Neutrino Physics Course

Lecture XV

1/6/2021

LMU

Spring 2021

Seesaw mechanism:

how to test it?

$\exists\ N_{i_L} \quad (i = 1, 2, 3)$

$$N_{i_L} \equiv C\, \overline{\nu_{R\,i}}^{\,T}$$

SM singlets

$$N_L^T M_D C \nu_L \quad \leftarrow \text{Dirac mass}$$

$$M_D \equiv y_D v$$

$$\Downarrow$$

$$+\ \frac{1}{2} N_L^T C M_N N_L$$

$$\Downarrow$$

$$\boxed{\begin{aligned} &\underline{M}_\nu = - M_D^T \frac{1}{\underline{M}_N} M_D \\[2ex] &\Leftrightarrow \quad \underline{M}_N \gg M_D \end{aligned}}$$

seesaw

1. $$\Theta_{\nu N} = \frac{1}{\underline{M}_N} \underline{M}_D \ll 1$$

$$\Downarrow$$

impossible to produce N

2. $$\underline{M}_D = i \sqrt{\underline{M}_N}\, O \sqrt{\underline{M}_\nu}$$

$$O^T O = 1 = O O^T, \quad O \in C$$

$$\Downarrow$$

$M_N, M_\nu \not\Rightarrow$ predict m_D (Y_D)

3. $d = 5$

neutral singlet

$$(\ell \Phi) \frac{1}{\Lambda} (\Phi e)$$

$\updownarrow$ $\boxed{\text{Higgs boson}}$

1. $y_{u,d,e} \ll 1$

$\Rightarrow$ no direct Higgs production

2. $m_f \Rightarrow y_f$ ✓ works

3. SSB $\Rightarrow$ Higgs boson ✓

How to produce h? | gauge symmetry

$\Downarrow$

through W, Z !

$$g\, M_W\, W^+ W^-\, h$$

$\Downarrow$

if W existence $\leftrightarrow$ fundamental principle $\leftrightarrow$ new gauge bosons

$\Downarrow$

1. and 3. = cured!

$\Downarrow$ iff

$$M_D = f(M_N, M_\nu)$$

$\Leftrightarrow$ $\mathcal{O}$ = uniquely fixed

$$\boxed{\Rightarrow \text{ a full self-contained theory of } \nu \text{ mass}}$$

$\Downarrow$ (while looking for a theory)

$$\underline{M}_\nu = - M_D^T \frac{1}{\underline{M}_N} \underline{M}_D$$

matrices

$$\boxed{\begin{aligned} \underline{M}_N &= M_N^T \\ M_\nu &= \underline{M}_\nu^T \end{aligned}}$$

<u>charged fermions</u>

$$\mathcal{L}_y(e) = \bar{l}_L^i \, Y_e^{ij} \, \Phi \, e_R^j + h.c.$$

$$\Downarrow$$

$$\Phi_u = \begin{pmatrix} 0 \\ v+h \end{pmatrix}$$

$$\mathcal{L}_Y(e) = \overline{l^0_{iL}}\, M^l_{ij}\, l^0_{iR} \left(1 + \frac{h}{v}\right) + h.c.$$

$$\downarrow$$

$$\overline{f^0_{iL}}\, \underbrace{M^f_{ij}}_{M_f \in \mathbb{C}}\, f^0_{jR} \left(1 + \frac{h}{v}\right) + h.c.$$

$$\downarrow$$

$f^0_{k,R} \neq$ physical states

("weak" states)

$$\mathcal{L}_{kin} = i\, \overline{f^0_{iL}}\, \gamma^\mu \partial_\mu f^0_{iL} + L \leftrightarrow R$$

$$= i\, \overline{f_{iL}}\, \gamma^\mu \partial_\mu f_{iL} + L \leftrightarrow R$$

$$\Leftrightarrow \left[\begin{aligned} f^0_{L,R} &= U_{L,R}\, f_{L,R} \\ U_{L,R}^{\dagger}\, U_{L,R} &= 1 \end{aligned} \right]$$

physical

$$\mathcal{L}_y = \bar{f}_L \underbrace{U_L^{f\dagger} M_f\, U_R^f}\, f_R \left(1 + \frac{h}{v}\right) + h.c.$$

$$\longrightarrow \quad D_f = \text{diagonal}\,(m_{1f}, \ldots)$$

$$(m_{if} > 0)$$

$$U_L^{f\dagger} M_f M_f^{\dagger} U_L^f = D_f^2$$

$$U_R^{f\dagger} M_f^{\dagger} M_f U_R^f = D_f^2$$

$$\mathcal{L}_Y = \left(\bar{f}_L D_f f_R + \bar{f}_R D_f f_L \right) \left(1 + \frac{h}{v} \right)$$

