LMU Neutrino Physics Course

Lecture IV

23/4/2021

Spring 2021

$\boxed{\text{Parity}}$

Laws of nature

$$\vec{F} = m\vec{a}$$

Symmetry: ROTATION

Galilean inv.: $\vec{x}' = \vec{x} + \vec{v}t$

$$t' = t$$

Parity: $\vec{x}' = -\vec{x}$

$$\vec{a} \to -\vec{a}, \quad \vec{F} \to -\vec{F}$$

$$\vec{F} = q(\vec{E} + \vec{v} \times \vec{B}) \qquad \vec{v} \to -\vec{v}$$

$$\Leftrightarrow \quad \vec{E} \to -\vec{E}, \quad \vec{B} \to \vec{B}$$

$$\Longleftrightarrow \quad \underbrace{\vec{A}_i \to -\vec{A}_i\,, \quad A_0 \to A_0}_{\text{Parity}}$$

$$A_\mu j^\mu \qquad (\vec{A}\cdot\vec{J})$$

$$j^\mu = \bar{\psi}\gamma^\mu \psi$$

$$\psi \xrightarrow{P} ? \quad \Rightarrow \quad \psi \to \eta_p \gamma^0 \psi$$

phase

$$\eta_p^2 = 1 \Rightarrow \eta_p = \pm 1$$

$$\boxed{\eta_p = +1}$$

$$\Downarrow$$

- $\psi = \begin{pmatrix} u_L \\ u_R \end{pmatrix} \quad \gamma^0 = \begin{pmatrix} 0 & 1 \\ 1 & 0 \end{pmatrix}$

$$\Rightarrow \quad \boxed{u_L \longleftrightarrow u_R}$$

- $\boxed{t' \to -t \qquad T}$ $\boxed{\begin{array}{c} \vec{a} \to \vec{a} \\ \vec{F} \to \vec{F} \end{array}}$

time reversal

$\varepsilon_{\cancel{T}} \simeq 10^{-3}$

- C : $p \to \bar{p}$ $\quad (q \to -q)$

$\vec{F} \to \vec{F}, \quad \vec{E} \to -\vec{E}, \vec{B} \to \vec{B}$

$j_\mu A^\mu \qquad j_\mu \xrightarrow{C} -j_\mu$

$$\boxed{\begin{array}{c} j^\mu \equiv \bar{\psi}\gamma^\mu Q e_\mu \psi \\ \Leftrightarrow \quad A^\mu \to -A^\mu \end{array}}$$

$$\psi \to \psi^c = \eta_c C \bar{\psi}^T = \eta_c i\gamma_2 \psi^*$$

(p) $(\bar{p})$

$$\eta_c = +1$$

$$(\eta_c^2 = 1)$$

$$\boxed{\cdot \quad P,\ C = \text{good}}$$

Fermi '34 $\quad P, C$ good

$$\boxed{u_L \leftrightarrow u_R}$$

$P: \quad u_L \leftarrow u_R \iff \psi \to \gamma^0 \psi$

$\psi_L \leftrightarrow \psi_R$

$C: \quad \psi \to C\bar{\psi}^T = i\gamma_2 \psi^*$

$$L \Rightarrow \begin{pmatrix} u_L \\ 0 \end{pmatrix} \to \begin{pmatrix} 0 \\ +i\sigma_2 u_L^* \end{pmatrix} \Leftarrow R$$

$$(\psi_L)^c \equiv (\psi^c)_R$$

$$\boxed{P,\ C: \quad L \leftrightarrow R}$$

weak int. $\Leftarrow$

P, C maximally broken

$CP: \quad \psi \to \gamma_0 C \bar{\psi}^T = \gamma_0 C \gamma_0 \psi^*$

$$C = i\gamma_2\gamma^0$$

$$\psi \longrightarrow +i\underbrace{\gamma_0\gamma_2}_{\text{diagonal}}\psi^*$$

$$= i\begin{pmatrix} 0 & 1 \\ 1 & 0 \end{pmatrix}\begin{pmatrix} 0 & \sigma_2 \\ -\sigma_2 & 0 \end{pmatrix}\psi = \begin{pmatrix} -i\sigma_2 & 0 \\ 0 & i\sigma_2 \end{pmatrix}\psi$$

$$\boxed{\begin{aligned} u_L &\rightarrow -i\sigma_2 u_L^* \\ u_R &\rightarrow i\sigma_2 u_R^* \end{aligned}}$$

$$\epsilon_{CP} = 10^{-3}$$

K-meson decays

$$\boxed{K^0 \leftrightarrow \overline{K^0}}$$ (neutral)

