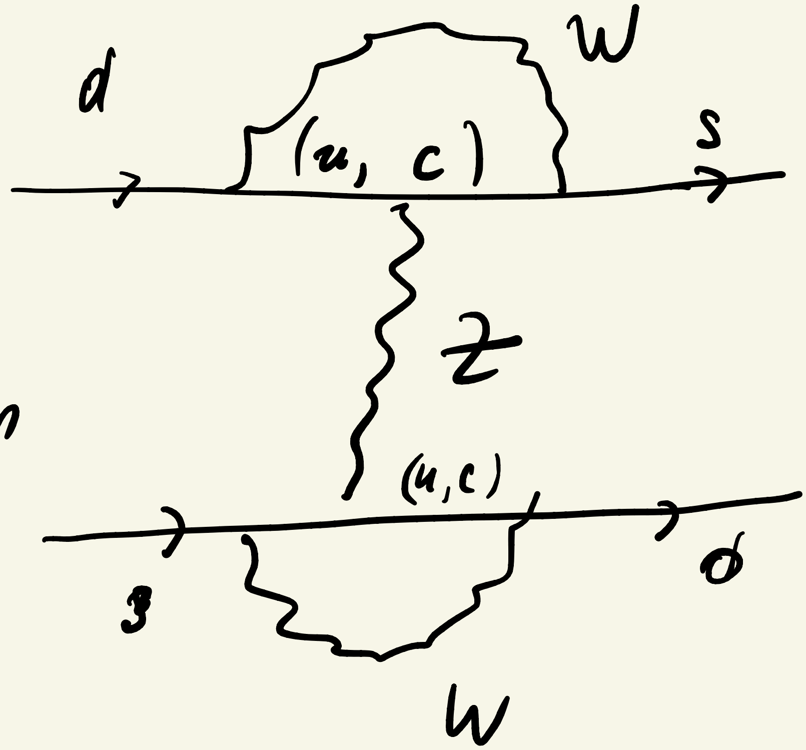


$= 0$ (tree)

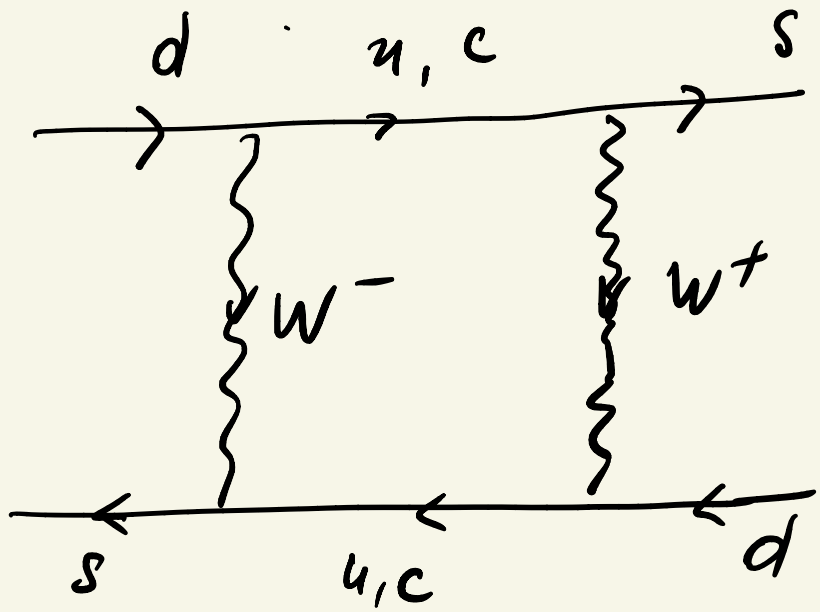
$\Downarrow$

$\boxed{n_g = 2}$

2-loop



1 loop: $K - \bar{K}$ mixing $(d\bar{s} \to s\bar{d})$



GIM

Glashow
Iliopoulos
Maiani '69

$\Downarrow$

lazy person's approach

$$\mathcal{H}_{eff}^{\Delta S=2} = \frac{G_F}{\sqrt{2}} \frac{\alpha}{4\pi} \sin^2\theta_c \, \Delta m^2 \, \underbrace{\bar{s}\gamma_\mu L d \, \bar{s}\gamma^\mu L d}_{d=6}$$