$$\begin{matrix} \equiv & & \parallel \\ (m_f > 0) & & (m_f > 0) \end{matrix}$$

diagonal mass matrix

$\underline{M} = (n \times n)$ complex $\rightarrow 2n^2$ real

$U = n^2$ real elements

$U_{L,R} \rightarrow 2n^2$ real

$\Downarrow$

$$\boxed{\mathcal{L}_y = m_f \bar{f} f \left(1 + \frac{h}{v} \right)}$$

$\Downarrow$

$$y_f = \frac{m_f}{v} = \frac{g}{2}\frac{m_f}{M_W} \qquad \left(M_W = \frac{g}{2}v\right)$$

$$\Downarrow$$

$$\boxed{\text{where did } V_L^{f}, V_R^{f} \text{ go?}}$$

$$\bullet\; e Q_f A_\mu \left(\bar{f}_L^{\,0} \gamma^\mu f_L^{0} + \bar{f}_R^{\,0} \gamma^\mu f_R^{0}\right)$$

$$\parallel \qquad f = u\,;\, d\,;\, e$$

$$e Q_f A_\mu \left(\bar{f}_L \underbrace{V_L^{f\,+} V_L^{f}}_{1} \gamma^\mu f_L + L \leftrightarrow R\right)$$

$$= \left(e Q_f A_\mu \bar{f} \gamma^\mu f\right)$$

$$\cdot \frac{g}{\cos\theta_W} Z_\mu \left[\bar{f}_L^{\,0} \gamma^\mu f_L^0 \left(t_3 - q_f \sin^2\theta_W \right) + L \leftrightarrow R \right]$$

$$= \frac{g}{\cos\theta_W} Z_\mu \left[\bar{f}_L \gamma^\mu f_L \left(- '' - \right) + L \leftrightarrow R \right]$$

<u>summary</u>

$f^0_{L,R}$ = "weak" basis

$f_{L,R}$ = "mass" basis

mixings ($V_{L,R}$) go away

$\boxed{\text{except}}$

$$W_\mu^+ \text{"} \bar{u}_L^0 \gamma^\mu d_L^0 \text{"} =$$

$$= W_\mu^+ \left[\bar{u}_L^0 \gamma^\mu d_L^0 + \bar{c}_L^0 \gamma^\mu s_L^0 + \bar{t}_L^0 \gamma^\mu b_L^0 \right]$$

$$= W_\mu^+ \bar{u}_L \gamma^\mu \underbrace{U_{uL}^+ U_{dL}}\, d_L$$

$$\boxed{V_{CKM} = \text{CKM unitary matrix}}$$

$V^+ V = 1$ (9 real elements)

$\Downarrow$

3 angles + $\boxed{\text{6 phases ?}}$

$V \in R \Rightarrow O \to 3$ Euler angles

θ_C

$\parallel\!\!|$

$\theta_{12}, \quad \theta_{23}, \quad \theta_{13}$

1-2 gen $\qquad$ 2-3 gen $\qquad$ 1-3 gen

$\sim 15^\circ \qquad \sim 10^{-2}$ (4?) $\qquad \simeq 10^{-3}$ (4?)

KM phase

quarks

where do 5 phases go?

- $f^0_{L,R} \longrightarrow f_{L,R} \therefore \quad \bar{f}_L m_f f_R + h.c.$

$$= m_f \bar{f} f$$

$$\Rightarrow \quad f_L^i \to e^{i\alpha_i} f_L, \quad f_R^i \to e^{i\alpha_i} f_R^i$$

$$m_f^i \bar{f}_i f_i \to m_f^i \bar{f}_i e^{-i\alpha_i} e^{i\alpha_i} f_i =$$

$$= m_f^i \bar{f}_i f_i \quad \text{(quarks)}$$

$$\Downarrow$$

<u>n generations</u> $\quad V_{CKM}$

angles $\swarrow$ $\quad \searrow$ phases

$V = O \quad (O^T O = 1)$

$$\Big[\underbrace{\tfrac{1}{2} n (n-1)}_{\text{angles}} + \underbrace{\tfrac{1}{2} n (n+1)}_{\text{phases}} \Big] = n^2$$

$\#$ phases $= \frac{1}{2} n(n+1) - 2n$

$\uparrow$

phases of up and down quarks

- $n = 2 \Rightarrow 3 - 4 = -1$
- $n = 3 \Rightarrow 6 - 6 = 0 \leftarrow$

$\Uparrow$

KM are wrong!! ??

