Neutrino Physics Course

Lecture XIX

15/6/2021

$\boxed{\text{P in LR: SSB}}$

$$G_{LR} = SU(2)_L \times SU(2)_R \times U(1)_{B-L}$$

$$\Delta_L \qquad \big\downarrow M_R \qquad \Delta_R$$

$$SU(2)_L \times U(1)_Y$$

$$\Phi \text{ (bi-doublet)} \quad \big\downarrow M_L = M_W$$

$$U(1)_{em}$$

$\cdot M_R$: break $SU(2)_R \times U(1)_{B-L}$

$$Y/2 = T_{3R} + \frac{B-L}{2} \qquad \big\downarrow U(1)_Y$$

$$\boxed{\langle \Delta_R \rangle : \quad \begin{aligned} M_{W_R} &\propto \langle \Delta_R \rangle \\ M_{Z_R} &\propto \langle \Delta_R \rangle \end{aligned}}$$

Δ_R : $\langle \Delta_R \rangle$ leaves only SM particles massless

$\begin{pmatrix} \nu \\ e \end{pmatrix}_L \quad e_R \quad \cancel{\nu_R}$

$\begin{pmatrix} u \\ d \end{pmatrix}_L \quad u_R, \ d_R$

$$\boxed{\begin{array}{c} \Leftrightarrow \ \nu_R \text{ must be heavy} \\ \Downarrow \\ \text{fixes } \Delta_R \text{ completely} \end{array}}$$

$\Updownarrow$

(SM: $m_f \neq 0 \Rightarrow$ Higgs is ϕ)

$\Downarrow$

if ν_R is heavy $\left(m_{\nu_R} \propto \langle \Delta_R \rangle \right)$

$\Rightarrow$ ν_R couples to Δ_R

$\Rightarrow$ $\boxed{\begin{array}{c} l_R \text{ couples to } \Delta_R \\ (\text{but } \underline{\text{not}}\ l_L) \end{array}}$

- l_R - and only l_R - couples to Δ_R

$\cancel{l_R \Delta_R}$

$\Downarrow$ two l_R

(a) $\cancel{\overline{l_R}\,\Delta_R\,l_R}$; (b) $l_R\,\Delta_R\,l_R$

$$l_R \equiv R\,l \Rightarrow \overline{l_R}\,l_R = (R\,l)^{+}\gamma^0 R\,l$$
$$= l^{+} R\,\gamma^0 R\,l$$
$$= l^{+}\gamma^0 L R\,l = 0$$

$\overline{l_R}\,\gamma^\mu\,l_R$ = Lorentz ok,

$\overline{l_L}\,l_R$ = Lorentz inv.

$\Downarrow$

$$\underbrace{l_R \quad \Delta_R \quad l_R}_{} = \left\{ \begin{array}{l} \text{Lorentz} \\ SU(2)_R \\ B-L \end{array} \right\} \underline{\text{inv}}$$

B-L: $-1 \quad 2 \quad -1$

$\Downarrow$

- $\boxed{(B-L)\,\Delta_R = 2\,\Delta_R} \Leftarrow (B-L)\,l_R = -\,l_R$ ✓

- $l_R\, l_R = 2_R \times 2_R = \cancel{1_R} + 3_R$

we must break $SU(2)_R$

$(M_{W_R},\ M_{Z_R} \propto \langle \Delta_R \rangle)$

$\Downarrow$

$$\Delta_R = U_R\, \Delta_R\, U_R^{+} \quad \text{(adjoint)}$$

$$\mathrm{Tr}\,\Delta_R = 0\,, \quad \Delta_R \stackrel{?}{=} \Delta_R^{+}$$

$$(B-L)\,\Delta_R = 2\,\Delta_R$$
$$(B-L)\,\Delta_R^{+} = -2\,\Delta_R$$
$$\Rightarrow \quad \Delta_R \neq \Delta_R^{+}$$

$$\Delta_R = \text{"complex"}$$

$$\Updownarrow$$

$$\Delta_R = \Delta_R^1 + i\,\Delta_R^2$$

$$\therefore \quad \Delta_R^1 = (\Delta_R^1)^\dagger$$

$$\Delta_R^2 = (\Delta_R^2)^\dagger$$

$$\Downarrow$$

$$\mathcal{L}_y(\Delta) = Y_{\Delta_R}\, l_R^T\, C\, i\sigma_2\, \Delta_R\, l_R +$$

