Neutrino Physics Course

Lecture VIII

LMU

Spring 2021

$$\boxed{\begin{array}{c}\text{Maxwell} \rightarrow \text{Proca} \\ \rightarrow \text{Maxwell}\end{array}}$$

$$\Downarrow$$

Higgs

Proca:

$$\mathcal{L}_P = -\frac{1}{4}F^2 + \frac{1}{2}m_A^2 A^2$$

$$\Downarrow$$

$$\partial_\mu A^\mu = 0$$

$$\Downarrow$$

$$\Delta_{\mu\nu} = -i\,\frac{g_{\mu\nu} - k_\mu k_\nu / m_A^2}{k^2 - m_A^2}$$

1.2.3 in HW3

4. $m_A = 0 \iff$ Take Maxwell

$$\mathcal{L}_M = -\frac{1}{4} F_{\mu\nu} F^{\mu\nu} =$$

$$= \frac{1}{2} A_\mu \left(\Box g^{\mu\nu} - \partial^\mu \partial^\nu \right) A_\nu$$

$$\downarrow \; A_\mu = e^{ipx} \epsilon_\mu(p)$$

$$-\frac{1}{2} \epsilon_\mu \underbrace{\left(p^2 g^{\mu\nu} - p^\mu p^\nu \right)}_{\text{\underline{no} inverse} \iff \text{no propagator}} \epsilon_\nu$$

$$\left(p^2 g^{\mu\nu} - p^\mu p^\nu \right) p_\nu = 0$$

$$\downarrow$$

zero mode

$$P_{\mu\nu} = g^{\mu\nu} - \frac{p_\mu p_\nu}{p^2}$$

$$P_{\mu\nu} P^{\nu\alpha} = P_\mu{}^\alpha$$

$$\boxed{P^2 = P} \quad \text{(projector)}$$

$$P \cdot P^{-1} = 1\!\!1/P \quad (P^{-1} = ?)$$

$$\Rightarrow P^2 P^{-1} = P \Rightarrow \boxed{1\!\!1 = P}$$

only $P = 1$ has inverse!

$$\Downarrow$$

$$\delta \mathcal{L} = \mathcal{L}_{gf} = -\frac{1}{2\xi}(\partial_\mu A^\mu)^2$$

$$\Delta_{\mu\nu} = -i \frac{g_{\mu\nu} + (\xi - 1)\frac{k_\mu k_\nu}{k^2}}{k^2}$$

- $m_A = 0 \Rightarrow h = \pm 1 \quad (h \equiv \vec{S} \cdot \hat{p})$

$\underline{\text{NO}} \longrightarrow A_\mu$ (4 d.o.f.)

$$A_\mu \rightarrow A_\mu - \partial_\mu \alpha(x)$$

- $\xi = 1$ Feynman
- $\xi = 0$ Landau

Dyson

$$\boxed{\Rightarrow \text{gauge invariance} = \xi\text{-independence}}$$

5. $\boxed{g A_\mu \bar{\psi}_L \gamma^\mu \psi_L}$

$\Leftrightarrow$ ψ_L: carries $U(1)$ charge

ψ_R : carries 0 charge

$$\boxed{\begin{gathered} D_\mu = \partial_\mu - ig A_\mu Q \\ Q\psi_L = \psi_L, \quad Q\psi_R = 0 \end{gathered}}$$

$\Downarrow$ step back

$$\mathcal{L}_D = i\bar{\psi}\gamma^\mu \partial_\mu \psi - m\bar{\psi}\psi$$

$$= i\bar{\psi}_L \gamma^\mu \partial_\mu \psi_L + L \leftrightarrow R - m\left(\bar{\psi}_L \psi_R + L \leftrightarrow R\right)$$

$\Downarrow$ two current

$\boxed{\bar{\psi} = \psi^\dagger \gamma^0}$

(V) vector : $j_\mu = \bar{\psi}\gamma_\mu\psi \Rightarrow \partial_\mu j^\mu = 0$

