# LMU Neutrino Course

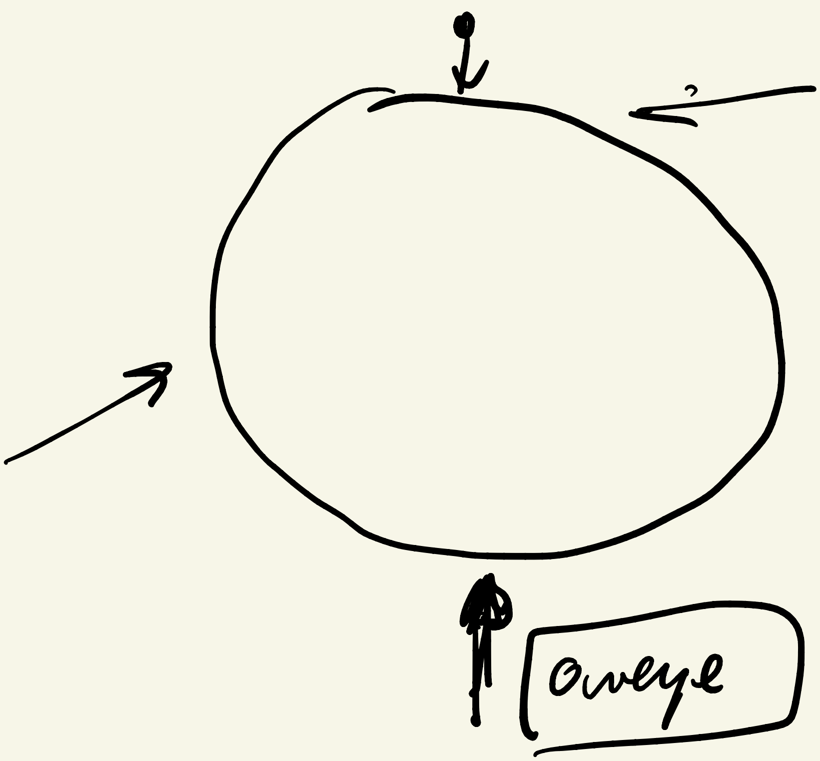
## Lecture V

27/4/2021

Spring 2021

# Neutrino mass

Atmospheric neutrino oscillations



$\pi^- \to \mu^- + \bar{\nu}_\mu$

$\begin{pmatrix} \nu_\mu \\ \mu \end{pmatrix}, \begin{pmatrix} \nu_\tau \\ \tau \end{pmatrix}$

$$P(\nu_\mu \to \nu_\tau) = \sin^2 2\theta \sin^2 \frac{\Delta m^2 L}{4E}$$

$$\nu_\mu = \cos\theta_A \, \nu_1 + \sin\theta_A \, \nu_2$$

$$\boxed{E \simeq \text{GeV}}$$

$$\nu_\tau = -\sin\theta_A \, \nu_1 + \cos\theta_A \, \nu_2$$

$\nu_{1,2}$ = mass eigenstates

- $\theta_A \simeq 45^\circ \Rightarrow \Delta m^2_A \simeq 10^{-3}\, eV^2$

($L_{osc} \simeq 500$ km)

## long baseline experiments

- K2K (KEK → Kamioka)

  250 km

- T2K (~300 km)

- MINOS (Fermilab – Minnesota

  750 km

- OPERA (CERN – Gran Sasso)

~800 km

$$\boxed{\nu_\mu \longrightarrow \nu_\tau \leftarrow \text{appear}}$$

- <u>Solar neutrino</u> <u>SNO</u>

$\nu_e \leftarrow$ disappear $\boxed{\nu_e \to \nu_\mu}$

$\nu_e + \nu_\mu + \nu_\tau$ = arrive safely

$$\Delta m^2_\odot = 10^{-5}\, eV^2, \; \theta_\odot \simeq 30^\circ$$

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. $\nu_\mu$ = produced

$\Delta m^2 < \sigma_{m^2}$ = basic QM

uncertainty of

measuring masse !