$L \leftrightarrow R$ $\quad Y_{\Delta_L}\, l_L^T\, C\, i\sigma_2\, \Delta_L\, l_L + h.c.$

$$P: \quad Y_{\Delta_L} = Y_{\Delta_R} \equiv Y_\Delta$$

$$l_R^T \, C \, i\sigma_2 \, \Delta_R \, l_R \to$$

$$\to l_R^T \, U_R^T \, i\sigma_2 \, U_R \, \Delta_R \, U_R^\dagger U_R \, C \, l_R$$

$$= l_R^T \, i\sigma_2 \, \underbrace{U_R^\dagger U_R}_{1} \, \Delta_R \, \underbrace{U_R^\dagger U_R}_{1} \, C \, l_R \quad \checkmark$$

$$\Downarrow$$

$$\boxed{\mathcal{L}_Y = y_\Delta \, l_R^T \, i\sigma_2 \, C \, \Delta_R \, l_R + R \to L + \text{h.c.}}$$

$$\Delta \longrightarrow U \Delta U^\dagger = e^{i\vec{\theta}\cdot\vec{T}} \, \Delta \, e^{-i\vec{\theta}\cdot\vec{T}}$$

$$= \Delta + i\vec{\theta}\cdot[\vec{T}, \Delta] + \cdots$$

$$\hat{\vec{T}}\Delta = [\vec{T}, \Delta] \qquad \vec{T} \equiv \vec{\sigma}/2$$

$$\Delta = \begin{pmatrix} a & b \\ c & d \end{pmatrix} \qquad \sigma_3 = \begin{pmatrix} 1 & 0 \\ 0 & -1 \end{pmatrix}$$

$$\hat{T}_3 \Delta = \frac{1}{2}\left[\begin{pmatrix} a & +b \\ -c & -d \end{pmatrix} - \begin{pmatrix} a & -b \\ +c & -d \end{pmatrix}\right]$$

$$= \begin{pmatrix} 0 \cdot a & +b \\ -c & 0 \cdot d \end{pmatrix}$$

$$Q_{em} = \hat{T}_{3L} + \hat{T}_{3R} + \frac{B-L}{2}$$

$$\Downarrow$$

$$Q_{em}\,\Delta_R = \hat{T}_{3R}\,\Delta_R + \Delta_R$$

$$Q_{ew}\ \Delta_R = \begin{pmatrix} 1.a & 2.b \\ 0.c & 1.d \end{pmatrix}$$

$$\Downarrow$$

$$\boxed{\Delta_R = \begin{pmatrix} \frac{1}{\sqrt{2}}\delta_R^{+} & \delta_R^{++} \\ \delta_R^{0} & -\delta_R^{+}/\sqrt{2} \end{pmatrix}}$$

$$\Downarrow$$

$$\boxed{\Delta_L = \begin{pmatrix} \frac{1}{\sqrt{2}}\delta_L^{+} & \delta_L^{++} \\ \delta_L^{0} & -\delta_L^{+}/\sqrt{2} \end{pmatrix}}$$

<u>M_R</u>: (1) $\langle \Delta_L \rangle = 0$ (?)

(2) $\langle \Delta_R \rangle = \begin{pmatrix} 0 & 0 \\ v_R & 0 \end{pmatrix}$ (?)