$\boxed{GIM}$

$$\boxed{\begin{matrix} K_0 \xleftrightarrow{\;\;CP\;\;} \bar{K}_0 \end{matrix}}$$

$K^+ \leftrightarrow K^-$

$K^0 \leftrightarrow \bar{K}^0$

$\underline{CP} \Rightarrow$

$$K^+ = \frac{K_0 + \bar{K}_0}{\sqrt{2}} \quad (\to K^+)$$

$$K^- = \frac{K^0 - \bar{K}^0}{\sqrt{2}} \quad (\to -K^-)$$

$K^+ = u\bar{s}$ $\qquad\qquad$ $K^- = \bar{u}s$

$K^0 = d\bar{s}$ $\qquad \Downarrow \qquad$ $\bar{K}^0 = \bar{d}s$

$$\mathcal{M}_{n\bar n} = \begin{array}{c} n^0 \\ \bar n^0 \end{array}\begin{pmatrix} m_n & \delta m_n \\ \delta m_n & m_n \end{pmatrix}$$

$$n^+ = \begin{pmatrix} 1 \\ 1 \end{pmatrix} \frac{1}{\sqrt{2}} \qquad \Downarrow$$

$$n^- = \begin{pmatrix} 1 \\ -1 \end{pmatrix} \frac{1}{\sqrt{2}}$$

$$\mathcal{M}_{n\bar n} \begin{pmatrix} 1 \\ 1 \end{pmatrix} = \overbrace{(m_n + \delta m_n)}^{m_+} \begin{pmatrix} 1 \\ 1 \end{pmatrix}$$

$$\mathcal{M}_{n\bar n} \begin{pmatrix} 1 \\ -1 \end{pmatrix} = \underbrace{(m_n - \delta m_n)}_{m_-} \begin{pmatrix} 1 \\ 1 \end{pmatrix}$$

$$\frac{m_+ - m_-}{m_+ + m_-} = \boxed{\frac{\delta m_n}{m_n}} \simeq \underline{10^{-14}} \, !$$

$$\boxed{m_+ \simeq m_- \simeq 100 \text{ MeV}}$$

$$\frac{\delta m_K^{SM}}{m_K} = \langle K^0 | H_{eff} | \bar{K}^0 \rangle \frac{1}{m_K}$$

$$= \frac{G_F}{\sqrt{2} m_K} (\alpha/4\pi) \underbrace{\langle K^0 | (\bar{s} \gamma^\mu L d)^2 | \bar{K}^0 \rangle}$$

$m_{d,s} = 0$

$$\Downarrow$$

$$= \frac{G_F}{\sqrt{2}} (\alpha/4\pi)\, m_K^3 / m_K$$

$$= 10^{-5} \cdot 10^{-3} \cdot 10^{-2} \simeq \underbrace{10^{-10}}_{\text{SM prediction}}$$

Lat,

I forgot the mixing!

$$V_\ell \equiv V_{CKM} = V_c = \begin{pmatrix} \cos\theta_c & \sin\theta_c \\ -\sin\theta_c & \cos\theta_c \end{pmatrix}$$

$$\bar{u}\gamma^\mu d \cos\theta_c + \bar{u}\gamma^\mu s \sin\theta_c$$

$$\bar{c}\gamma^\mu d\,(-\sin\theta_c) + \bar{c}\gamma^\mu s \cos\theta_c$$

$$\left(\sin^2\theta_c \simeq 1/10 - 1/100\right)$$

$$\boxed{\text{still off by } 1/10^3 !}$$

digression about flavor

$$P(\nu_e \to \nu_\mu) = \sin^2 2\theta \, \sin^2 \frac{\Delta m^2 L}{4E}$$

$$\nu_e = \cos\theta \, \nu_1 + \sin\theta \, \nu_2$$

$$\nu_\mu = -\sin\theta \, \nu_1 + \cos\theta \, \nu_2$$

$$\theta_{max} = 45^0$$

Oscillation $\Leftrightarrow$

$$\theta \neq 0$$

$$\Delta m^2 \neq 0$$

$$\boxed{\text{Flavor violation} \Leftrightarrow \Delta m^2 \neq 0}$$

$$\Downarrow$$

$$\mathcal{H}_{eff}^{\Delta S=2} = \frac{G_F}{\sqrt{2}} \frac{\alpha}{4\pi} \sin^2\theta_c \frac{m_c^2 - m_u^2}{M_W^2} \quad \bar{s}\gamma^\mu L d \;\; \bar{s}\gamma_\mu L d$$

