Neutrino Physics Course

Lecture IX

11/5/2021

LMU

Spring 2021

Higgs mechanism

Spontaneous Symmetry Breaking (SSB) of gauge symmetries

- effective theory (weak int)

$$\left(\frac{1}{\Lambda_F^2}\right) J_\mu^{w} J_w^{\mu}$$

bad high energy

- $$\mathcal{L}_{fund} = \mathcal{L}_{proca} = g/\sqrt{2}\, W_\mu^+ J_\mu^w + h.c.$$

$$M_W \neq 0$$

U(1) gauge theory

$$\mathcal{L}_p = -\frac{1}{4} F_{\mu\nu} F^{\mu\nu} + \frac{1}{2} m_A^2 A_\mu A^\mu$$

$$\Downarrow$$

$$\Delta_{\mu\nu} \propto \frac{g_{\mu\nu} - \frac{k_\mu k_\nu}{m_A^2}}{k^2 - m_A^2} \xrightarrow[k\to\infty]{} \frac{1}{m_A^2}$$

<u>not</u> soft $\Leftarrow$ m_A stays even when $k \to \infty$

$$\Downarrow$$

Higgs = soft breaking (SSB)

<u>SSB</u> : symmetric theory

vev $\langle \psi \rangle \neq 0$

↳ vacuum expectation value

<u>but</u> some symmetries = sacred

$U(1)_{em}$, $\boxed{\text{Lorentz symmetry}}$

$\Downarrow$

$\psi = ?$ $\langle A_\mu \rangle \neq 0$???

$\langle e \rangle \neq 0$???

$\langle \nu \rangle \neq 0$???

$\underset{(i)}{\Rightarrow}$ $\boxed{\psi = \text{scalar}}$ (fundamental)

(iii) $\underbrace{\Psi = \langle \bar{q} q \rangle, \ \langle \bar{l} l \rangle}_{\text{Lorentz scalar}}$

complicated dynamics

NO

$$\Downarrow$$

$$\phi(x) \to e^{i\alpha(x) Q} \phi(x)$$

$$Q\phi = \phi$$

$$\boxed{D_\mu = \partial_\mu - i g A_\mu Q}$$

$$F_{\mu\nu} = \frac{i}{g}\left[D_\mu, D_\nu\right] = \underbrace{\partial_\mu A_\nu - \partial_\nu A_\mu}_{\text{Maxwell}}$$

$$\Downarrow$$

$$\mathcal{L}_{Higgs} = -\frac{1}{4} F_{\mu\nu}F^{\mu\nu} + \frac{1}{2}(D_\mu \phi)^\dagger (D^\mu \phi)$$

$$- V(\phi)$$

$$A_\mu \; (d=2), \quad \phi \; (d=2)$$

$$V = \frac{\lambda}{4}\left(|\phi|^2 - v^2\right)^2 + h.o.t.$$

$$\boxed{\lambda > 0}$$

crucial

$$= \frac{\lambda}{4}|\phi|^4 - \frac{\mu^2}{2}|\phi|^2 + \text{const.}$$

?