(1) $\not{P}$ spontaneous

$$\langle \Delta_L \rangle = 0, \quad \langle \Delta_R \rangle \neq 0$$

(2) $\Delta_R \rightarrow$ how to "know" to conserve Q_{em}?

$$\text{(1)} \quad \phi_L \overset{D = P}{\longleftrightarrow} \phi_R$$

$$\therefore \quad \langle \phi_L \rangle = 0, \quad \langle \phi_R \rangle \neq 0$$

$$D: \quad V = -\frac{\mu^2}{2}\left(\phi_L^2 + \phi_R^2\right) \leftarrow P$$

(equal)

$$+ \frac{\lambda}{4}\left(\phi_L^4 + \phi_R^4\right) \leftarrow P$$

equal

$$+ \frac{\lambda'}{2} \phi_L^2 \phi_R^2$$

$$= -\frac{\mu^2}{2}\left(\phi_L^2 + \phi_R^2\right) + \frac{\lambda}{4}\left(\phi_L^2 + \phi_R^2\right)^2$$

$$+ \frac{\lambda' - \lambda}{2} \phi_L^2 \phi_R^2$$

(a) $\lambda' = \lambda \Rightarrow$ flat direction

$$\boxed{\langle \phi_L^2 \rangle + \langle \phi_R^2 \rangle = \mu^2/\lambda} \quad (*)$$

<u>not</u> realistic

(b) $\lambda' - \lambda$ determines the breaking b1. $\lambda' - \lambda > 0$

$$\Rightarrow \boxed{\langle \phi_L \rangle = 0, \ \langle \phi_R \rangle \neq 0} \ (\text{from } (*))$$

$$\langle \phi_R \rangle = 0, \ \langle \phi_L \rangle \neq 0$$

b2. $\lambda' - \lambda < 0$ & maximizes

$$\Rightarrow \langle \phi_L \rangle \neq 0 \neq \langle \phi_R \rangle$$

$$\Downarrow$$

$$\langle \phi_L \rangle = \langle \phi_R \rangle$$

$$\frac{\partial V}{\partial \phi_L} = -\mu^2 \phi_L + \lambda \phi_L^3 + \lambda' \phi_L \phi_R^2$$

$$= (-\mu^2 + \lambda \phi_L^2 + \lambda' \phi_R^2) \phi_L = 0$$

$$\frac{\partial V}{\partial \phi_R} = -\mu^2 \phi_R + \lambda \phi_R^3 + \lambda' \phi_R \phi_L^2$$

$$= \left(-\mu^2 + \lambda \phi_R^2 + \lambda' \phi_L^2\right) \phi_R = 0$$

$\Rightarrow$ • $\langle \phi_L \rangle = \langle \phi_R \rangle = 0 \Longleftrightarrow$ local maximum

• $\langle \phi_L \rangle \neq 0 \neq \langle \phi_R \rangle$

$\Rightarrow \quad \langle \phi_L \rangle = \langle \phi_R \rangle$

• $\langle \phi_L \rangle = 0$, $\langle \phi_R \rangle \neq 0$

or vice-versa

$$\Downarrow$$

$$\boxed{\lambda' - \lambda > 0}$$

$$\Downarrow$$

$$\boxed{\langle \phi_L \rangle = 0, \quad \langle \phi_R \rangle \neq 0}$$

$\downarrow$ gauge case

- $\phi_{L,R}$ = doublet

$\Rightarrow \quad \phi_L^\dagger \phi_L, \quad \phi_R^\dagger \phi_R$

$$\lambda \left[(\phi_L^\dagger \phi_L)^2 + L \leftrightarrow R \right]$$

$\lambda' \, \phi_L^\dagger \phi_L \, \phi_R^\dagger \phi_R$

- $\phi_{L,R}$ = triplet

$$\mathrm{Tr}\, \Delta_{L,R}^\dagger \Delta_{L,R}$$

$$\phi_L^2 \longrightarrow \left(\mathrm{Tr}\, \Delta_L^\dagger \Delta_L\right)^2$$