CP:
$$\left.\begin{aligned} K_{(+)} &= \frac{K_0 + \bar{K}_0}{\sqrt{2}} \\ K_{(-)} &= \frac{K_0 - \bar{K}_0}{\sqrt{2}} \end{aligned}\right\} \text{physical}$$

$$K_{(+)} \to \pi^+ + \pi^- \cancel{+ \pi^0}$$

$$K_{(-)} \to \pi^+ + \pi^- + \pi^0, \quad \pi^0 + \pi^0 + \pi^0$$

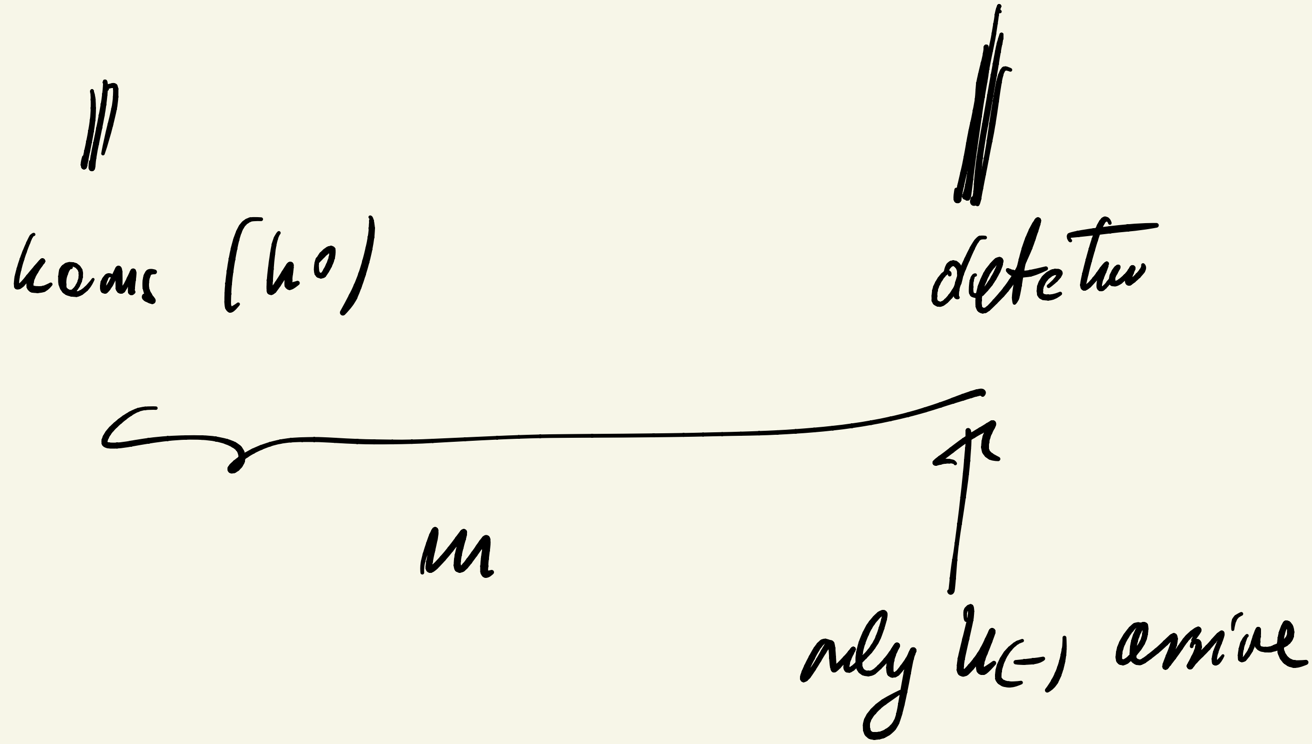
pions = P odd

π^0 = CP odd $\quad \pi^0 \xrightarrow{CP} -\pi^0$

$$m_{(+)} \cong m_{(-)} \Leftarrow \boxed{\frac{\Delta m_K}{\Sigma m_K} \simeq 10^{-14}}$$

$\tau_{(+)} \simeq 10^{-10}$ sec $\simeq$ cm $\leftarrow$

$\tau_{(-)} \simeq 10^{-8}$ sec $\simeq$ 1m



$$\boxed{K^{(-)} \longrightarrow 3\pi \quad (CP\ \text{odd})}$$

$1/1000 \quad K^{(-)} \longrightarrow 2\pi$

$\Rightarrow$ CP broken!

