Lecture III

Neutrino Physics Course

LMU

Spring 2021

V − A theory of weak interactions

Marshak, Sudarshan '57

Feynman, Gell-Mann '58

$\Downarrow$

$$\mathcal{H}^{(W)}_{eff} = \frac{G_F}{4\sqrt{2}} J^W_\mu \bar{J}^\mu_W \qquad \cancel{P} \qquad \cancel{C}$$

$$J^\mu_W = \bar{p}_L \gamma^\mu n_L + \bar{\nu}_L \gamma^\mu e_L$$

$$f_L \equiv L f$$

$$\exists \ e_R, \ p_R, \ n_R \ ; \ \nexists \ \nu_R$$

u u

d

p

u d

d

n

"V-A was the key"

$$\Downarrow$$

$$\boxed{J^\mu_W = \bar{u}_L \gamma^\mu d_L + \bar{\nu}_L \gamma^\mu e_L}$$

$$\Updownarrow$$

$$J^\mu_{em} = \bar{f} \gamma^\mu Q_{em} f = \varepsilon \bar{f} \gamma^\mu f$$

$$= \bar{f}_L \gamma^\mu Q_{em} f_L + \bar{f}_R \gamma^\mu Q_{em} f_R$$

$$\Leftrightarrow \boxed{Q_L = Q_R}$$

$$\Leftrightarrow \boxed{Q f^c = -f}$$

$$\boxed{(f^c)_R \equiv C \bar{f}_L^{\,T}}$$

- $f_L,\ f_R \iff f_L,\ (f^c)_L$
  $\qquad\qquad\qquad f_R,\ (f^c)_R$

- chirality = ?

(i) $m \neq 0 \Rightarrow$ no meaning

$$m \bar{f} f = m(\bar{f}_L f_R + \bar{f}_R f_L)$$

(iii) $m = 0 \Rightarrow$ helicity

<u>Proof:</u> $\not{p}\,\Psi = 0$

$$\gamma^i = \begin{pmatrix} 0 & \sigma^i \\ -\sigma^i & 0 \end{pmatrix}$$

$$\gamma^0 = \begin{pmatrix} 0 & I \\ I & 0 \end{pmatrix}$$

$$\Downarrow$$

$$\begin{pmatrix} 0 & E - \vec{p}\cdot\vec{\sigma} \\ E + \vec{p}\cdot\vec{\sigma} & 0 \end{pmatrix} \begin{pmatrix} u_L \\ u_R \end{pmatrix} = 0$$

$$\Downarrow$$

$$(E \mp \vec{p}\cdot\vec{\sigma})\, u_{R,L} = 0$$

$\vec{p} \equiv p\hat{p}$ $\qquad \Downarrow \quad E = |\vec{p}| \equiv p$

$$\hat{p}\cdot\vec{\sigma}\, u_{R,L} = \pm\, u_{R,L}$$

$$\vec{S} = \vec{\sigma}/2\,, \quad h \equiv \vec{S}\cdot\hat{p}$$

$$\Downarrow$$

$$\boxed{h\, u_L = -\tfrac{1}{2}\, u_L \qquad h\, u_R = +\tfrac{1}{2}\, u_R}$$

$$\psi_L = \begin{pmatrix} u_L \\ 0 \end{pmatrix} \notin LH$$

$$\Rightarrow h\, \psi_L = -\tfrac{1}{2}\, \psi_L \quad !$$

$$(\psi^c)_R \equiv C\, \bar{\psi}_L^T = C\gamma_0\, \psi_L^* = i\gamma_2\, \psi_L^*$$