$$\mathcal{L}_{un} = \frac{g}{\sqrt{2}} W_\mu^+ (\bar{u}\, \bar{c}\, \bar{t} \cdots)_L \gamma^\mu\, V \begin{pmatrix} d \\ s \\ b \\ \vdots \end{pmatrix}_L$$

$\Downarrow$

$$
n\begin{cases}
u \to e^{i\alpha_u}\, u \\
c \to e^{i\alpha_c}\, c \to e^{i\alpha_u}\, e^{i(\alpha_c-\alpha_u)}\, c \\
t \to e^{i\alpha_t}\, t \to e^{i\alpha_u}\, e^{i(\alpha_t-\alpha_u)}\, t \\
\quad \vdots
\end{cases}
$$

$$
n\begin{cases}
d \to e^{i\alpha_d}\, d \to e^{i\alpha_u}\, e^{i(\alpha_d-\alpha_u)}\, d \\
\quad \vdots \\
b \to e^{i\alpha_b}\, b \to e^{i\alpha_u}\, e^{i(\alpha_b-\alpha_u)}\, b
\end{cases}
$$

$$\vdots$$

$$u, c, t \;\cdots \to e^{i\alpha_u}\; u, c, t \;\cdots$$

$$d, s, b \;\cdots \to e^{i\alpha_u}\; d, s, b \;\cdots$$

$$
(\bar{u}\;\bar{c}\;\bar{t}\cdots)_L \begin{pmatrix} d \\ s \\ b \\ \vdots \end{pmatrix}_L \to (\bar{u}\;\bar{c}\;\bar{t}\cdots)_L\, e^{-i\alpha_u}\, e^{i\alpha_u} \begin{pmatrix} d \\ s \\ b \\ \vdots \end{pmatrix}_L
$$

$$= (\bar{u}\,\bar{c}\,\bar{t}\ldots)_L \begin{pmatrix} d \\ s \\ b \\ \vdots \end{pmatrix}_L$$

$\boxed{\Rightarrow d_n \neq \text{there}}$

$$\Downarrow$$

$$\boxed{\begin{aligned} \#\ \text{phases} &= \frac{n}{2}(n+1) - (2n-1) \\ &= \frac{n^2 + n - 4n + 2}{2} = \frac{n^2 - 3n + 2}{2} \\ &= \frac{(n-1)(n-2)}{2} \end{aligned}}$$

- $n = 2 \Rightarrow 0$ phases

$\boxed{\cdot\ n=3 \Rightarrow 1 \text{ phase}}$ $\boxed{KM}$

$\delta_{KM} \simeq 45^\circ$

$2n$ phase freedom $(\to 2n-1)$

$\Updownarrow$

$m_f \bar{f} f \Leftrightarrow$ phase of f = not physical

charged fermions!!!

$\cdot$ neutral $f \longrightarrow m_M f^T C f$

?

$(m_M > 0)$

$$m_M f^T C f \longrightarrow m_M e^{2i\alpha} f^T C f$$

$$f \to e^{i\alpha} f$$

stay tuned

$$V_{CKM} = \underbrace{U_{Lu}^{+} U_{Ld}}$$

- $U_{Lu} = 1 \Rightarrow V_{CKM} = U_{Ld}$
- $U_{Ld} = 1 \Rightarrow V_{CKM} = U_{Lu}^{+}$

SM can only measure V_{CKM}!

$$\boxed{\text{Leptons}}$$

$$\boxed{\underline{M}_\nu = -\underline{M}_D^T \frac{1}{\underline{M}_N} \underline{M}_D}$$

$\underline{M}_\ell$ (charged lepton)

CKM $\longrightarrow$ PMNS

up $\longrightarrow$ neutrino

down $\longrightarrow$ charged lepton

$$\boxed{\underline{M}_\ell = D_\ell}$$

$$\Downarrow$$

$$\boxed{V_{PMNS} \equiv \underset{\substack{\uparrow \\ \text{convention}}}{U_{\nu L}}} \qquad \boxed{U_{eL} \overset{\downarrow}{=} 1}$$

$$\left.\begin{aligned} \underline{M}_N &= U_{LN}\, m_N(D_N)\, U_{LN}^T \\ \underline{M}_\nu &= U_{L\nu}\, m_\nu(D_\nu)\, U_{L\nu}^T \end{aligned}\right\} \text{convention}$$