$$\boxed{\text{dimensional argument}}$$

$$M_f = U_{Lf}^\dagger \underbrace{m_f\,(d_f)}_{\text{diagonal}} U_{Rf}$$

$$\Downarrow$$

$f = u$: $\quad m_u = \begin{pmatrix} m_u & 0 \\ 0 & m_c \end{pmatrix}$

$\delta m = 0 \quad \Rightarrow \quad m_u = m_0 \begin{pmatrix} 1 & 0 \\ 0 & 1 \end{pmatrix}$

$(m_c = m_u)$

$$= m_0 \mathbb{1}$$

$$\Downarrow$$

$$V^+ m_u V = m_0 V^+ \mathbb{1} V$$

$$V^+ V = 1$$

$$\Downarrow$$

$$V^+ m_u V = m_0 \mathbb{1} = m_u$$

$$M_u = U_{Lu}^+ m_u U_{LR}$$

$$= U_{Lu}^+ V^+ m_u V U_{L,R}$$

$$U_{LU} \to V U_{LU} = \mathbb{1}$$

$$U_{RU} \to V U_{RU}$$

$$\boxed{V = U_{LU}^{+}}$$

or better:

$$V_c = U_{LU}^{*} U_{Ld} \to U_{LU}^{+} V^{+} U_{Ld}$$

choose: $V = U_{Ld} U_{LU}$

$$V_c \to U_{LU}^{+} U_{Ld} U_{Ld}^{+} U_{Ld}$$

$$= \mathbb{1}$$

$$\boxed{\text{if } \Delta m_u = 0 \Rightarrow V_c = 1}$$

$$\Downarrow \qquad m_u \simeq O(\mathrm{MeV}) = 0$$

$$\boxed{H_{eff}^{\Delta S=2} = \underbrace{\frac{G_F}{\sqrt{2}} \frac{\alpha}{4\pi} \sin^2\theta_c}_{10^{-11}} \frac{m_c^2}{M_W^2} \left(\bar{s}\gamma_\mu L d\right)^2}$$

$$\Downarrow$$

$$\left(m_c/M_W\right)^3 \simeq 10^{-3}$$

$$\Downarrow$$

$$\boxed{m_c \simeq 3\ \mathrm{GeV}}$$

GIM

Gaillard, Lee '74

$$\boxed{m_c = 1.5\ \text{GeV}}$$

Conclusion:

$\cancel{P}$ maximal $\nexists$

Higgs theory of the origin of mass

Imagine

$$\boxed{P = \cancel{good}}$$

$$\Downarrow$$

$$G_{SM} = SU(2)_L \times U(1)_Y$$

$$\Downarrow$$

$$\begin{pmatrix} u \\ d \end{pmatrix}_L \longleftrightarrow \begin{pmatrix} u \\ d \end{pmatrix}_R$$

$$\Downarrow \quad \begin{matrix} Y_L = Y_R \Longleftrightarrow \\ Q_L = Q_R \end{matrix}$$

$$\text{"}\mathcal{L}_Y\text{"} = (\bar{u}\ \bar{d})_L\, M \begin{pmatrix} u \\ d \end{pmatrix}_R + h.c.$$

$SU(2)_L \times U(1)$ singlet

$$\Downarrow$$

$$M_u = M_d$$

$$\Downarrow$$

$$\boxed{\begin{array}{l} m_u = m_d \\ U^u_{L,R} = U^d_{L,R} \Rightarrow V_{CKM} = 1 \end{array}}$$

