Neutrino Physics Course

Lecture XI

18/5/2021

LMU

Spring 2021

SM and Origin of Mass

(Charged) Fermion Masses

Q. Cartan vs # of invariants

A. Adjoint repr.

$$\Sigma \to U \Sigma U^{\dagger}$$

$$\to \text{diagonal}$$

SU(3) $\qquad \Sigma = \begin{pmatrix} a & & \\ & b & \\ & & -(a+b) \end{pmatrix}$

$$\Downarrow$$

$\mathrm{Tr}\,\Sigma^2, \quad \mathrm{Tr}\,\Sigma^3$

$\mathrm{Tr}\,\Sigma^4 \neq$ independent

Cartan $= \{T_3, T_8\}$

$\hookrightarrow \begin{pmatrix} \mathrm{Tr}\,T_a = 0 \\ T_a = T_a^\dagger \end{pmatrix}$

$\Sigma = c_a T_a \qquad \Sigma = \Sigma^\dagger$

$\qquad\qquad\qquad\quad \mathrm{Tr}\,\Sigma = 0$

$\downarrow$

diag $= c_3 T_3 + c_8 T_8$

SM: starts as ew theory

$\Rightarrow$ theory at origin of mass

(Higgs mechanism)

$$SU(2)_L \times U(1)_Y = G_{SM}$$

T_a, $\quad Y/2$

$A_a \quad B$

$\Downarrow$

$$\bullet \; i \bar{f} \gamma^\mu D_\mu f = \frac{g}{\sqrt{2}} W_\mu^+ \bar{u}_L \gamma^\mu d_L + \text{h.c.}$$

$$f_L = \begin{pmatrix} u \\ d \end{pmatrix}_L \qquad W_\mu^\pm = \frac{(A_1 \mp i A_2)_\mu}{\sqrt{2}}$$

$$+ \; e A_\mu \bar{f} \gamma^\mu Q_{em} f \; +$$

$$+ \frac{g}{\cos\theta_W} Z_\mu \bar{f} \gamma^\mu \left[T_3 - Q \sin^2\theta_W \right] f$$

- $\frac{1}{2} (D_\mu \Phi)^+ (D^\mu \Phi)$

$$\Phi_{un} = \begin{pmatrix} 0 \\ v+h \end{pmatrix}$$

$$\Phi_{general} = e^{i G_i / v \, T_i} \begin{pmatrix} 0 \\ v+h \end{pmatrix}$$

$$\boxed{T_i \begin{pmatrix} 0 \\ v+h \end{pmatrix} \neq 0}$$

$$T_i = \frac{\sigma_i}{2}$$

Higgs

"would be" NG bosons

$$\Phi_{\text{general}} \to U\,\Phi_{\text{general}} =$$

$$= e^{-i G^i/v\, T_i}\,\Phi_{\text{general}} = \Phi_{\text{un}}$$

$$\boxed{A_\mu^{i\,P} = A_\mu^{i\,H} - \frac{\partial_\mu G^i}{v}}$$

Longitudinal A

$$\Downarrow \quad \Phi = \Phi_{\text{un}} = \begin{pmatrix} 0 \\ v+h \end{pmatrix}$$

$$= \left(1 + \frac{h}{v}\right)\begin{pmatrix} 0 \\ v \end{pmatrix} = \underbrace{\left(1+\frac{h}{v}\right)}\Phi_0$$

$$\Downarrow$$

$$\frac{1}{2} M_Z^2 Z_\mu Z^\mu \left(1 + \frac{h}{v}\right)^2$$

$$+ M_W^2 \underbrace{W_\mu^+ W^{\mu -}}_{\frac{1}{2}\left(A_1^\mu A_{1\mu} + A_2^\mu A_{2\mu}\right)} \left(1 + \frac{h}{v}\right)^2$$

$$\boxed{\begin{aligned} M_W &= \frac{g}{2} v \\ M_Z &= \frac{M_W}{\cos\theta_W} \end{aligned}}$$