$$\boxed{E^2 = p^2 + m^2}$$

$$\sigma_{m^2} = \sigma_{E^2} + \sigma_{p^2}$$

$$= E\sigma_E + \cdots$$

$$\pi^- \longrightarrow \mu^- + \bar{\nu}_\mu$$

$m_\pi = 140\,\text{MeV}$

$m_\mu \simeq 100\,\text{MeV}$

$$\sigma_E \simeq \Gamma_\pi$$

$$\sigma_{m^2} \simeq E\,\Gamma_\pi \simeq 10\,\text{MeV} \cdot \Gamma_\pi$$

$$\Gamma_\pi \simeq G_F^2 \frac{m_\pi^5}{8\pi} \simeq 10^{-10} \cdot \frac{10^{-5}}{10}\,\text{GeV}$$

$G_F = 10^{-5}\,\text{GeV}^{-2}$ $\qquad \simeq 10^{-16}\,\text{GeV}$

$\Rightarrow \delta_{m^2} \simeq 10^{-2} \cdot 10^{-16}\,\text{GeV} \simeq 10^{-18}\,\text{GeV}^2$

$$\boxed{\delta_{m^2} \simeq \text{eV}^2}$$

$$\boxed{\Delta m^2_A \simeq 10^{-3}\,\text{eV}^2 \ll \delta_{m^2}\,!}$$

$\boxed{\text{Genuine effect : QM}}$

. all other explanations fail

neutron decay

violation of Lorentz invariance

$\Downarrow$ Holy Grail

$\boxed{\text{measure the mass}}$

$\boxed{\bullet \text{ direct } \beta \text{ decay}}$ $\boxed{\text{any mass}}$

$n \to p + e + \bar{\nu}_e$

- $0\nu 2\beta$ : neutrinoless double beta

Majorana mass = central

experiment today

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$\boxed{\bullet \text{ direct}}$ $^3H \to {}^3He + e + \bar{\nu}_e$