$$U_L^\dagger\, \underline{M}\, U_R = D$$

$$\underline{M}^T = \underline{M} \Rightarrow U_R^T M^T U_L^* = U_R^T M U_L^*$$

$$\Rightarrow \boxed{U_R = U_L^*}$$

$$\Rightarrow \boxed{\underline{M} = U_L D U_L^T} \quad \boxed{\text{symmetric}}$$

$$\Downarrow$$

$$\boxed{V_{PMNS} = U_{L\nu}^*} \Rightarrow \boxed{U_{L\nu} = V_{PMNS}^*}$$

Convention

$$\underline{M}_\nu = V_{PMNS}^* \, m_\nu \, V_{PMNS}^\dagger$$

$$U_\ell = 1$$

- $m_f \longrightarrow y_f = \frac{m_f}{v}$

- $\underline{M}_\nu, \underline{M}_N \longrightarrow y_D\ (M_D) = f(M_\nu, M_N)$

$$\underline{M}_\nu = \left\{ \begin{array}{l} \text{neutrino masses } m_\nu \ (1,2,3) \\ \text{Mixings in } V_{PMNS} \end{array} \right\}$$

low energies

measure ???

⓪ W^- decays

$W^- \longrightarrow e^- + \bar{\nu}$ NO!

1 $\boxed{\text{OSCILLATIONS!}}$

$$\theta_{12}^{\nu} \simeq 30^\circ, \quad \theta_{23}^{\nu} \simeq 45^\circ, \quad \theta_{13}^{\nu} \simeq 10^\circ$$

phases = ???

$$\Delta m_{\odot}^2 \simeq 10^{-5}\,\mathrm{eV}^2, \quad \Delta m_{\odot}^2 \simeq 10^{-3}\,\mathrm{eV}^2$$

2 · KATRIN

$$n \to p + e + \bar{\nu}_e$$

direct

$$m_\nu \le 1\,\mathrm{eV}$$

3 $0\nu 2\beta$ $\leftarrow$ $\boxed{\text{indirect}}$

$\boxed{\text{Oscillations}}$

$M_\nu \leftarrow$ "weak" basis (0)

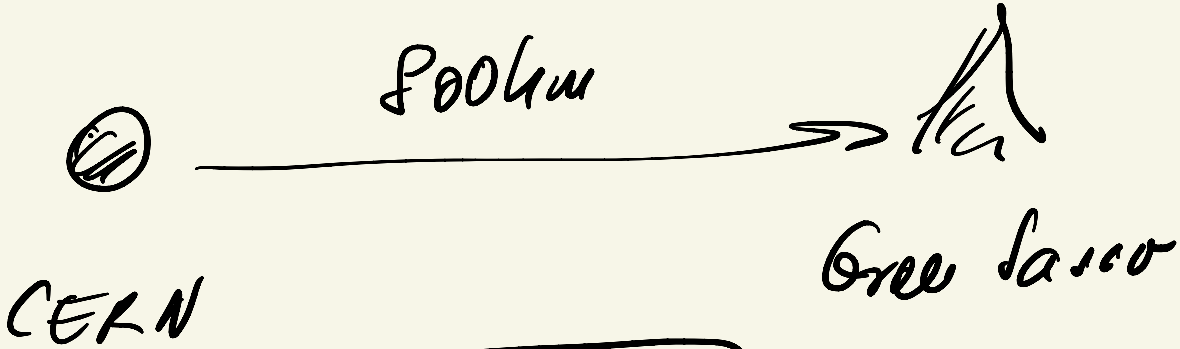
$m_\nu \leftarrow$ "mass" basis

$$\begin{pmatrix} \nu_e \\ e \end{pmatrix}^0, \begin{pmatrix} \nu_\mu \\ \mu \end{pmatrix}^0, \begin{pmatrix} \nu_\tau \\ \tau \end{pmatrix}^0$$

$$e^0 = e, \ \mu^0 = \mu, \ \tau^0 = \tau$$

$$M_\nu = V^*_{PMNS} \, m_\nu \, V^+_{PMNS}$$

$$\boxed{\nu_\mu \to \nu_\tau \quad f(\theta_{12}, \theta_{23} -)}$$



$$\boxed{\nu_\mu \to \nu_\tau}$$

$$\begin{pmatrix} \nu_\mu \\ \nu_\tau \end{pmatrix} = \begin{pmatrix} \cos\theta_A & \sin\theta_A \\ -\sin\theta_A & \cos\theta_A \end{pmatrix} \begin{pmatrix} \nu_1 \\ \nu_2 \end{pmatrix}$$

oscillates