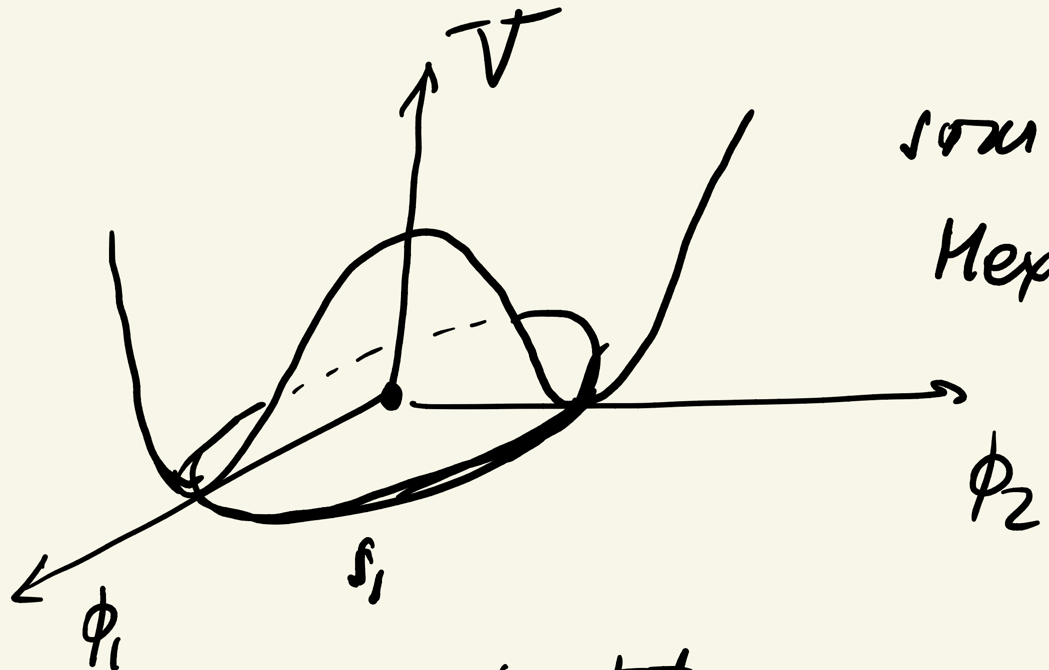
$$+ \frac{1}{\Lambda^2}|\phi|^6$$

$$\mu^2 \equiv \lambda v^2 \qquad \boxed{v = \text{mass}}$$

$$\phi = (\phi_1 + i\phi_2) \to e^{i\alpha}(\phi_1 + i\phi_2)$$

$U(1) = SO(2)$

$$\begin{pmatrix}\phi_1\\ \phi_2\end{pmatrix} = \begin{pmatrix}\cos\theta & \sin\theta \\ -\sin\theta & \cos\theta\end{pmatrix}\begin{pmatrix}\phi_1\\ \phi_2\end{pmatrix}$$



sombrero = Mexican hat

ground state $\equiv$ vacuum

$\mathcal{M}_0 \equiv$ vacuum manifold

$$\mathcal{M}_0 = \{\ \phi_0 :\ V = V_{min} = 0 \}$$

$$= \{\ \phi_0 :\ |\phi_0|^2 = v^2 \} = S_1$$

• $\phi_0 = v$ (S_1 symmetry = all points equivalent)

$$\phi = v + h + iG$$

↑ we live near the ground state

$$\Downarrow$$

$$V = \frac{\lambda}{4}\left((v+h)^2 - G^2 - v^2\right)^2 =$$

$$= \frac{\lambda}{4}\left(2vh + h^2 + G^2\right)^2$$

$$= \frac{\lambda}{4}\left(h^2 + G^2\right)^2 + vh\left(h^2 + G^2\right)$$

$$+ \frac{2\mu^2}{2} h^2 + 0 \cdot G^2$$

$m_h \propto v$

$$\boxed{m_h^2 = 2\mu^2 = 2\lambda v^2}$$

$$\boxed{m_G = 0}$$

$h =$ Higgs boson $\qquad G = ?$

Goldstone ?

- $Q\,\phi_0 \neq 0 \quad \nexists \quad Q\,\phi_0 = \phi_0$

$$e^{i\alpha Q}\,\phi_0 \neq \phi_0$$

Q breaks $U(1)$ spont. on ϕ_0

- "kinetic" energy

$$\frac{1}{2}\left|D_\mu \phi\right|^2 = \frac{1}{2}\Big|\Big(\partial_\mu h + i\,\partial_\mu G -$$

$$- ig\,A_\mu(v + h + iG)\Big|^2$$

$$= \frac{1}{2}\Big|\left(\partial_\mu h + g\,A_\mu G\right) + i\left(\underline{\underline{\partial_\mu G}} - g\,A_\mu(v+h)\right)\Big|^2$$

$$= \frac{1}{2}(\partial_\mu h)^2 + \frac{1}{2}(\partial_\mu G)^2 -$$

$$(**) \; \underbrace{- \partial_\mu G \, A^\mu v g} + \frac{1}{2} g^2 A_\mu A^\mu (v+h)^2$$