$$Q = M_i - M_f - m_e$$

$$E_e = m_e + T \text{ (kinetic)}$$

$$\frac{d\Gamma}{dE_e} \propto E_e E_\nu p_\nu p_e$$

$$M_i = M_f + E_e + E_\nu$$

$$= M_f + m_e + T + E_\nu$$

$$\Rightarrow \quad Q = T + E_\nu \Rightarrow \boxed{E_\nu = Q - T}$$

$$p_\nu = \sqrt{E_\nu^2 - m_\nu^2}$$

$$\Downarrow$$

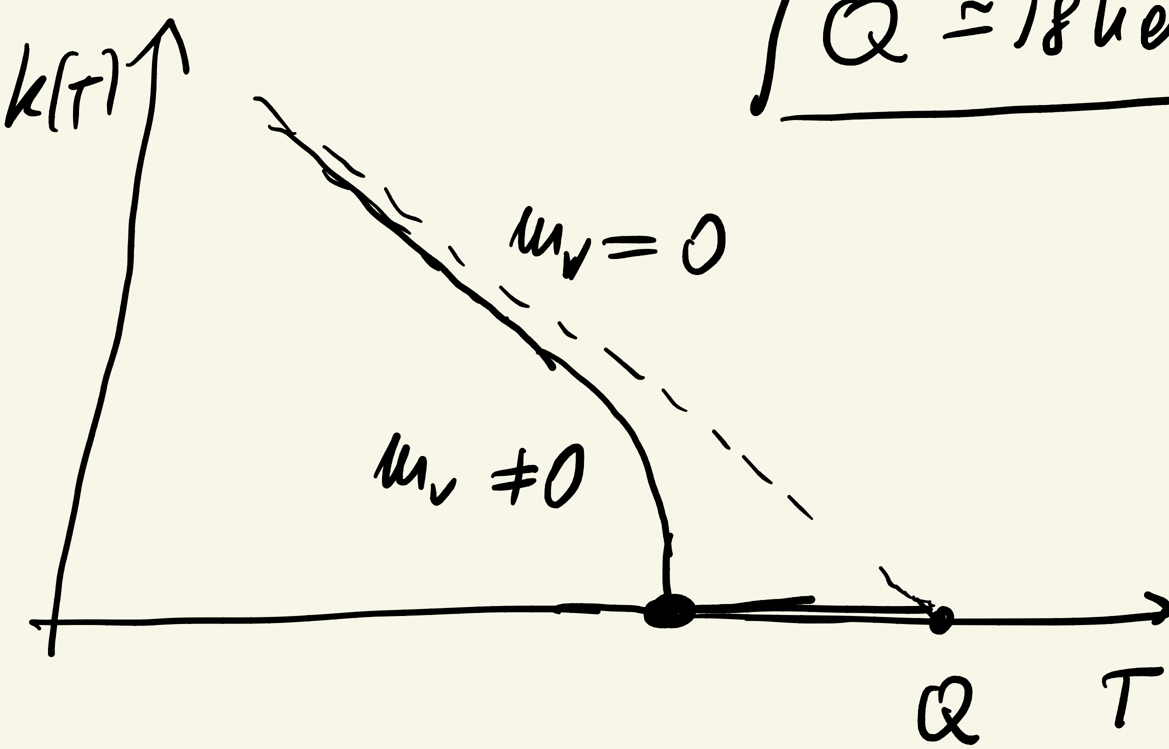
$$\frac{d\Gamma}{dE_e} \propto \underbrace{(Q-T)\sqrt{(Q-T)^2 - m_\nu^2}}_{\parallel}$$

$K^2$ (K = Kurie function)

exp $\Downarrow$

$m_\nu = 0 \Rightarrow K = Q - T$

$\boxed{Q \simeq 18\,\text{keV}}$



KATRIN

direct

$m_\nu \leq 1\ \text{eV}$

$\Delta \tilde{m}_A \simeq 10^{-3}\ \text{eV} \rightarrow m_\nu \gtrsim 1/25\ \text{eV}$

0.2 eV ? end of Katrin

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Nature of neutrino mass

Dirac (D)

Majorana (M)

## Dirac

$$\psi \to S\,\psi$$

$$S = \exp\left(i\,\theta_{\mu\nu}\,\Sigma^{\mu\nu}\right)$$

$$\Sigma^{\mu\nu} = \frac{1}{4i}\left[\gamma^\mu, \gamma^\nu\right]$$

$$\psi = \begin{pmatrix} u_L \\ u_R \end{pmatrix} \Rightarrow$$

$$u_{L,R} \to e^{\,i\vec{\sigma}/2\,\left(\underbrace{\vec{\theta}}_{\text{ROT}} \,\pm\, i\,\underbrace{\vec{\varphi}}_{\text{BOOST}}\right)}\, u_{L,R}$$

$$\theta_i = \theta_{0i}$$

$$\varphi_i = \varepsilon_{ijk}\,\theta_{jk}\,1/2$$

• $u_L^+ u_L \neq$ invariant

$u_R^+ u_R \neq$ — " —

$$\Rightarrow \boxed{u_L^+ u_R = \text{invariant}}$$

$\parallel$

Dirac $\Rightarrow$ $m_D(u_L^+ u_R + h.c.)$

$$= m_D(\bar{\psi}_L \psi_R + h.c.) = m_D \bar{\psi}\psi$$

$$\psi_{L/R} \equiv L(R)\,\psi$$

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• Majorana $\boxed{u_L^T i\sigma_2 u_L}$