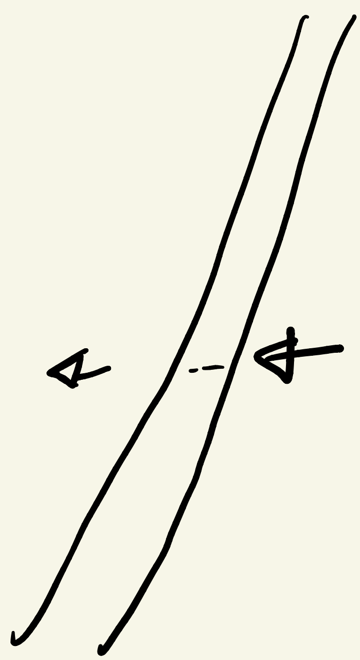
etc.

To be studied:

$\langle \Delta_R \rangle \neq 0$

$\langle \Delta_L \rangle = 0$

domain 1



$\langle \Delta_L \rangle \neq 0$

$\langle \Delta_R \rangle = 0$

domain 2

$$\boxed{\text{size of domain} = \text{size of causal contact}}$$

$$(2)\ \langle \Delta_R \rangle \stackrel{?}{=} \begin{pmatrix} 0 & 0 \\ v_R & 0 \end{pmatrix}$$

neutral component

$$V = -\frac{\mu^2}{2} T_r\, \Delta^+\Delta + \frac{\lambda}{4}\left(T_r\, \Delta^+\Delta\right)^2$$

$$+ \frac{\lambda'}{4} T_r\ \Delta^+\Delta\, \Delta^+\Delta$$

$$+ \frac{\lambda''}{4} \mathrm{Tr}\, \Delta^2 \cdot \mathrm{Tr} (\Delta^+)^2$$

** Prove that there are only two Invariants

$$\Leftrightarrow \mathrm{Tr}\, \Delta^+ \Delta \Delta^+ \Delta = a (\mathrm{Tr}\, \Delta^+ \Delta)^2 + b\, \mathrm{Tr}\, \Delta^2\, \mathrm{Tr} (\Delta^+)^2$$

$$\Downarrow$$

$$V = -\frac{\mu^2}{2} \mathrm{Tr}\, \Delta^+ \Delta + \frac{\lambda_1}{4} (\mathrm{Tr}\, \Delta^+ \Delta)^2 + \frac{\lambda_2}{4} \mathrm{Tr}\, \Delta^2\, \mathrm{Tr} (\Delta^+)^2$$

$$\langle \Delta \rangle \neq 0$$

$$\Delta = \Delta_1 + i\,\Delta_2 \qquad \Delta_i^+ = \Delta_i$$

$$\Downarrow$$

$$\Delta_1 = \Delta_1^+ \quad + \quad \Delta_1 \to U \Delta U^+$$

$$\Downarrow \qquad\qquad \Downarrow$$

$$\langle \Delta_1 \rangle = \begin{pmatrix} a & 0 \\ 0 & -a \end{pmatrix}, \quad a \in \mathbb{R}$$

$$\langle \Delta_2 \rangle = \begin{pmatrix} b & c \\ c^* & -b \end{pmatrix}, \quad b \in \mathbb{R}$$

$$c \in \mathbb{C}$$

$$\langle \Delta \rangle = \begin{pmatrix} a & 0 \\ a & -a \end{pmatrix}$$

$$SU(2) \xrightarrow[\langle\Delta_1\rangle]{} U(1)\ (T_3)$$

$$\hat{T}_3 \langle\Delta_1\rangle = [T_3, \langle\Delta_1\rangle] = 0$$

$$\Rightarrow U_3 = e^{i\theta 2T_3} = e^{i\theta\sigma_3} = \text{unitaria}$$

$$U_3 = \cos\theta + i\sin\theta\,\sigma_3 = \begin{pmatrix} e^{i\theta} & 0 \\ 0 & e^{-i\theta} \end{pmatrix}$$

$$\Downarrow$$

$$\langle\Delta_2\rangle \to \begin{pmatrix} e^{i\theta} & 0 \\ 0 & e^{-i\theta} \end{pmatrix} \begin{pmatrix} b & c \\ c^* & -b \end{pmatrix} \begin{pmatrix} e^{-i\theta} & 0 \\ 0 & e^{i\theta} \end{pmatrix}$$

$$= \begin{pmatrix} b & c e^{2i\theta} \\ c^* e^{-2i\theta} & -b \end{pmatrix} \Rightarrow \begin{matrix} c e^{2i\theta} = c \\ c^* e^{-2i\theta} = c \end{matrix}$$