$$\Downarrow$$

$$\boxed{\epsilon_{CP} \simeq 10^{-3}}$$

$\boxed{CPT = \text{symmetry}}$ $\boxed{\text{Lorentz inv. QFT}}$

$$\left.\begin{array}{l} CP = \text{good} \\ T = \text{good} \end{array}\right\} \underline{\underline{10^{-3}}}$$

Back to neutrino

(Majorana) $\psi_L^T C \psi_L = \underbrace{u_L^T i\sigma_2 u_L}$

$u_L = \begin{pmatrix} \uparrow \\ \downarrow \end{pmatrix}$

$J = 0$

$= \uparrow\downarrow - \downarrow\uparrow$

SM : $SU(2)_L \times U_Y(1)$

$\Rightarrow \quad \underbrace{\nu_L^T C \nu_L}$ (allowed by Lorentz)

NOT by SM

forbidden by $SU(2)_L$

$\nu_L^T \; C \; \nu_L$ breaks $SU(2)_L$

$T_3: \quad \frac{1}{2} + \frac{1}{2} = 1$

$$\boxed{\begin{array}{c}\text{"Nature" is invariant } G_{SM} \\ \Updownarrow \\ \mathcal{L}_{SM} = \text{invariant under } G_{SM}\end{array}}$$

$W^+, \; W^-, \; A, \; Z$

$m_W \neq m_A$

$\Rightarrow$ SU(2) is broken

. Higgs mechanism : $\mathcal{L}$ symm

but vacuum (ground state)

$\Downarrow$

$\boxed{\text{SM predicts } m_\nu = 0}$

$\boxed{\text{but } m_\nu \neq 0 \Leftarrow \text{ oscillations}}$

Solar oscillations ⟸ Bruno Pontecorvo

'50s

'60s

Davis

Bahcall

$$\nu_e \longrightarrow \nu_\mu$$

$$\begin{pmatrix} \nu_e \\ e \end{pmatrix}_L \qquad \begin{pmatrix} \nu_\mu \\ \mu \end{pmatrix}_L$$

$$\begin{pmatrix} u \\ d \end{pmatrix}_L \qquad \begin{pmatrix} c \\ s \end{pmatrix}_L \cdots$$

10 ν_e a day – expect

5 ν_e a day – observe

Davis says

Bahcall computes

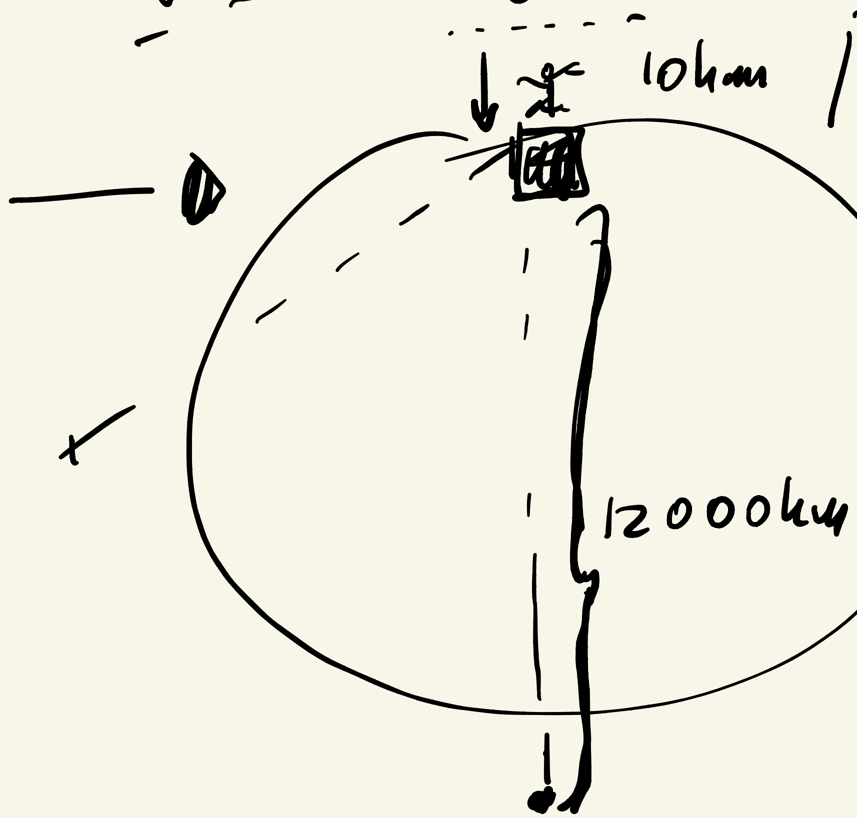
70's and 80's

Solar Neutrino Puzzle (SNP)

'90s -- '2000

- proton decay Kamiokande

atmospheric neutrinos



which flavor?