$\underbrace{\qquad\qquad}$

inv. both under Rot., Boost

$C \equiv i\gamma_2\gamma_0$

$\Downarrow$

$$\frac{1}{2} m_M \left( u_L{}^T i\sigma_2 u_L + h.c. \right)$$

$$\Updownarrow \qquad = \frac{1}{2} m_M \left( \psi_L{}^T C \psi_L + h.c. \right)$$

$$\mathcal{L}_M = i \bar{\psi}_L \gamma^\mu \partial_\mu \psi_L - \frac{1}{2} m_M - '' -$$

$$\psi_M = \psi_L + C \bar{\psi}_L{}^T = \begin{pmatrix} u_L \\ i\sigma_2 u_L^* \end{pmatrix}$$

pseudo-real

$$\bar{\psi}_M \gamma^\mu \partial_\mu \psi_M = 2 \bar{\psi}_L \gamma^\mu \partial_\mu \psi_L$$

$$\bar{\psi}_M \psi_M = \psi_L{}^T C \psi_L + h.c.$$

$$\Downarrow$$

mass

$$\mathcal{L}_M = \frac{1}{2} \left[ \bar{\psi}_M \gamma^\mu \partial_\mu \psi_M - m_M \bar{\psi}_M \psi_M \right]$$

$\Downarrow$ Dirac (analogy)

$$\boxed{E^2 = \bar{p}^2 + m_\mu^2 \nwarrow \text{mass}}$$

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chirality = crucial

$$\begin{pmatrix} \nu \\ e \end{pmatrix}_L \leftarrow \boxed{\text{weak int.}}$$

change the $\gamma$- basis:

$$g_{\mu\nu} = \mathrm{diag}(+, - - -)$$

$$\gamma^0 = \begin{pmatrix} 0 & \sigma_2 \\ \sigma_2 & 0 \end{pmatrix} \quad (\gamma_0^T = -\gamma_0)$$

$$\gamma^1 = \begin{pmatrix} i\sigma_1 & 0 \\ 0 & i\sigma_1 \end{pmatrix}, \quad \gamma^2 = \begin{pmatrix} 0 & \sigma_2 \\ -\sigma_2 & 0 \end{pmatrix}$$

$$\gamma^3 = \begin{pmatrix} i\sigma_3 & 0 \\ 0 & i\sigma_3 \end{pmatrix}$$

$$\Sigma_{\mu\nu} \equiv \underbrace{\frac{1}{4i} \overbrace{[\gamma_\mu, \gamma_\nu]}^{\text{real}}}_{\text{imaginary}}$$

$$\Rightarrow \boxed{i\,\theta_{\mu\nu}\Sigma^{\mu\nu} = \text{real}}$$

$$\Rightarrow S = \text{real}$$

$$\boxed{\psi \in \mathbb{R} \Rightarrow S\psi \in \mathbb{R}}$$

Majorana spinor $\Longleftrightarrow$ $h$ (Higgs)

$$p = \bar{p}$$

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Q. why not electron??

A. $e$ = charged $\Rightarrow \psi_e \to e^{i\alpha Q_e} \psi_e$

(i) $\Rightarrow \psi_e \neq$ real

(Majorana basis)

(ii) $\psi_e \neq \psi_L + C\bar{\psi}_L^T$

$\qquad\qquad\qquad\qquad e^{-i\alpha Q} \psi_L$

$\qquad \hookrightarrow e^{i\alpha Q} \psi_L$

$\left( m_e^{M} / m_e^{D} \leq 10^{-20} \; ? \right)$

$\boxed{\text{Majorana mass term — breaks charges !}}$

$$\Downarrow$$

charges — local ($Q_{em}$, $T_{3W}$ ..)

charges — global ($B$, $L$, $F$)

baryons + leptons

($p$, $n$, ...) $e, \mu, \tau$

quarks $\nu_e, \nu_\mu, \nu_\tau$

(nature)

fund. interactions : $\Delta B = \Delta L = 0$

$$W \curvearrowright \begin{pmatrix} u \\ d \end{pmatrix}_L \qquad \begin{pmatrix} \nu \\ e \end{pmatrix}_L \curvearrowleft W$$

$$\boxed{\begin{aligned} B &= \text{baryon number} \\ L &= \text{lepton} \quad -\!\!\shortparallel\!\!- \\ F &= \text{fermion} \quad -\!\!\shortparallel\!\!- \end{aligned}}$$