$M_W = M_Z = 0 \Rightarrow$ need Higgs

$$\Downarrow$$

$$\mathcal{L}_Y + \Delta\mathcal{L}_Y = (\bar{u}\,\bar{d})_L\, M \begin{pmatrix} u \\ d \end{pmatrix}_R$$

$$+ (\bar{u}\,\bar{d})_L\, Y_\Sigma\, \Sigma \begin{pmatrix} u \\ d \end{pmatrix}_R + h.c.$$

$$\begin{pmatrix} u \\ d \end{pmatrix}_{L,R} \to U \begin{pmatrix} u \\ d \end{pmatrix}_{L,R} \quad \boxed{UU^\dagger = 1}$$

$$\Downarrow$$

$$\to (\bar{u}\,\bar{d})_L\, M \quad U^+ \underset{\checkmark}{U} \begin{pmatrix} u \\ d \end{pmatrix}_R$$

$$+ (\bar{u}\,\bar{d})_L\, Y_\Sigma \quad U^+ \Sigma' U \begin{pmatrix} u \\ d \end{pmatrix}_R + h.c.$$

$$U^+ U = 1 \,, \qquad U^+ \Sigma' U = 1$$

$$\Downarrow$$

$$\boxed{\Sigma' = U \Sigma U^+}$$ adjoint

$$\boxed{\Sigma^+ = \Sigma, \; \mathrm{Tr}\,\Sigma = 0}$$

$$\Downarrow$$

$$\Sigma_0 \to U \Sigma_0 U^+ = \text{diag}$$

$$\Downarrow$$

$$\boxed{\Sigma_0 = v \begin{pmatrix} 1 & 0 \\ 0 & -1 \end{pmatrix}}$$

$$\Downarrow$$

$$M_u = M + Y_\Sigma v$$

$$M_d = M - Y_\Sigma v$$

$$M_u \neq M_d$$

neutral int. $\Leftrightarrow$ conserve flavor

Higgs sector SM

$$M_f = Y_f v$$

$$\Rightarrow M_f = \text{diagonal} \Leftrightarrow Y_f = \text{diagonal}$$

$$\Downarrow$$

new P carry SM

$$[Y_\Sigma, M_u] = 0 = [Y_\Sigma, M_d]$$

$$\Downarrow$$

$$[Y_\Sigma, M] = 0$$

$$\Downarrow$$

$$[M_u, M_d] = 0$$

$$\Downarrow$$

M_u, M_d = diagonal
simultaneously

$$U_{Lu} = U_{Ld} \quad , \quad U_{Ru} = U_{Rd}$$

$$\Downarrow$$

$$\boxed{V_{CKM} = 1}$$

- $\Sigma' =$ adjoint

$$SU(2) \xrightarrow[\Sigma_0]{} U(1)$$

$$SU(2)_L \times U(1)_Y \to U(1)_{T_{3L}} \times U(1)_Y$$

$$\Downarrow$$

$$\boxed{M_Z = 0}$$

$$\Downarrow$$

new Higgs !

$$\underbrace{\text{take } \Phi \text{ on top of } \Sigma!}$$

$$M_W^2 = g^2\left[\Sigma_0^2 + \Phi_0^2\right]$$

$$M_Z^2 = g^2\, \Phi_0^2 \qquad M_W \neq M_Z$$

Φ — <u>no</u> coupling to f

$$\Downarrow$$

$$\boxed{\begin{gathered}\underline{\text{no}} \text{ prediction, except} \\ V_{CKM} = 1\end{gathered}}$$

$$\Downarrow$$

$\not P$ maximal = great

for q, l

$\not P$ maximal = failure for ν

$$P : \begin{pmatrix} \nu \\ e \end{pmatrix}_L \longleftrightarrow \begin{pmatrix} \nu \\ e \end{pmatrix}_R$$

$$\nu_L \longleftrightarrow \nu_R$$

$$\Downarrow$$

$$\boxed{m_\nu \neq 0}$$

$$\Downarrow$$

you need $\mathcal{L}$

you need $\cancel{\mathcal{L}}$ unexcited

$\Downarrow$

$\boxed{\text{SSB}}$