$$\Downarrow$$

$$(+ O(h^2))$$

$$M_W^2 W_\mu^+ W^{\mu -} + g M_W h W_\mu^+ W^{\mu -}$$

$$+ \frac{1}{2} M_Z^2 Z_\mu Z^\mu + \frac{g}{\cos\theta_W} M_Z h Z_\mu Z^\mu$$

$\uparrow$ Γ mass of Z

Z coupling

$$h \to W^+ + (W^{-*} = \text{off-shell})$$

$m_h = 125$ GeV

$M_W = 80$ GeV

<u>Counting of states</u>

$U(1)$ $\quad Q\,\Phi_0 \neq 0 \Rightarrow M_A \neq 0$

$(M_A \propto \Phi_0)$

$SU(2)$ $\quad T_a\,\Phi_0 \neq 0 \quad (a = 1, 2, 3)$

$\Downarrow$

$$M_A^a \propto \Phi_0 \qquad a = 1, 2, 3$$

$SU(2) \times U(1)$ $\qquad T_a \Phi_0 \neq 0, \quad \frac{Y}{2} \Phi_0 \neq 0$

$$Q_{em} \Phi_0 = 0$$

$$\Rightarrow \quad Q_{em} = T_3 + Y/2$$

$$\Downarrow$$

3 broken generators $\Rightarrow$

3 massive g.b W^+, W^-, Z

1 unbroken $\Rightarrow$ massless A

Theorem:

$$\boxed{T_a \Phi_0 \neq 0 \Leftrightarrow A_a : M_{A_a} \propto \Phi_0 \qquad 1 \longleftrightarrow 1}$$

$$\Uparrow$$

$$\boxed{\text{gauge symmetry} = \text{local} \qquad \alpha = \alpha(x)}$$

- <u>global</u> $\alpha = \text{const.}$

- <u>no</u> gauge bosons $D_\mu \to \partial_\mu$

- G_i are real, physical states

$$\Downarrow$$

$$m_{G_i} = 0 \qquad \text{(NG mechanism)}$$

local symmetry	global symmetry
$T_a \Phi_0 \neq 0$	$T_a \Phi_0 \neq 0$
$M_{A_a} \propto \Phi_0$	$m_{G_a} = 0$

(I) <u>Proca theory</u>

$$M_A \neq 0, \quad (+\ G)$$

[redacted]

- $g A_\mu \bar{\psi}_L \gamma^\mu \psi_L$ (<u>no</u> ψ_R)

$(g \to 0) \Rightarrow$ <u>decoupling !</u>

(I) Higgs theory

$$\mathcal{L} = \frac{1}{2}(D_\mu \phi)^* (D^\mu \phi) - V(\Phi)$$

$$- \frac{1}{4} F_{\mu\nu} F^{\mu\nu} - (y\, \bar{\Psi}_L \phi \Psi_R + h.c.)$$

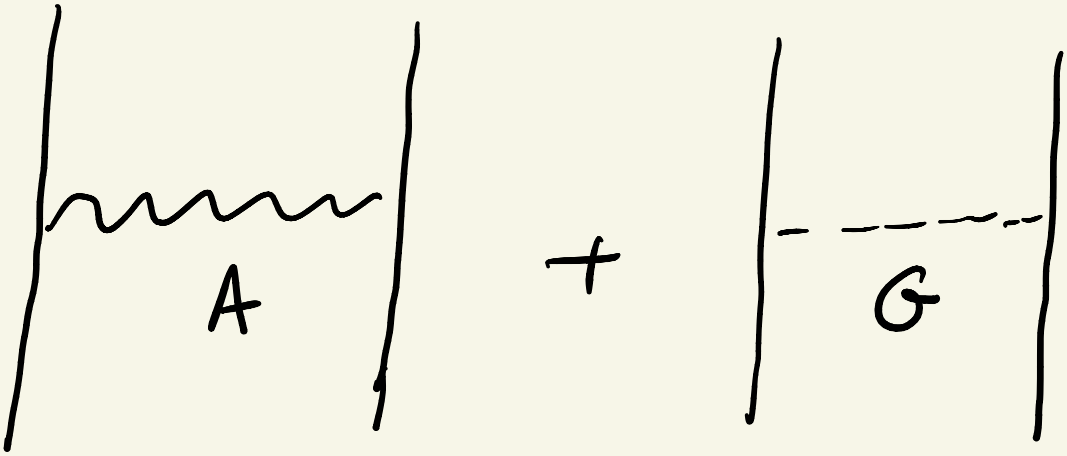
$$+ i \bar{\Psi}_L \gamma^\mu D_\mu \Psi_L + i \bar{\Psi}_R \gamma^\mu D_\mu \Psi_R$$

$$D_\mu \Psi_L = (\partial_\mu - i g A_\mu) \Psi_L \qquad D_\mu \Psi_R = \partial_\mu \Psi_R$$

$$D_\mu \phi = (\partial_\mu - i g A_\mu) \phi$$

$$\phi = v + h + i G$$

$\Downarrow$ scattering

$\propto g^2$  $\propto y^2$

(a) (b)