• GUT : $\left( \begin{array}{c} \Delta B \neq 0 \neq \Delta L \\ \Delta F = 0 \end{array} \right)$

$$\boxed{\bullet \ \text{in nature} : \quad m_M \Rightarrow \Delta L = 2}$$

- neutrino = Majorana

  (natural) $m_\nu / m_e \lesssim 10^{-6}$

- $m_\nu \ll m_e$ - miracle

unlike quarks: $m_u \simeq m_d$

$$\begin{pmatrix} m_c \simeq m_s \\ m_t \simeq m_b \end{pmatrix}$$

$\nu$ = Majorana - separates $\nu$ from $e$

$\Downarrow$

$$\boxed{\text{implications of } m_\nu^M}$$

$\Longleftrightarrow \Delta L = 2$

$\Downarrow$

e produced without $\nu$ !

$$\beta: \quad \underbrace{Co \longrightarrow Ni + e + \bar{\nu}_e}_{\Delta L = 0}$$

weak int. = conserve L

$(\Delta L = 0)$

$\Downarrow$

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$$\Delta L = 2 \;\not\Leftarrow\; m_\nu^{M}$$

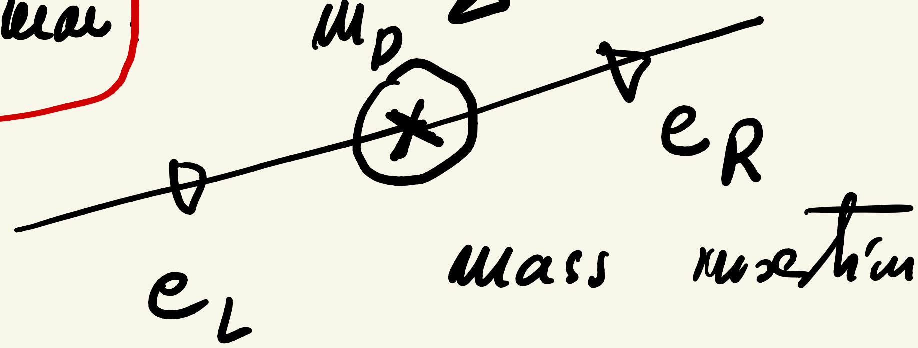
bilinear

$\Downarrow$

2 neutrons produced $\bar{e}_R e_L$

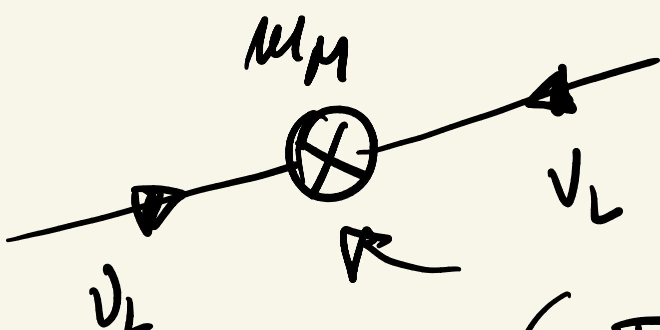
• $\boxed{\text{Feynman}}$

$\boxed{\text{Dirac}}$




$$S_F(e) \propto \frac{1}{\not p - m_D} = \frac{\not p + m_D}{p^2 - m^2}$$

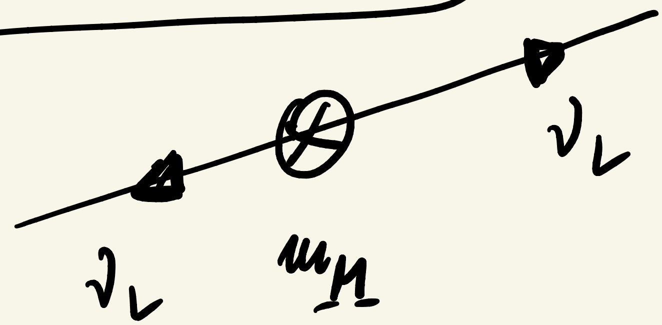
$\boxed{\text{Majorana}}$ $\nu_L$ , $\underline{\text{no}}$ $\nu_R$



