Neutrino Physics Course

Lecture XIV

28/5/2021

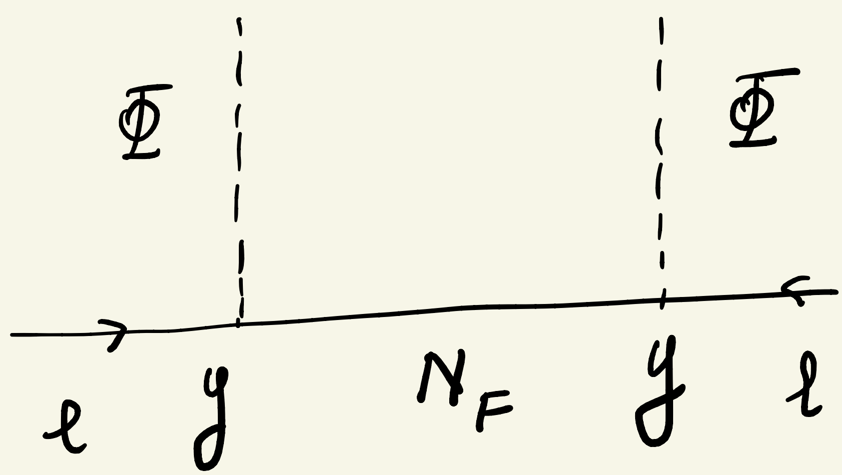
Seesaw mechanism for neutrino mass

<u>$d=5$</u> $\qquad \Delta L = 2$

$$\mathcal{L}_{eff}^{\Delta L = 2} = \frac{1}{\Lambda_L} (\ell \Phi)\, (\Phi \ell)$$

$$\downarrow \qquad \downarrow$$

$$N_F \qquad N_F$$



$$\downarrow \quad \frac{1}{\not{p} - m_N} \qquad (p \to 0)$$

$$\rightarrow \boxed{-\frac{y^2 (\ell \Phi)(\Phi e)}{m_N}}$$

$\Downarrow$ fundamental $(d=4)$ approach

$$\mathcal{L}_y (d=4) = y_D\, N_L^T\, C\, \Phi\, i\sigma_2\, \ell + h.c.$$

$$\left(N_L^T \equiv C\, \bar{\nu}_R^{\,T} \right) \quad +$$

$$+ \frac{1}{2} m_N\, N_L^T C\, N_L + h.c.$$

$$\uparrow$$

$$N_M = N_L + C\, \bar{N}_L^{\,T}$$

$$m_N\, \bar{N}_M\, N_M = m_N \left(N_L^T C N_L + h.c. \right)$$

$$\mathcal{L}_{kin} = i\, \bar{N}_M\, \gamma^\mu \partial_\mu N_M = 2\, \bar{N}_L\, \gamma^\mu \partial_\mu N_L$$

$$\Downarrow$$

$$\mathcal{L}_M = \frac{1}{2}\left[i \bar{N}_M \gamma^\mu \partial_\mu N_M - m_M \bar{N}_M N_M \right]$$

overall

<u>neutrino</u> $\Phi = \begin{pmatrix} 0 \\ v+h \end{pmatrix}$

$$\mathcal{L}_\nu^{(y)} = N_L^T C \nu_L m_D \nu_L \left(1 + \frac{h}{v}\right)$$

$$+ \frac{1}{2} N_L^T C N_L m_N \quad + h.c.$$

$$= \frac{1}{2}\left[\left(N_L^T C \nu_L + \nu_L^T C N_L\right) m_D \left(1 + \frac{h}{v}\right) \right.$$
$$\left. + N_L^T C N_L m_N \right]$$

$$\Downarrow$$

$$
\mathcal{M}_{\nu N} = \begin{array}{cc} \nu_L \\ N_L \end{array} \begin{pmatrix} 0 & m_D \\ m_D & m_N \end{pmatrix}
$$