$= \text{independent of } \xi$

$(\xi = 1)$ $\Downarrow$

$$\frac{g\left(\bar{\psi}_L \gamma^\mu \psi_L\right)^2}{q^2 - M_A^2} + g^2 \frac{m_f^2 / M_A^2}{q^2 - M_A^2}\left(\bar{\psi}\gamma_5\psi\right)^2$$

$\boxed{(\xi = 1 \text{ gauge})}$

$$D(G) \propto \frac{1}{k^2 - M_A^2}\ ; \quad \Delta_{\mu\nu} \propto \frac{g_{\mu\nu}}{k^2 - M_A^2}$$

$$D(G) = \frac{i}{k^2 - \xi M_A^2}$$

$$\Uparrow$$

$$\Delta_{\mu\nu} = \frac{-i}{k^2 - M_A^2}\left[g_{\mu\nu} + (\xi - 1)\frac{k_\mu k_\nu}{k^2 - \xi M_A^2}\right]$$

$$(b) \quad y\left[\bar{\psi}_L (v + h + iG)\psi_R + \bar{\psi}_R (v + h - iG)\psi_L\right]$$

$$= y(v+h)\left(\bar{\psi}_L \psi_R + \bar{\psi}_R \psi_L\right) + iGy\left(\bar{\psi}_L \psi_R - \bar{\psi}_R \psi_L\right)$$

$$= y(v+h)\,\bar{\psi}\psi - iGy\,\bar{\psi}\gamma_5\psi$$

$$\Downarrow$$

$$m_f = yv \,, \quad M_A = gv$$

$$\Downarrow$$

$$\boxed{y = g\,\frac{m_f}{M_A}}$$

$$\Rightarrow \boxed{h\,\bar{f} f \; g\,\frac{m_f}{M_A}}$$

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local $\longrightarrow$ global

$M_A \neq 0 \xrightarrow[(?)]{} m_G = 0$

$$D_\mu = \partial_\mu - i g Q A_\mu$$

$g = 0 \;\Downarrow$

$\boxed{\delta = 0 \text{ limit}}$

$(a) \longrightarrow 0$, $(b) \not\longrightarrow 0$

$$A \to A_b + \frac{\cancel{g^2}\, m_f^2}{\cancel{g^2}\, v^2} \frac{1}{q^2} \left(\bar\psi \gamma_5 \psi\right)^2$$

$$\boxed{\text{massless particle} = \\ = \text{NG boson}}$$

(i) $\exists$ massless G $(m_G = 0)$

(ii) has $\bar\psi \gamma_5 \psi$ int. !!!

(iii) $y = \dfrac{m_f}{v}$

decouples when $v \to \infty$

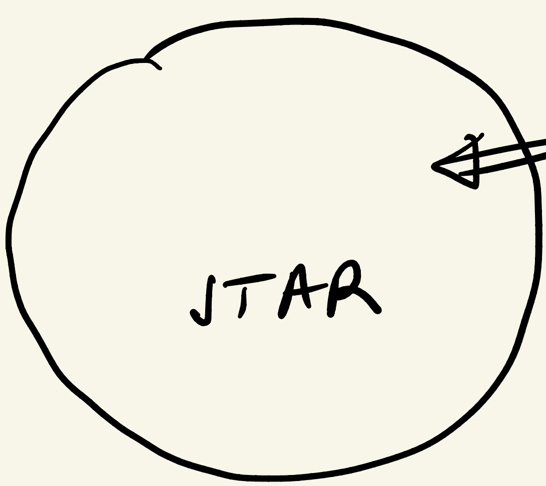
v = "cut-off"

massless messenger G

$$\Downarrow$$

change gravity !?

($V_{gr}(r) \Leftarrow$ changed ?)



many protons + neutrons

$N_\odot \simeq 10^{57}$

= heavy ($M = N$ GeV)

• $\bar{\psi}\gamma_5\psi$ G

$\uparrow$ spin-dependent

<u>NR limit</u>

$$\left\{ \begin{aligned} \gamma^0 &= \begin{pmatrix} 1 & 0 \\ 0 & -1 \end{pmatrix} \\ \gamma_5 &= \begin{pmatrix} 0 & 1 \\ 1 & 0 \end{pmatrix} \end{aligned} \right\}$$

$$\psi = \begin{pmatrix} u \\ \frac{\vec{p}\cdot\vec{\sigma}}{2m} u \end{pmatrix} \leftarrow \text{small}$$

$$\gamma^\mu p_\mu \psi = m\psi$$

$$\Rightarrow \begin{pmatrix} E-m & \vec{p}\cdot\vec{\sigma} \\ -\vec{p}\cdot\vec{\sigma} & E+m \end{pmatrix} \begin{pmatrix} u \\ v \end{pmatrix} = 0$$