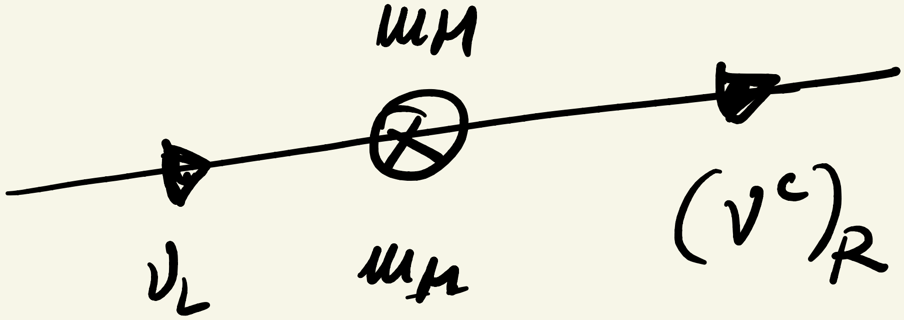

$$m_M(\nu_L{}^T C \nu_L + h.c.)$$

$$= m_M(\nu_L{}^T C \nu_L + \nu_L{}^\dagger C^\dagger \nu_L{}^*)$$

neutrino "annihilator"




$\Leftrightarrow$




$$m_M \, \overline{(\nu^c)_R} \, \nu_L = m_M \, \overline{C \bar{\nu}_L^{T}} \, \nu_L$$

$$= m_M \, \nu_L^T C \, \nu_L \qquad (\Delta L = 2)$$

$$\overline{C \bar{\nu}_L^{T}} = \left(C \gamma_0 \nu_L^*\right)^\dagger \gamma^0 = \nu_L^T \gamma_0 C^\dagger \gamma_0$$