(A) axial-
(vector) : $j_\mu^5 = \bar{\psi}\gamma_\mu\gamma_5\psi \Rightarrow \partial_\mu j_5^\mu = 2m\bar{\psi}\gamma_5\psi$

(V) $\psi \to e^{i\alpha}\psi$ symmetry

Noether: $\partial_\mu j^\mu = 0 \Rightarrow j^\mu = \bar{\psi}\gamma^\mu\psi$

$\Leftrightarrow \psi_L \to e^{i\alpha}\psi_L,\ \psi_R \to e^{i\alpha}\psi_R$

(A) $\psi \to e^{i\beta\gamma_5}\psi \Leftrightarrow \psi_L \to e^{i\beta}\psi_L,\ \psi_R \to e^{-i\beta}\psi_R$

$$\bar{\psi}_L\psi_R \to \bar{\psi}_L e^{-i\beta}e^{-i\beta}\psi_R =$$

$$= e^{-2i\beta}\bar{\psi}_L\psi_R$$

mass breaks <u>chiral</u> symmetry

Dirac: $i\gamma^\mu \partial_\mu \psi = m\psi$

$$\Rightarrow \boxed{\partial_\mu \bar\psi \gamma^\mu \gamma_5 \psi = +2i\bar\psi\gamma_5\psi}$$

- $m = 0 \Rightarrow$ chiral symmetry

$\Downarrow$ Noether

$$\partial_\mu j^\mu_5 = 0$$

QED $\quad e A_\mu j^\mu_{em}$

$$\partial_\mu j^\mu_{em} = 0 \Rightarrow j_\mu^{em} = \bar\psi \gamma_\mu Q_{em} \psi$$

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fixing the sign

$$i\gamma^\mu \partial_\mu \psi = m\psi$$

$$\gamma^\mu \partial_\mu \psi = -im\psi$$

$$\partial^\mu \bar\psi \gamma_\mu \gamma_5 \psi = (\partial_\mu \bar\psi)\gamma_\mu \gamma_5 \psi$$

$$+ \bar\psi \partial^\mu \gamma_\mu \gamma_5 \psi = (\partial_\mu \bar\psi)\gamma^\mu \gamma_5 \psi \quad (a)$$

$$\left(- \bar\psi \gamma_5 \partial^\mu \gamma_\mu \psi = im \bar\psi \gamma_5 \psi\right) \quad (b)$$

$$= (a) + im \bar\psi \gamma_5 \psi = 2im \bar\psi \gamma_5 \psi$$

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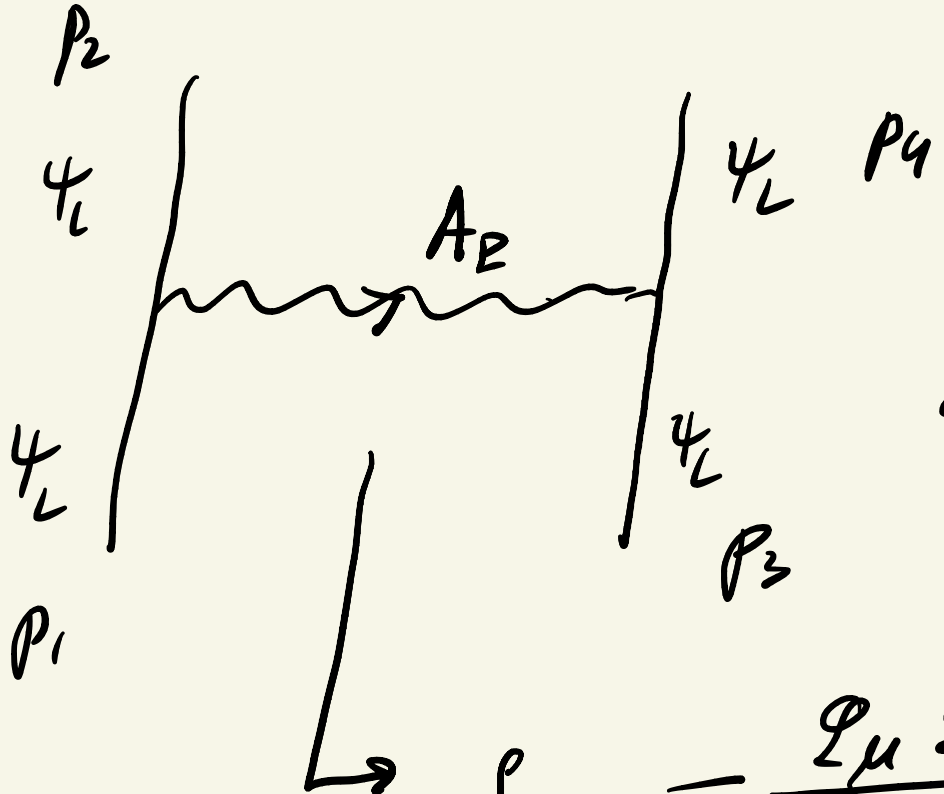
5. of HW 3 - chiral gauge theory