$+$ interactions

$$\phi \to e^{i\alpha(x)} \phi, \quad A_\mu \to A_\mu - \frac{1}{g}\partial_\mu \alpha$$

$$\Rightarrow \boxed{m_A = g v}$$

$m_A \propto v$

only mass

$$A_\mu^P = A_\mu^M + (\partial_\mu G)/m_A$$

$(d=3)$

Maxwell $(d=2)$

$\Downarrow$

$G \neq$ Goldstone boson

$G \neq$ particle

phantom

$$\boxed{m_A = g v}$$

$$\frac{1}{2}|D_\mu \phi|^2 = \cdots + \frac{1}{2} m_A^2 \left(A_\mu - \frac{\partial_\mu G}{m_A}\right)^2$$

$$\frac{1}{2} m_A^2 A_\mu^\rho A^\mu_\rho$$

$$\left.\begin{aligned} A_\mu &\to A_\mu - \frac{1}{g}\partial_\mu \alpha \\ G &\to G - m_A \alpha \end{aligned}\right\} \text{gauge invariance}$$

initially A_μ^{μ} $(d=2)$; ϕ $(d=2)$

finally $A_\mu^{?}$ $(d=3)$; h $(d=1)$

$\hookrightarrow$ $(*)$ implies $\mathcal{L}_{gf}$!!

$$\mathcal{L}_{g.f} = -\frac{1}{2\xi}\left(\partial_\mu A^\mu + \xi\, m_A G\right)^2$$

$$= -\frac{1}{2\xi}(\partial_\mu A_\mu)^2 - \frac{1}{2}\xi\, m_A^2 G^2$$

$$\Downarrow$$

$$m_G = \sqrt{\xi}\, m_A$$

$$- m_A\, \partial^\mu A_\mu G = + m_A (\partial^\mu G) A_\mu$$

$(**)$

$$\Downarrow$$

$$\mathcal{L} = -\frac{1}{4} F_{\mu\nu} F^{\mu\nu} + \frac{1}{2} |D_\mu \phi|^2 - V(\phi)$$

$$= -\frac{1}{4} F^2 + \underbrace{\frac{1}{2} (\partial_\mu h)^2 - \frac{1}{2} m_h^2 h^2}$$

$$+ \underbrace{\frac{1}{2} (\partial_\mu G)^2 - \xi m_A^2 G^2} \qquad (m_G = \sqrt{\xi}\, m_A)$$

$$- \frac{1}{2\xi} (\partial_\mu A^\mu)^2 + \frac{1}{2} m_A^2 A_\mu A^\mu$$

$$- \frac{1}{2} \mathcal{L}_{int}$$

$$= \frac{1}{2} A_\mu \left[(\Box + m_A^2) g^{\mu\nu} - \left(1 - \frac{1}{\xi}\right) \partial^\mu \partial^\nu \right] A_\nu$$

$$+ \frac{1}{2} (\partial_\mu h)^2 - \frac{1}{2} m_h^2 h^2 + \frac{1}{2} (\partial_\mu G)^2 - \frac{1}{2} \xi m_A^2 G^2$$

$$- \mathcal{L}_{int}$$

$$\Downarrow$$

$$D(h) = \frac{i}{k^2 - m_h^2} \leftarrow (\text{no } \xi \text{ dep.})$$

$$D(G) = \frac{i}{k^2 - \xi m_A^2} \leftarrow \xi \text{ dep.}$$

$$\Delta_{\mu\nu}(A) = -i \, \frac{g_{\mu\nu} + (\xi - 1)\dfrac{k_\mu k_\nu}{k^2 - \xi m_A^2}}{k^2 - m_A^2}$$

all is well !!