$\Downarrow$

muon neutrinos

$\boxed{\text{Oscillations}}$

$$\pi^- \longrightarrow \mu^- + \bar{\nu}_\mu$$

$$\longrightarrow e^- + \bar{\nu}_e \qquad (m_e \ll m_\mu)$$

$m_\pi = 140\,\text{MeV}$

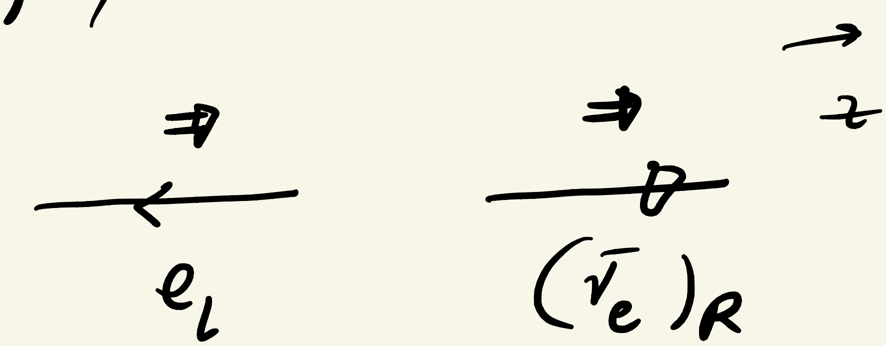
$m_\mu = 100\,\text{MeV}$ $\qquad m_e = 0.5\,\text{MeV}$

$\boxed{\text{Q. Why } \underline{\text{no}} \text{ electron mode?}}$

$\boxed{\text{A. helicity!}}$

$\mathcal{L} \ni \mu_L, \nu_{\mu L}$ (are LH)

π^- at rest:



$$h\, e_L = -\frac{1}{2}\, e_L \qquad h(\nu^c)_R = +\frac{1}{2}\, \nu^c_R$$

$$h \equiv \vec{S}\cdot\hat{p}$$

$$J_z^{in} = S_z^{in} + l_z^{in} = 0 + 0 = 0$$

$$J_z^{f} = S_z^{fin} + l_z^{fin} = +1 + 0 = 1$$

$$\Rightarrow \boxed{\begin{array}{l}\pi^- \not\to e + \bar{\nu}_e \\ \quad \text{when } m_e \to 0\end{array}}$$

$$\Downarrow$$

$$\Gamma(\pi^- \to e + \bar{\nu}_e) \propto m_e^2$$

$$\Downarrow$$

$$R(\mu/e) \equiv \frac{\Gamma(\pi^- \to \mu^- + \bar{\nu}_\mu)}{\Gamma(\pi^- \to e + \bar{\nu}_e)} \simeq \left(\frac{m_\mu}{m_e}\right)^2$$

$= 10^4$

$$\boxed{\text{atmosphere} = \bar{\nu}_\mu \quad (\underline{\text{no}}\ \bar{\nu}_e)}$$

$\Downarrow$

observation : # of ν_μ ($\bar{\nu}_\mu$)

depends strongly on L

(oscillating pattern)

$$\begin{pmatrix} \nu_e \\ e \end{pmatrix}_L \Leftrightarrow W^+_\mu \, \bar{\nu}_{e_L} \gamma^\mu e_L \quad \frac{g}{\sqrt{2}}$$

$$\begin{pmatrix} \nu_\mu \\ \mu \end{pmatrix}_L \Leftrightarrow W^+_\mu \, \bar{\nu}_{\mu_L} \gamma^\mu \mu_L \quad g/\sqrt{2}'$$