$$\frac{1}{2} g A_\mu \bar\psi \gamma^\mu \frac{1+\gamma_5}{2}\psi =$$

$$= \frac{1}{2} g A_\mu \bar\psi \gamma^\mu \psi + \frac{1}{2} g A_\mu \bar\psi \gamma^\mu \gamma_5 \psi$$

$$= \frac{1}{2} g A_\mu j^\mu_V + \frac{1}{2} g A_\mu j^\mu_A$$

$$\partial^\mu j^\mu_V = 0, \quad \partial^\mu j_\mu^A = 2i\, m_f \bar{\psi} \gamma_5 \psi$$



$$q \equiv p_1 - p_2 = p_4 - p_3$$

$$① \quad \frac{g_{\mu\nu} - \frac{q_\mu q_\nu}{m_A^2} \; ②}{q^2 - m_A^2}$$

$$① \to \bar{\psi}_L \gamma^\mu \psi_L \, \bar{\psi}_L \gamma_\mu \psi_L \, \frac{g^2}{q^2 - m_A^2}$$

② $g^2 \, \bar{\psi}_L(p_2)\gamma^\mu \psi_L(p_1) \; \bar{\psi}_L(p_4)\gamma^\nu \psi_L(p_3)$

$$\frac{1}{q^2 - m_A^2} \quad q_\mu q_\nu / m_A^2$$

$$q_\mu \bar{\psi}_L(p_2)\gamma^\mu \psi_L(p_1) = q_\mu \bar{\psi} \gamma^\mu \frac{1+\gamma_5}{2} \psi$$

$$= \underbrace{\tfrac{1}{2} q_\mu \bar{\psi}\gamma^\mu \psi}_{\substack{\parallel \\ 0}} + \underbrace{\tfrac{1}{2} q_\mu \bar{\psi}\gamma^\mu\gamma_5 \psi}_{} = \pm m \bar{\psi}\gamma_5\psi$$

$$\partial^\mu j_\mu^V = 0 \qquad \partial^\mu j_\mu^A = 2i\bar{\psi}\gamma_5\psi m$$

$$② \longrightarrow \frac{m_f^2/m_A^2}{q^2 - m_A^2} (\bar{\psi}\gamma_5\psi)^2$$

$$\boxed{m_A \to 0 \Rightarrow \infty \; !}$$

$$\boxed{\text{No limit: } m_A \to 0}$$

- <u>fermion propagator</u>

$$S_F = \frac{i}{\not p - m_f} \xrightarrow{m_f \to 0} \frac{i}{\not p}$$

bad

$$\Delta_{\mu\nu}(k) = -i \, \frac{g_{\mu\nu} - \frac{k_\mu k_\nu}{m_A^2}}{k^2 - m_A^2} \xrightarrow{k \to \infty} \frac{1}{m_A^2}$$

$$\Updownarrow$$

Proca = 3 d.o.f. "Maxwell" = 2 d.o.f.