$(r \in \mathbb{R})$

$\Downarrow$

$$\langle \Delta_2 \rangle = \begin{pmatrix} b & r \\ r & -b \end{pmatrix}$$

$\Downarrow$

$$\boxed{\begin{aligned} \langle \Delta \rangle &= \langle \Delta_1 \rangle + i \langle \Delta_2 \rangle \\ &= \begin{pmatrix} z & ir \\ ir & -z \end{pmatrix} \quad \begin{array}{l} z = a + ib \\ r \in \mathbb{R} \end{array} \end{aligned}}$$

$$\langle \Delta \rangle \equiv \langle \Delta_R \rangle \overset{?}{=} \begin{pmatrix} 0 & 0 \\ \vartheta_R & 0 \end{pmatrix}$$

$\boxed{\text{PUZZLE!}}$

$$\langle \Delta^+ \rangle = \begin{pmatrix} z^* & -iv \\ -iv & -z^* \end{pmatrix}$$

$$\Downarrow$$

$$\boxed{\mathrm{Tr}\, \Delta^+ \Delta = \left(|z|^2 + v^2 \right) 2}$$

$$\mathrm{Tr}\, \Delta^2 = \left(z^2 - v^2 \right) 2$$

$$\Downarrow$$

$$V = -\mu^2 \left(|z|^2 + v^2 \right) + \lambda_1 \left(|z|^2 + v^2 \right)^2$$

$$+ \lambda_2 \left(z^2 - v^2 \right) \left(z^{*2} - v^2 \right)$$

$\cdot\ \lambda_2 = 0 \Rightarrow$ $\boxed{\text{flat } |z|^2 + v^2 = \mu^2 / \lambda_1}$

$\Downarrow$ $\boxed{\left(z^2 - v^2 \right) \left(z^{*2} - v^2 \right) \geq 0}$

(i) $\underline{\lambda_2 > 0}$ $\Rightarrow$ $\boxed{z^2 = r^2}$

$$\Rightarrow \boxed{\langle \Delta \rangle = \begin{pmatrix} \pm 1 & i \\ i & \mp 1 \end{pmatrix} r \quad \text{must be!!!}}$$

<u>chose</u>: $\langle \Delta \rangle = \begin{pmatrix} 1 & i \\ i & -1 \end{pmatrix} r$

(ii) $\underline{\lambda_2 < 0}$ $\Rightarrow$ $r = 0$ or $z = 0$

$$\Rightarrow \langle \Delta \rangle = r \begin{pmatrix} 0 & i \\ i & 0 \end{pmatrix} \quad \underline{\underline{\text{or}}}$$

$$\Big\} \quad \langle \Delta \rangle = \begin{pmatrix} v & 0 \\ 0 & -v \end{pmatrix}$$

equivalent

$$\neq \begin{pmatrix} 0 & 0 \\ v & 0 \end{pmatrix}$$

$$\Downarrow$$

(i) must be true!

$$\langle \Delta \rangle = \begin{pmatrix} 1 & i \\ i & -1 \end{pmatrix} v$$

$$\neq \begin{pmatrix} 0 & 0 \\ v & 0 \end{pmatrix}$$

$$\begin{pmatrix} 1 & 1 \\ i & -1 \end{pmatrix} \propto U \begin{pmatrix} 0 & 0 \\ 1 & 0 \end{pmatrix} U^{+}$$

find $U = ?$