$$= \nu_L^T \left(-C^\dagger\right) = \nu_L^T C$$

$\Downarrow$ $\Delta L = 2$

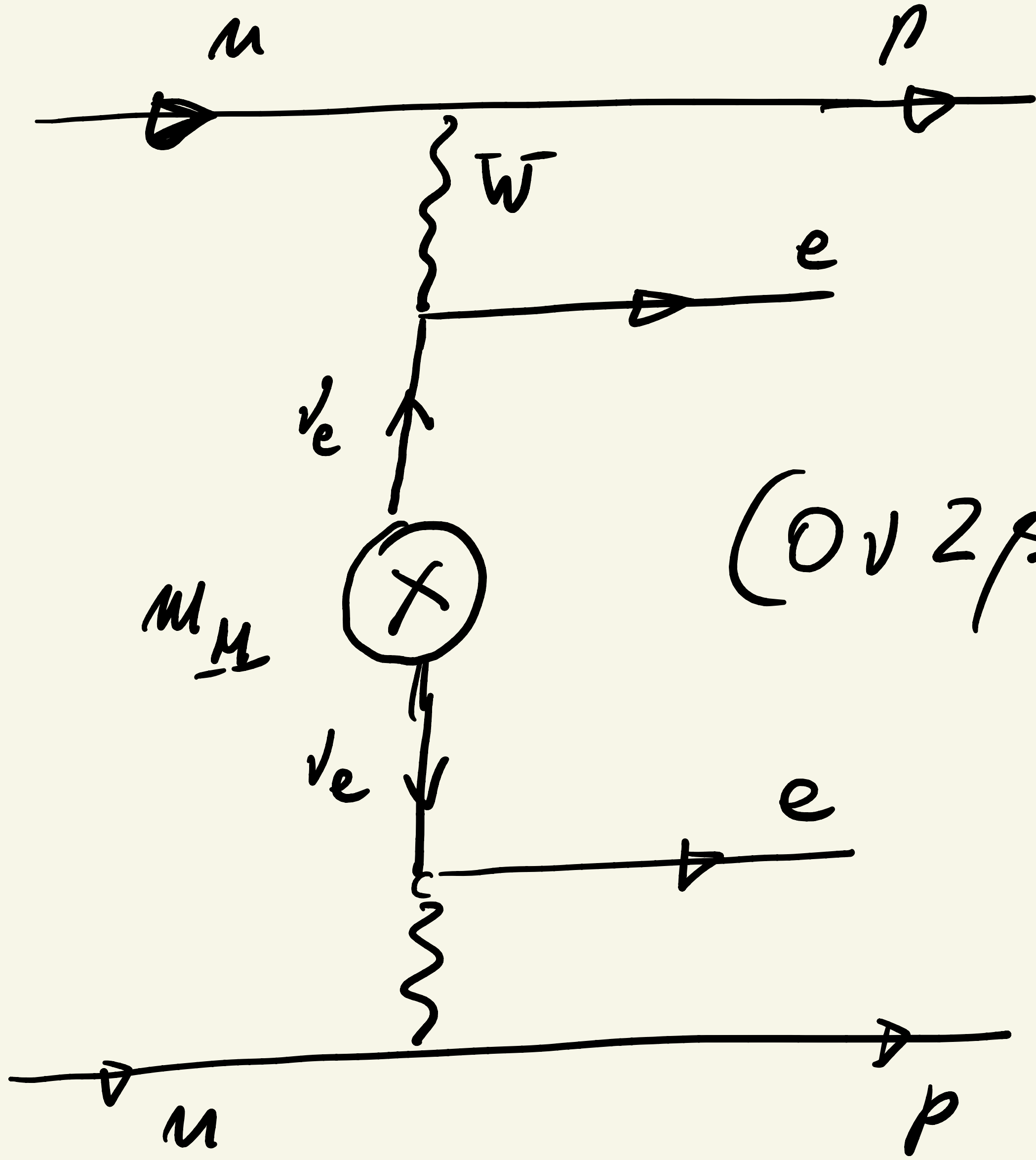
double $\beta$ decay

Goeppert - Meyer '35

$Ge \not\to Ar + e + \bar{\nu}_e$

$m_{Ar} > m_{Ge}$ forbidden

$$\boxed{\begin{array}{c} Ge \to Se + e + e + \bar{\nu}_e + \bar{\nu}_e \\ m_{Ge} > m_{Se} \\ \tau_{2\beta} \simeq 10^{21}\, yr \end{array}}$$




$(0\nu 2\beta)$

1937 Majorana

1938 Racah

Furry

· EXO,

GERDA *

NEMO

MAJORANA

$$\boxed{\tau_{0\nu 2\beta} \gtrsim 10^{25}\ \mathrm{yr}}$$

$\Rightarrow \boxed{m_\nu^{\mu} < eV}$

$(\lesssim 0.3\,eV)$

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$\boxed{\text{generations}}$ =

$\nu_e \leftrightarrow \nu_\mu$ (generation) (flavor)

flavor: $\left.\begin{matrix} u \\ d \end{matrix}\right\}$ different flavors

$N_F = 2\,N_{gen}$

$\Delta F \neq 0$ (flavor) | $K \to \pi + \pi$

$s \to d$ $(\Delta F \neq 0)$

$$d \to u + e + \bar{\nu}_e$$

$$\begin{cases} \Delta F \neq 0 & (\text{flavor}) \\ \Delta B = \Delta L = 0 \end{cases} \quad \boxed{\text{nature}}$$

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- generations: $1 \leftrightarrow 2 \leftrightarrow 3$

$$m_1 \neq m_2 \neq m_3$$

$$m_\nu = 0 \iff m_{\nu_1} = m_{\nu_2} = m_{\nu_3}$$

- $m_\nu \leq 10^{-6}\, m_e \Rightarrow m_\nu = 0$

$0 <$ small

$\Downarrow$

$\boxed{\text{naturalness}}$