Fermion Majorana:

$$m_M \psi_L^T C \psi_L$$

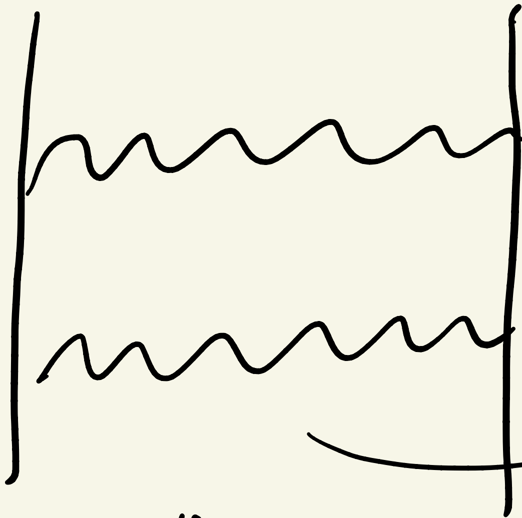
$m_M \to 0$: smooth (Weyl)

$$\boxed{\psi_M = \psi_L + (\psi^c)_R}$$

· infrared divergence in QED

Feynman: $$\Delta^M_{\mu\nu}(A) = \frac{-i g_{\mu\nu}}{q^2 - m_A^2}$$

$m_A \to 0$ at the end! :)



$\int d^4k$

propagators

$\Downarrow$

$$\int^{\infty} d^4k \; \underbrace{\frac{1}{\Delta_F^2}\,\frac{1}{S_F^2}}_{\substack{\downarrow\, k \to \infty \\ 0}} = \text{finite}$$

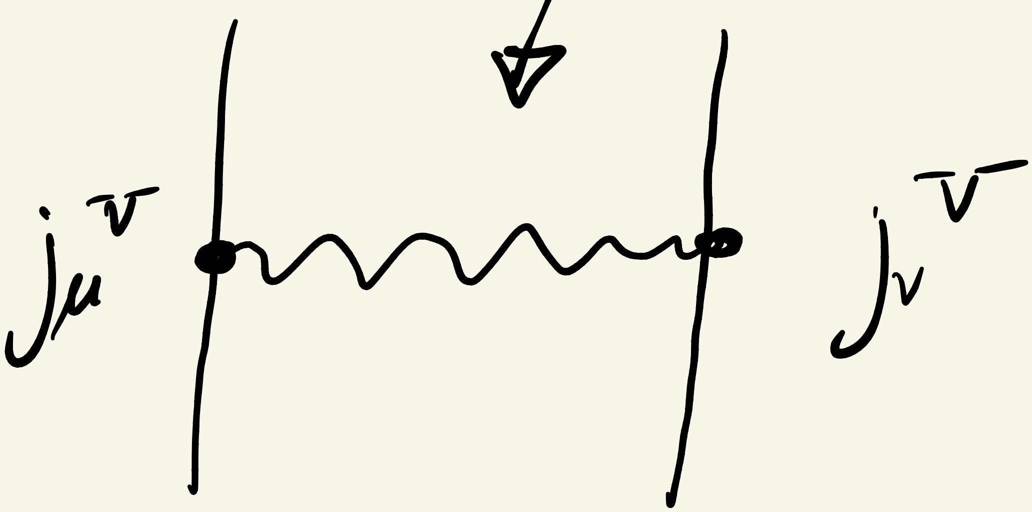
<u>QED</u>

$A_\mu \, j_\mu^V$ = <u>vector!</u>

$$\partial_\mu j^\mu_V = 0$$

$$\Leftrightarrow \quad q_\mu \, j^\mu_V(q) = 0$$

$$\Delta_{\mu\nu}(q) = \frac{g_{\mu\nu} - \frac{q_\mu q_\nu}{m_A^2}}{q^2 - m_A^2}$$



$q_\mu \, j^\mu_V = 0 \Rightarrow$ goes away

1960 Veltman, van Dam

if j_μ = chiral (axial)

$\Rightarrow$ no limit $m_A \to 0$

$\Downarrow$

chiral case ($\partial^\mu j_\mu \neq 0$)