$$
\qquad\qquad\quad \nu_L \qquad N_L
$$

$$
\boxed{m_N \gg m_D} \quad (\text{assumption})
$$

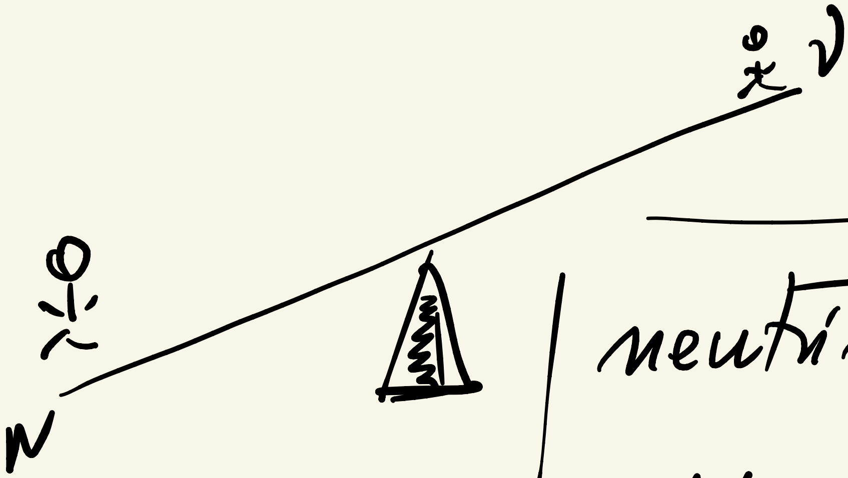
$$
\Downarrow
$$

$$
m_H \simeq m_N + O\left(m_D^2/m_N\right) \quad \longleftarrow \text{heavy}
$$

$$
m_L \simeq -\frac{m_D^2}{m_N} \quad \leftarrow \text{light state}
$$

$$
\mathrm{tr}\, \mathcal{M}_{\nu N} = m_H + m_L \simeq m_N \quad \checkmark
$$

$$
\det \mathcal{M}_{\nu N} = m_H \cdot m_L \simeq -m_D^2 \quad \checkmark
$$



neutrino light $\Leftarrow$
N heavy

$$\nu_L' = \nu_L + \Theta N$$
$$N_L' = N_L - \Theta \nu_L$$

$$\boxed{\Theta = \frac{m_D}{m_N}}$$

$$\Downarrow$$

$$W_\mu^+ \bar{\nu}_L \gamma^\mu e_L = W_\mu^+ \Big[\bar{\nu}_L' \gamma^\mu e_L$$
$$+ \Theta \bar{N}_L' \gamma^\mu e_L \Big]$$

N' = physical

$$\boxed{N \equiv N'}$$

$$\sigma(N) \propto \theta^2 = \left(\frac{m_D}{m_N}\right)^2 = \left(\frac{m_\nu}{m_N}\right)$$

$$\Downarrow$$

$$\boxed{\text{no way of producing } N}$$

$$\Downarrow \quad \boxed{\text{generations}}$$

$$N_L^{T\,i}\, C\, y_D^{ij}\, \nu_L^j \left(1 + \frac{h}{v}\right) v + h.c.$$

y_D = matrix $\qquad i, j = 1, \dots, N_g$

$$= N_L^T\, C\, y_D\, \nu_L v\, (1 + h/v) + h.c.$$

$$= N_L^T C \underline{M}_D \nu_L \left(1 + h/v\right) + h.c.$$

$$= \frac{1}{2}\left[N_L^T C M_D \nu_L + N_L^T C M_D \nu_L\right]\left(1 + h/v\right) + h.c.$$

$$= \frac{1}{2}\left[N_L^T C \underline{M}_D \nu_L + \nu_L^T (-1)(C^T) M_D^T N_L\right] + h.c.$$

$$= \frac{1}{2}\left[N M_D \nu + \nu M_D^T N\right]\left(1 + h/v\right) + h.c.$$

$$\Downarrow \qquad h \bar{\nu} N \left(M_D/v = y_D\right)$$

$$\underline{M}_{\nu N} = \begin{array}{c} \nu_L \\ N_L \end{array}\begin{pmatrix} 0 & M_D^T \\ M_D & M_N \end{pmatrix}$$