$$v = \frac{\vec{p}\cdot\vec{\sigma}}{E+m} u \simeq \frac{\vec{p}\cdot\vec{\sigma}}{2m} u$$

$$\Downarrow$$

$$\bar{\psi}\gamma_5\psi \propto u^\dagger \frac{\vec{\sigma}\cdot\vec{p}}{2m} u =$$

$$= \underbrace{\left(u^\dagger \frac{\vec{\sigma}}{2} u\right)} \cdot \vec{p} = \langle \vec{S} \rangle \cdot \vec{p}$$

$$\langle \vec{S} \rangle_{\text{nuclei}} \simeq O(1)$$

$$\langle \vec{S} \rangle_{\text{star}} \simeq O(1)$$

$$\boxed{\text{spin is not coherent!}}$$

$$\Downarrow$$

$$\boxed{\begin{array}{l}\text{G exchange} = \text{negligible} \\ \qquad\qquad \text{compared to gravity}\end{array}}$$

$\Downarrow$

Sun "radiates" G

$$\Rightarrow \boxed{v \gtrsim 10^{10}\ \text{GeV}}$$

$\Uparrow$

Red giants

$\Downarrow$ $\boxed{\text{back to SM}}$

$q_L \equiv \begin{pmatrix} u \\ d \end{pmatrix}_L \qquad u_R \qquad d_R$

$\ell_L \equiv \begin{pmatrix} \nu \\ e \end{pmatrix}_L \qquad e_R$

$$\mathcal{L}_y = \bar{q}_L\, y_d\, \Phi\, d_R + \bar{\ell}_L\, \Phi\, y_e\, e_R$$

$$+\bar{Q}_L\, y_u\, i\sigma_2\, \Phi^*\, u_R + h.c.$$

$$\Phi_u = \begin{pmatrix} 0 \\ v+h \end{pmatrix}$$

$$\Rightarrow \mathcal{L}_y = \Big[\bar{d}_L\, y_d\, d_R + \bar{e}_L\, y_e\, e_R + \bar{u}_L\, y_u\, u_R + h.c.\Big](v+h)$$

$$= \underbrace{y v \bar{f} f}_{\Downarrow} + y h \bar{f} f$$

$$m_f = v y \qquad y = \frac{m_f}{v} = \frac{g}{2}\frac{m_f}{M_W}$$

$$\left(M_W = \frac{g}{2} v\right)$$

$$\Downarrow$$

$h \to \bar{f} f$ probe directly

$$\Gamma(h \to \bar{f} f) = \frac{y^2}{8\pi} m_h \qquad (m_h \gg m_f)$$

$$\propto \left(\frac{m_f}{M_W}\right)^2 m_h$$

Origin of mass:

probing $h \to f\bar{f}$

LHC

$$\left.\begin{matrix} W^+, W^-, Z \\ t, b, \tau \end{matrix}\right\} \leftarrow \text{Higgs}$$

- $e, u, d \quad (m \to 0)$

  hard to probe Higgs!

- μ, c, s

Direct test of Higgs

Universe was hot

$$\boxed{T \gg v \simeq M_W \simeq 100\ \text{GeV}}$$

$$\left(M_W = \frac{g}{2} v\right)$$

$$g \simeq 0.6$$

π

$e = g \sin\theta_W$

2

$e \simeq 1/3$ $\qquad \theta_W \simeq 30^\circ$

$\boxed{\text{Weinberg '74}}$

$$V_T = V_{T=0} + a T^2 \Phi^\dagger \Phi$$

$$a = g^2 + \lambda + |y|^2 > 0$$

$$\text{high } T \Longrightarrow \Phi_0(T) = 0$$

$$V_{T=0} = \frac{\lambda}{4}\left(\Phi^\dagger\Phi - v^2\right)^2$$

$$= \frac{\lambda}{4}\left(\Phi^\dagger\Phi\right)^2 - \frac{\lambda v^2}{2}\Phi^\dagger\Phi + \text{const.}$$

symm. breaking

$$a T^2 \gg -v^2\lambda \Longrightarrow \langle \Phi_0 \rangle_T = 0$$

$T \simeq 100\,\text{GeV} \simeq 10^{13}\,\text{eV} = 10^{5}\,\text{MeV}$

early universe !

Origin of mass (f)

$h \bar{f} f \quad \frac{m_f}{v}$

$\Rightarrow \quad \Gamma(h \to f \bar{f}) \propto m_f^2$

NOT asking : why m_f is m_f?