(ξ = finite)

$$\xi \to \infty \implies D(G) \to 0 \quad (\text{no } G)$$

$$\Delta_{\mu\nu}^{(A)} \to \Delta_{\mu\nu}^{P} = -i \, \frac{g_{\mu\nu} - \frac{k_\mu k_\nu}{m_A^2}}{k^2 - m_A^2}$$

$$\boxed{\begin{array}{l}\text{"unitary" gauge} \\ \text{physical gauge}\end{array}} \Leftrightarrow \xi \to \infty$$

(after the computation)

- $\phi = (v+h)\, e^{iG/v} =$ (diff. variables)

$$= v + h + iG + \text{h.o.t.}$$

$$\phi \to e^{i\alpha(x)} \phi$$

$$A_\mu \to A_\mu - \frac{1}{g} \partial_\mu \alpha(x)$$

$$\Downarrow$$

$$G/v \to G/v + \alpha(x)$$

choose: $\alpha(x) = -G/v$

$$\boxed{\phi \to \phi_{un} = (v+h)}$$

$\hookrightarrow$ unitary

$$A_\mu \to \underbrace{A_\mu + \frac{1}{m_A}\partial_\mu G}_{\text{Proca}}$$

$$\boxed{A^P \text{ "eats" } G}$$

$\boxed{G = \text{would have been Goldstone boson}}$

- $\xi = 1$

$$\Delta_{\mu\nu}^{(A)} = -i \, \frac{g_{\mu\nu}}{k^2 - m_A^2}$$

$$D(G) = \frac{i}{k^2 - m_A^2}$$

- $\xi \to \infty$

unitary

$$\boxed{\begin{aligned} \Delta_{\mu\nu}^{(A)} &\to -i \, \frac{g_{\mu\nu} - \frac{k_\mu k_\nu}{m_A^2}}{k^2 - m_A^2} \\ D(G) &\to 0 \end{aligned}}$$

- $\xi = 0$

$$\Delta_{\mu\nu}^{(A)} = -i \, \frac{g_{\mu\nu} - \frac{k_\mu k_\nu}{k^2}}{k^2 - m_A^2}$$

$$D(G) \to \frac{i}{k^2}$$

G = would have been - - -

$\Downarrow$

$\boxed{\begin{array}{c}U(1)\\ \text{global}\end{array}}$ $\alpha = \text{const.} \quad (g=0)$

$$\mathcal{L} = \frac{1}{2}(\partial_\mu \phi)^+(\partial^\mu \phi) - V(\phi)$$

$$\cancel{-\frac{1}{4}F_{\mu\nu}F^{\mu\nu}}$$

$$D_\mu \equiv \partial_\mu - ig A_\mu Q \longrightarrow \partial_\mu$$

$$\boxed{\phi = (v+h)e^{i\theta/v}}$$

$$\phi \to e^{i\alpha}\phi \Rightarrow \theta/v \to \theta/v + \alpha \neq 0$$

$$\Downarrow$$

θ = physical field

$$\phi^+\phi = (v+h)^2$$

$$\Rightarrow V = \frac{\lambda}{4}(2vh+h^2)^2$$

$$\Rightarrow \boxed{m_h^2 = 2\lambda v^2} \qquad \Rightarrow \boxed{m_\theta = 0}$$

$\boxed{\text{but } G = \text{physical}}$

Nambu – Goldstone boson (NG)

$Q\phi_0 \neq 0 \Rightarrow \exists$ NG boson G

$\therefore m_G = 0$

$\boxed{\exists h \therefore m_h^2 = 2\lambda v^2}$ 1961 Goldstone

$\cdot\ \alpha = \text{cnst.} \Rightarrow$ $\exists$ "Higgs"

$\exists$ NG

$\cdot\ \alpha = \alpha(x) \Rightarrow$ $\exists$ Higgs

$\nexists$ NG $(m_A \neq 0)$

eaten by A

<u>comment:</u>

- $$\phi = (v+h)\, e^{i\sigma/v} = v\,(1 + h/v)\, e^{i\sigma/v}$$

$$\partial_\mu \phi = e^{i\sigma/v}\left[\partial_\mu h + (v+h)\, i\, \frac{\partial_\mu \sigma}{v}\right]$$

$$|\partial_\mu \phi|^2 = (\partial_\mu h)^2 + (\partial_\mu \sigma)^2 \left(1 + \frac{h}{v}\right)^2$$

"bad" for computation

- instead: $\underbrace{\phi = v + h + i\sigma}_{\text{renormalizable}}$

$$V = \frac{\lambda}{4}\left[(v+h)^2 + \sigma^2 - v^2\right]$$

$$= \frac{\lambda}{4}\left(2vh + h^2 + G^2\right)^2 = \frac{\lambda}{4} G^4 + \dots$$

$$\Rightarrow \boxed{m_G = 0}$$

$$\mathcal{L}_{kin} = \frac{1}{2}(\partial_\mu h)^2 + \frac{1}{2}(\partial_\mu G)^2$$

Where does G^4 go ?
(when $p \to 0$)

• axion = popular "NG" boson

• gauge symmetries $\Rightarrow$ breaking must be spontaneous!