ν_τ, ν_e, ν_μ = neutrino (flavor eigenstates) flavors

Gribov, Pontecorvo '1968

ν_1, ν_2 = neutrino mass eigenstates

$$\nu_\mu = \nu_1 \cos\theta + \nu_2 \sin\theta$$

$$\nu_\tau = -\nu_1 \sin\theta + \nu_2 \cos\theta$$

"derive":

$$P(\nu_\mu \to \nu_\tau) \propto \overset{1}{\sin^2 2\theta} \; \sin^2 \frac{\Delta m^2}{\underset{4}{E}} L$$

$$\theta_{max} = 45^\circ$$

$$\Delta m \to 0 \Rightarrow P(\nu_\mu \to \nu_\tau) \to 0$$

$$\theta \to 0 \Rightarrow \quad -\text{''}- \quad \to 0$$

$$E \to \infty \Rightarrow \text{-}\|\text{-} \longrightarrow 0$$

$$\boxed{\begin{aligned} c &\equiv \cos\theta \\ s &\equiv \sin\theta \end{aligned}}$$

$$t=0: \; \nu_\mu = \cos\theta \, \nu_1 + \sin\theta \, \nu_2$$

$$\Rightarrow \; \nu_\mu(t) = \cos\theta \, e^{iE_1 t} \nu_1 + \sin\theta \, e^{iE_2 t} \nu_2$$

$$= e^{iE_1 t} \left[c \, \nu_1 + s \, e^{i\Delta E t} \nu_2 \right]$$

$$\langle \nu_\tau | \nu_\mu(t) \rangle = e^{iE_1 t} \left(-cs + cs \, e^{i\Delta E t} \right)$$

$$\Downarrow$$

$$P(\nu_\mu \to \nu_\tau) = \left| \langle \nu_\tau | \nu_\mu(t) \rangle \right|^2 =$$

$$= c^2 s^2 \left(1 - e^{-i\Delta E t} \right) \left(1 - e^{i\Delta E t} \right)$$

$$\Downarrow$$

$$= c^2 s^2 \left(2 - 2\cos \Delta E t \right)$$

$$1 - \cos\alpha = 2\sin^2 \alpha/2$$

$$\Downarrow$$

$$P(\nu_\mu \to \nu_\tau) = 4 c^2 s^2 \sin^2 \frac{\Delta E\, t}{2}$$

$$E = p + \frac{m^2}{2p} \qquad \left(E = \sqrt{p^2 + m^2}\right)$$

$$\Delta E = \frac{\Delta m^2}{2p} \simeq \frac{\Delta m^2}{2E}$$

$$\Downarrow$$

$$\boxed{P(\nu_\mu \to \nu_\tau) = \sin^2 2\theta \, \sin^2 \frac{\Delta m^2 L}{4E}}$$

$$\langle \sin^2\alpha \rangle = \langle \cos^2\alpha \rangle = 1/2$$

expected

$\uparrow \quad P(\nu_\mu \to \nu_\tau) = \tfrac{1}{2}\sin^2 2\theta$

<u>below</u> $\quad$ — " — $\quad = 1/2$ observed

$$\Rightarrow \boxed{\theta_A \simeq 45^\circ \equiv \theta_{23}}$$

$$\Rightarrow \boxed{\Delta m_A^2 \simeq 10^{-3}\,eV^2}$$

<u>SOLAR</u> $\quad$ (MSW) $\quad \boxed{\nu_e \to \nu_\mu}$

$$\Delta m_\odot^2 \simeq 10^{-5}\,eV^2$$

$$\theta_\odot \simeq 30^\circ \equiv \theta_{12}$$

$$\boxed{\theta_{13} \simeq 10^\circ}$$

leptonic mixings: 30°, 45°, 10°

$\neq$

quark mixings: 13°, 10^{-2}, 10^{-3}

oscillations $\Leftrightarrow$

mass difference Δm^2 be smaller

than δm^2 & QM uncertainty

$$\begin{pmatrix} \nu_e \\ e \end{pmatrix}_L = \begin{pmatrix} \cos\theta\, \nu_1 + \sin\theta\, \nu_2 \\ e \end{pmatrix}_L$$

$$\Leftrightarrow W_\mu^+ \left[\cos\theta\, \bar{\nu}_{1L} \gamma^\mu e_L + \sin\theta\, \bar{\nu}_{2L} \gamma^\mu e_L\right]$$

ν_1, ν_2 — usual weak int.

universe is "empty"

star = baryons

$$\frac{n_B}{n_\gamma} = 10^{-10}$$

$$\boxed{T_\gamma = E_\gamma \approx 10^{-4}\, eV}$$

photons + neutrinos = universe

$$\frac{400}{cm^3} + \frac{400}{cm^3}$$

challenge : observe cosmic neutrinos

$$\boxed{m_\nu \le 1\ \mathrm{eV}}$$

experiment!