convention $\qquad M_N = M_N^T$

$$\bullet \quad \psi_i^T (M)_{ij}\, C\, \psi_j =$$

$$= -\psi_j^T C^T (M^T)_{ji}\, \psi_i =$$

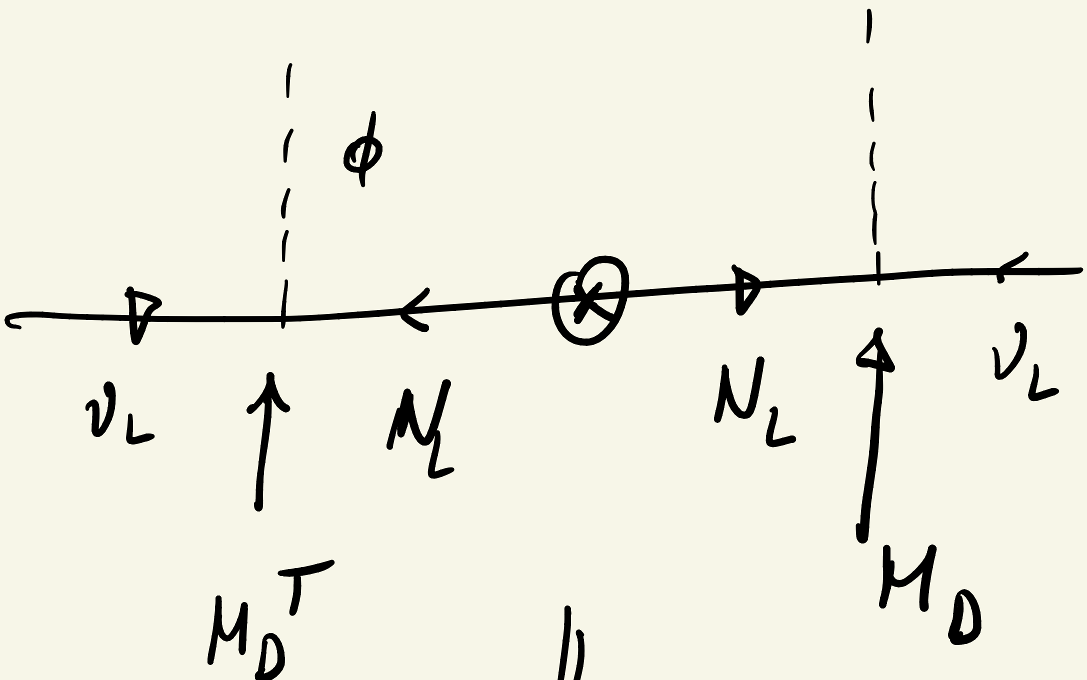
$$= +\psi_j^T C\, (M^T)_{ji}\, \psi_i =$$

$$= \psi_j^{\dagger} C\, (M)_{ji}\, \psi_i$$

$$\Rightarrow \boxed{M^T = M}$$

$$\boxed{\text{Majorana mass matrix} = \\ = \text{symmetric}}$$

effectively



$$\Downarrow$$

$$\left(- M_D^T \frac{1}{M_N} M_D \right) \overset{?}{=} M_\nu$$

$\Downarrow$ Explicit check

$$\mathcal{M}_{\nu N} = \begin{array}{cc} \begin{pmatrix} 0 & M_D^T \\ M_D & M_N \end{pmatrix} & \begin{array}{c} \nu \\ N \end{array} \\ \begin{array}{cc} \nu & N \end{array} & \end{array}$$

$$\begin{pmatrix} \nu \\ N \end{pmatrix} \longrightarrow U \begin{pmatrix} \nu \\ N \end{pmatrix} \qquad U^{\dagger} U = 1$$

$$\Downarrow$$

$$\mathcal{M}_{\nu N} \longrightarrow U^T \mathcal{M}_{\nu N} U = D_{\nu N}$$

$$D_{\nu N} = \text{block diagonal}$$

$$\boxed{\mathcal{M}_N \gg \mathcal{M}_D}$$

$$U = \begin{pmatrix} 1 & \theta' \\ \theta & 1 \end{pmatrix}, \quad U^{\dagger} = \begin{pmatrix} 1 & \theta^{\dagger} \\ \theta'^{\dagger} & 1 \end{pmatrix}$$

$$\boxed{\theta, \theta' \ll 1}$$

Pause: why U (unitary) and not O (orthogonal)?