$\alpha = \alpha(x)$

otherwise infinities

$\Rightarrow$ point of exact symmetry =

= special point

- global symmetry $\Rightarrow$ explicit
  $\alpha = $ const. breaking is allowed (and natural)

however, $\boxed{\text{spont.} \gg \text{explicit breaking}}$

Nambu: pions = NG bosons of chiral symmetry

$m_q = 0 \Rightarrow q \to e^{i\alpha\gamma_5} q$ symmetry

$m_u = m_d = 0 \Rightarrow \underbrace{SU(2)_L \times SU(2)_R}_{\text{chiral}}$

$\underbrace{\langle \bar{q} q \rangle}_{\text{breaks chiral symmetry}} = \langle \bar{q}_L q_R + h.c. \rangle \simeq \Lambda_{QCD}^3$

$L = R$

$$
\begin{array}{cc}
SU(2)_L \times SU(2)_R & (6 \text{ gen.}) \\
\downarrow \langle \bar{q} q \rangle & \\
SU(2)_{L+R} & (3 \text{ gen.})
\end{array}
$$

$$\boxed{\begin{array}{c} 6 - 3 = 3 \text{ NG} \\ \Rightarrow \text{pions} \end{array}}$$

(π^+, π^-, π^0) : $m_\pi \simeq 140\,\text{MeV}$

$m_\pi \ll m_p, m_n \simeq 10^3\,\text{MeV}$

$m_u \cong m_d =$ small $\neq 0$

$\rightsquigarrow m_q \simeq \text{MeV}$

$\Downarrow$

$\vec{\pi} = \boxed{\text{(pseudo)}}$ Goldstone bosons

$$\boxed{m_\pi^4 \simeq m_q \Lambda_{QCD}^3}$$

Dark Matter $\overset{?}{=}$ "stable" elem. particle

$\Downarrow$

$\tau_{DM} \gtrsim T_U$

$\Downarrow$

global symmetry (Z_2)

$\Downarrow$

(scalar) $\bar{F}f$

explicit breaking

$\hookrightarrow$ − scalar

$\boxed{\text{The only good global symmetry is the broken (spont.) global symmetry}}$

PQ U(1) global (chiral)

$$q_L \to e^{i\alpha} q_L, \quad q_R \to q_R$$

$\Downarrow$

$$\mathcal{L}_Y^{SM} = y_d \, \overline{Q_L} \, \Phi_d \, d_R \; + \qquad (1)$$

$$+ y_u \, \overline{Q_L} \, \Phi_u^* \, u_R \qquad (2)$$

$$(1) \; \Phi_d \to e^{i\alpha} \Phi_d \, , \qquad (2) \; \Phi_u \to e^{-i\alpha} \Phi_u$$

$$\langle \Phi_{u,d} \rangle \neq 0 \; \boxed{\text{breaks chiral}} \, .$$

$$\Rightarrow \exists \; \text{NG} \; \therefore \; m_a = 0$$

$\boxed{\text{axion}}$ (axial symmetry)

$$M_{PQ} \gtrsim 10^{10} \, \text{GeV}$$

$$y_a \simeq \frac{m_e}{M_{PQ}} \leq 10^{-10}$$

$$m_a = m_\pi \frac{\Lambda_{QCD}}{M_{Pl}} \leq m_\pi \times 10^{-10}$$

$$\leq 10^{-2}\,\text{eV}$$

axion

DM $\Rightarrow \boxed{M_{Pl} \geq 10^{12}\,\text{GeV}}$

- $Q\ \phi_0 \neq 0 \Rightarrow m_\sigma = 0$

(global)

$$y_\sigma = \frac{m_f}{\phi_0}$$

$\boxed{\text{suppressed by } \frac{1}{\text{scale}}}$