$k \to 0$ limit = sick

$$\frac{\frac{k_\mu k_\nu}{m_A^2}}{k^2 - m_A^2} \to \frac{1}{m_A^2}$$

contributes

$\Downarrow$ $\boxed{\text{what to do?}}$

Summary

Theory: based on symmetry principle

chiral symmetry: $m_f = 0$

$\Leftrightarrow \psi \to e^{i\gamma_5\beta}\psi$

mass = breaks (often) symmetry

$\Leftrightarrow$ soft breaking

energy E: $\left(\frac{m}{E}\right) \Rightarrow$

$$\boxed{E \to \infty \iff m \to 0}$$

breaking goes away

<u>wrong in general!</u>

$m_A \equiv m_{Proca} \to 0$ limit is

not smooth

$$\boxed{m_A \neq \text{soft breaking}}$$

<u>Proca theory</u>

$$\mathcal{L} = -\frac{1}{4} F_{\mu\nu} F^{\mu\nu} + \frac{1}{2} m_A^2 A_\mu A^\mu$$

$$\Downarrow$$

$$\boxed{\partial^\mu A_\mu = 0} \quad (\text{e.o.m.})$$

$\rightarrow$ physical degrees of freedom

QED : works because of unphysical degrees

$4\ A_\mu \longrightarrow 2$ physical

Cun'Folo : add a degree of freedom (unphysical)

$\Downarrow$

$$\mathcal{L}_s = -\frac{1}{4} F_{\mu\nu} F^{\mu\nu} + \cdots$$

($\boxed{\text{Stueckelberg}}$)

added G (scalar)

$\boxed{\text{5 degrees of freedom}}$

$$\frac{1}{2} m_A^2 \left(A_\mu - \frac{\partial_\mu G}{m_A}\right)\left(A^\mu - \frac{\partial^\mu G}{m_A}\right)$$

G ≠ not physical

$$\boxed{\begin{aligned} A_\mu &\to A_\mu - \partial_\mu \alpha(x) \\ G/m_A &\to G/m_A - \alpha(x) \end{aligned}}$$

gauge invariance

$$\hookrightarrow \underbrace{\frac{1}{2} m_A^2 A_\mu A^\mu} + \underbrace{\frac{1}{2} (\partial_\mu G)(\partial^\mu G)}$$

$$\left(- m_A^2 A_\mu \partial^\mu G = \underline{m_A^2 G \partial^\mu A_\mu} \right)$$

$$+ \Delta \mathcal{L} = \mathcal{L}_{gf} = -\frac{1}{2\xi} \left(\partial^\mu A_\mu + \xi m_A G \right)^2$$

$$= -\frac{1}{2\xi} (\partial^\mu A_\mu)^2 - \frac{1}{2} \underbrace{\xi m_A^2} G^2$$

$$m_G = \sqrt{\xi}\, m_A$$

$$- m_A G \partial^\mu A_\mu \qquad \Longleftrightarrow G \neq \text{physical}$$

$$\Downarrow$$

$$\mathcal{L}_s + \mathcal{L}_{gf} = \underbrace{\frac{1}{2}(\partial_\mu G)^2 - \frac{1}{2}\xi m_A^2 G^2}_{\text{scalar piece}}$$

$$+ \boxed{\begin{aligned} &-\frac{1}{4}F_{\mu\nu}F^{\mu\nu} + \frac{1}{2}m_A^2 A^\mu A_\mu \\ &\quad -\frac{1}{2\xi}(\partial_\mu A^\mu)^2 \\ &\quad \neq \text{Proca} \end{aligned}}$$

$$\Downarrow \quad \boxed{\text{HW?}}$$

$$\Delta^s_{\mu\nu}(A) = \frac{-i}{k^2 - m_A^2}\left[g_{\mu\nu} + (\xi - 1)\frac{k_\mu k_\nu}{k^2 - \xi m_A^2}\right]$$