$O^T M_{\nu N} O \neq$ not diagonal

$U^T M_{\nu N} U =$ diagonal (always)

$$\boxed{\begin{gathered} H = H^\dagger \quad \text{(hermitian)} \\ U H U^\dagger = D \quad (= \text{diagonal}) \end{gathered}}$$

general matrix M $\quad M \to U_L M U_R^\dagger$

$$\boxed{\begin{gathered} U_L \, M \, U_R^\dagger = D \\ U_L \neq U_R \end{gathered}}$$

$$H_1 = M M^+ \Rightarrow H_1^+ = H_1$$

$$H_2 = M^+ M \Rightarrow H_2^+ = H_2$$

$$H_1 \equiv M M^+ \longrightarrow U_L M U_R^+ U_R M^+ U_L^+ =$$

$$= U_L M M^+ U_L^+ = U_L H_1 U_L^+$$

$$= D_1$$

$$H_2 = M^+ M \longrightarrow U_R M^+ U_L U_L M U_R^+ =$$

$$= U_R H_2 U_R^+ = D_2$$

- $\mathrm{Tr}\, D_1 = \mathrm{Tr}\, H_1 = \mathrm{Tr}\, M M^+ = \mathrm{Tr}\, M^+ M = \mathrm{Tr}\, H_2$
$= \mathrm{Tr}\, D_2$

$$\det D_1 = \det H_1 = \det H_2 = \det D_2$$

$$\Downarrow$$

$$\boxed{\underline{2\times 2} \Rightarrow D_1 = D_2}$$

$\downarrow$ generalised to any dimension

$$\boxed{D_1 = D_2}$$

- $H = H^\dagger \Rightarrow U_R = U_L$

- $S = S^T \Rightarrow U_L \, S \, U_R^\dagger = D$

$$\Downarrow$$

$$U_R^* \, S^T \, U_L^\dagger = D$$

$$\Downarrow$$

$$U_L = U_R^*$$

$$\Downarrow$$

$$\boxed{U S' U^T = D}$$

all this 'justifies':

$$U^T M_{\nu N} U = D_{\nu N} \qquad (\theta, \theta' \ll 1)$$

$$U = \begin{pmatrix} 1 & \theta' \\ \theta & 1 \end{pmatrix} \qquad U^\dagger = \begin{pmatrix} 1 & \theta^\dagger \\ \theta'^\dagger & 1 \end{pmatrix}$$

$$U U^\dagger = \begin{pmatrix} 1 + \cancel{\theta' \theta'^\dagger} & \theta' + \theta^\dagger \\ \theta + \theta'^\dagger & 1 + \cancel{\theta \theta^\dagger} \end{pmatrix}$$

$$\boxed{\theta' = -\theta^\dagger}$$

$$\boxed{\Rightarrow U = \begin{pmatrix} 1 & \theta^\dagger \\ -\theta & 1 \end{pmatrix}}$$

$$\begin{pmatrix} 1 & -\theta^T \\ \theta^* & 1 \end{pmatrix} \begin{pmatrix} 0 & M_D{}^T \\ M_D & M_N \end{pmatrix} \begin{pmatrix} 1 & \theta^+ \\ -\theta & 1 \end{pmatrix}$$

$$= \begin{pmatrix} -\theta^T M_D & \overbrace{M_D{}^T - \theta^T M_N}^{0} \\ M_D & M_N \end{pmatrix} \begin{pmatrix} 1 & \theta^+ \\ -\theta & 1 \end{pmatrix}$$

$$= \begin{pmatrix} -\theta^T M_D & 0 \\ \underbrace{M_D - M_N \theta}_{0} & M_N \cancel{+ \left(-M_D{}^2 / M_N\right)} \end{pmatrix} = D_{JN}$$

$$M_D = M_N \theta$$

$$\boxed{\theta = \frac{1}{M_N} M_D}$$

$|\theta_{ij}| \ll 1$

(small)

$\Leftrightarrow \boxed{\theta^2 = 0}$

$\Downarrow$

$$D_{\nu N} \simeq \begin{pmatrix} M_\nu & 0 \\ 0 & M_N \end{pmatrix} \therefore$$

$$M_\nu = -\theta^T M_D = -M_D^T \frac{1}{M_N} M_D$$

$$\Leftrightarrow \quad M_\nu^T = -M_D^T \frac{1}{M_N^T} M_D =$$

$$= -M_D^T \frac{1}{M_N} M_D = M_\nu$$

$\boxed{\theta \ll 1}$

$$\Rightarrow \sigma(N) \propto \theta^2 = \mathcal{O}\left(\sim \frac{m_\nu}{m_N}\right)$$

seesaw problem

• charged fermions

$$m_f = y_f v$$

$$\uparrow$$

$$y_f h \bar{f} f$$

$$m_f \longrightarrow \Gamma(h \to f\bar{f}) \simeq m_f^2$$

mass $\longrightarrow$ yukawa $\rightarrow$ rates

• $(\nu, N) \leftarrow$ Probe the origin of mass of ν?

$$\boxed{M_D \text{ from } \underline{M}_\nu, \underline{M}_N}$$

$$\underline{M}_\nu = - M_D^T \frac{1}{\underline{M}_N} \underline{M}_D$$

$$\Downarrow$$

$$\boxed{\underline{M}_D = i\sqrt{\underline{M}_N}\ O\ \sqrt{\underline{M}_\nu}}$$

$$\Rightarrow\ M_D^T = i\sqrt{\underline{M}_\nu}\ O^T\sqrt{\underline{M}_N}$$

$$\Downarrow$$

$$M_\nu = + \sqrt{\underline{M}_\nu}\ O^T \cancel{\sqrt{M_N}\ \frac{1}{M_N}\sqrt{M_N}}\ O\sqrt{M_\nu}$$

$\checkmark$

$$= \sqrt{M_\nu}\ \boxed{O^T O}\sqrt{M_\nu}$$

$$= \sqrt{\underline{M}_\nu}\sqrt{M_\nu} = \underline{M}_\nu \quad \text{iff}$$

$$\boxed{O^T O = 1}$$

$\boxed{O \in \text{complex}}$ (in general)

$\cdot\ O \in \mathbb{R} \Rightarrow \theta = \begin{pmatrix} \cos\theta & \sin\theta \\ -\sin\theta & \cos\theta \end{pmatrix}$

$|O_{ij}| \leq 1$

$(O^T O)_{11} = \sum O_{1i} O_{1i} = 1$

$= \sum |O_{1i}|^2$

$\bullet\ O = \begin{pmatrix} \mathrm{ch}\, x & i\, \mathrm{sh}\, x \\ -i\, \mathrm{sh}\, x & \mathrm{ch}\, x \end{pmatrix}$

$\mathrm{ch}\, x = \frac{e^x + e^{-x}}{2} \to \infty$

$\Downarrow$

$$\theta = \frac{1}{M_N} \quad (M_D = \text{large})$$

$$\boxed{\sigma(N) \propto \theta^2 = O(1)}$$

$\Leftrightarrow$ neutrino light by accident

$$\boxed{\underline{M}_\nu = - \underbrace{M_D^T}_{\text{huge}} \frac{1}{M_N} \underbrace{M_D}_{(0)}}$$

$$M_D / M_N \ll 1 \quad \boxed{\text{cheat}}$$

need $\Downarrow$ $\boxed{\text{seesaw}}$

1. N to be "physical" & could be produced

2. predict $\underline{M}_D = f(\underline{M}_\nu, \underline{M}_N)$

$\underline{M}_D = v\, Y_D$ $\Downarrow$

$$y_D = f(\underline{M}_\nu, \underline{M}_N)$$

$\Leftrightarrow$ charged fermions

$$y_f = \frac{m_f}{v}$$

$$\boxed{\theta \ll 1 \ominus \theta \lesssim 1}$$

$$\theta = \frac{1}{M_N} M_D = i \frac{1}{M_N} \sqrt{M_N}\, O \sqrt{m_\nu}$$

$$= i \frac{1}{\sqrt{M_N}} O \sqrt{m_\nu} \quad \left(\sim \sqrt{m_\nu / M_N}\right)$$

$$|\theta^2| \simeq m_\nu / M_N \ll 1$$

$$\left(\lesssim 10^{-12}\right)$$

$$\boxed{\theta \simeq O(1) \Rightarrow O \sim O(10^6)}$$