$\boxed{\text{"Lazy person" approach}}$ /

- $\xi \to \infty \Rightarrow$ Proca

$$g_{\mu\nu} - \frac{k_\mu k_\nu}{m_A^2}$$

- $m_A \to 0 \Rightarrow$ Maxwell

<u>bottom line:</u>

$$\Delta^{\xi}_{\mu\nu} \xrightarrow[k \to \infty]{} O\left(1/k^2\right)$$

for any (finite) ξ

Proca = $\xi \to \infty$ after

guess $U(1)$ gauge theory

scalar: $\phi \to e^{i\alpha(x)}\phi$

$\phi = \rho(x)\, e^{i\theta(x)}$

dimensions

$\Rightarrow$ dimension of mass?

v = mass scale

$\phi = (\quad)\, e^{i\theta(x)/v \,\equiv\, \theta(x)}$

$\Rightarrow \phi \to e^{i\alpha(x)}\phi$

$\Leftrightarrow \quad \mathcal{L}/_0 \to \mathcal{L}/_0 + \alpha$

Stueckelberg = fundamental gauge theory

$$\boxed{\Leftrightarrow \quad G \neq \text{physical}}$$

$$\boxed{\xi = 0 \Leftrightarrow \partial^\mu A_\mu = 0}$$

QED

$$\mathcal{L}_M = -\frac{1}{4} F_{\mu\nu} F^{\mu\nu} =$$

$$= \frac{1}{2} A_\mu \left(\Box g^{\mu\nu} - \partial^\mu \partial^\nu \right) A_\nu$$

$$\partial^\nu A_\nu = 0 \Rightarrow \frac{1}{2} A_\mu \Box g^{\mu\nu} A_\nu$$

$$\Rightarrow \quad \Delta_{\mu\nu} = \frac{g_{\mu\nu}}{\Box} \to \frac{g_{\mu\nu}}{k^2}$$

$$\boxed{\mathcal{L}_{gf} = -\frac{1}{2\xi}\left(\partial_\mu A^\mu\right)^2} \quad \boxed{\xi = 0 \Leftrightarrow \partial^\mu A_\mu = 0}$$

$$M: \left(\Box g^{\mu\nu} - \partial^\mu \partial^\nu\right) A_\nu = 0 \quad (\text{e.o.m.})$$

$$P: \left[\left(\Box + m_A^2\right) g^{\mu\nu} - \partial^\mu \partial^\nu\right] A = 0 \quad / \partial_\nu$$

$$\Rightarrow \quad m_A^2 \partial^\mu A_\mu = 0$$

$$(\text{e.o.m.}) \quad \partial^\mu A_\mu = 0 \quad (m_A \neq 0)$$

M: $\quad A_\mu \to A_\mu - \partial_\mu \alpha(x)$

$$\partial^\mu A_\mu \to \partial^\mu A_\mu - \Box \alpha(x) = 0$$

$$\boxed{\Box \alpha(x) = \partial^\mu A_\mu}$$

- $m_A = 0 \Rightarrow h = \vec{S} \cdot \hat{p} =$

$=$ Lorentz invariant

$$\boxed{h = \pm 1} \quad (\text{physical})$$

$$\mathcal{L}_M = -\frac{1}{4} F_{\mu\nu} F^{\mu\nu}$$

$$\Rightarrow \boxed{A_0 = 0}$$

$$A_0 \to A_0 - \partial_0 \alpha(x) = 0$$

$$A_i \to A_i - \partial_i \alpha(x) -$$

$$\partial_i A_i \to \partial_i A^i - \nabla^2 \alpha(x) = 0$$

$$\partial_i A_i \to 0 \Leftrightarrow \boxed{\nabla^2 \alpha(x) = \partial_i A_i}$$

$$\boxed{A_\mu \; (4 - 2 = 2)}$$

$$\nabla^2 \alpha(x) = \rho(x) \qquad \rho \equiv \partial^i A_i$$

$$\alpha(x) = \frac{1}{\nabla^2} \rho(x) \quad \text{(Fourier)}$$