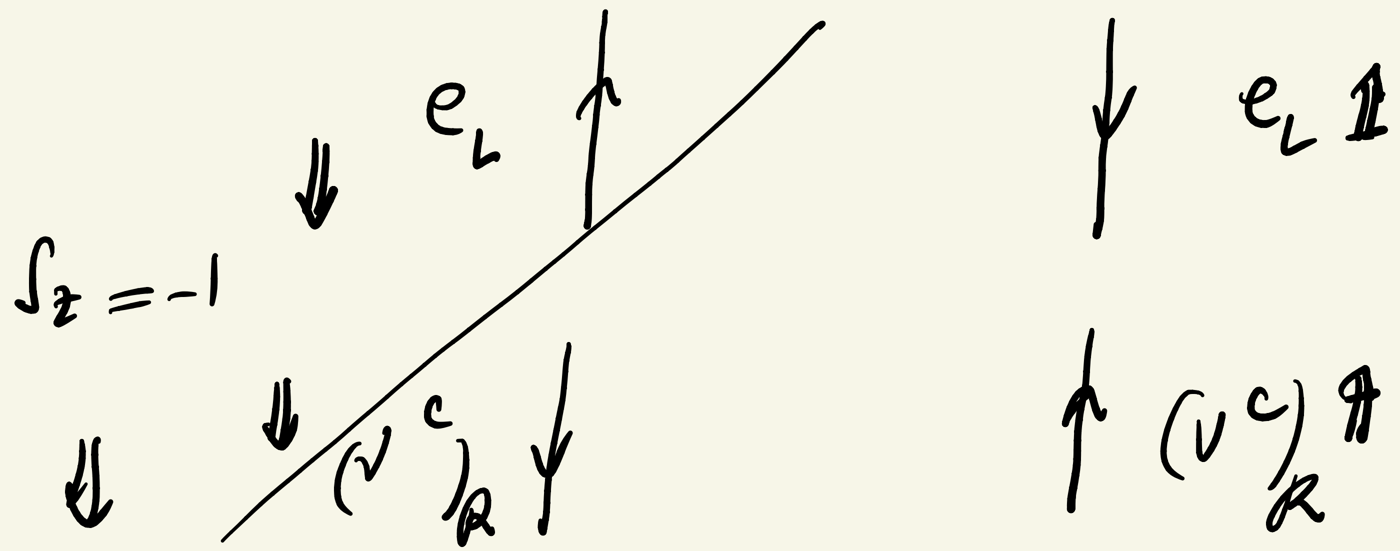
$$= \begin{pmatrix} 0 \\ -i\sigma_2 u_L^* \end{pmatrix}$$

$$\boxed{J_z^{\mathrm{in}} = +1}$$

$\cancel{P}$:

$$\boxed{m_e = 0}$$

$$\mathrm{Co} \to \mathrm{Ni} + e + \bar{\nu}\ (\equiv \nu^c)$$



$$J_z^f = \underset{\overset{\shortparallel}{0}}{L_z} + S_z = -1 \qquad\qquad J_z^f = +1$$

$$\boxed{\nu_L\,(e_L) \Longleftarrow \begin{array}{l}\text{Salam '57}\\ \text{Landau '57}\end{array}}$$

$$\Downarrow$$

$$\frac{g}{\sqrt{2}} W_\mu^+ J^\mu_W + h.c. \equiv \mathcal{L}_{int}^W$$

$$\Downarrow$$

$W_\mu^+, \; W_\mu^-, \; A_\mu$

(photon)

Minimal group?

'57 Schwinger

$\downarrow$

gauge theory $\Downarrow$

'60 Glashow

$G_{em} = \underline{U(1)_{em}}$: $\quad \partial_\mu \to D_\mu \equiv \partial_\mu - ie\, Q A_\mu$

$$f \to e^{i\theta(x)\, Q_{em}} f \qquad \boxed{A = \text{neutral}}$$

$$D_\mu f \to e^{i\theta(x)\, Q_{em}} D_\mu f$$

$$\Longleftrightarrow \quad A_\mu \to A_\mu - \frac{1}{e}\partial_\mu \theta(x)$$

$$\Downarrow$$

$$\mathcal{L}_0 = i\bar{f}\gamma^\mu\partial_\mu f - m\bar{f}f$$

$$\to i\bar{f}\gamma^\mu D_\mu f - m\bar{f}f$$
$$- \frac{1}{4}F_{\mu\nu}F^{\mu\nu}$$

<u>electro</u> - <u>weak</u> theory

(em + weak)

minimal

$$G_{ew} = SU(2)\ (g) \qquad (?)$$

$$= \underset{q_1}{U(1)} \times \cancel{\underset{q_2}{U(1)} \times \underset{q_3}{U(1)}}$$

$U(1)$ $\quad e\, A_\mu J^\mu_{em}$

$\Downarrow$

$Q_{em} A_\mu = 0$

but $W^\pm$!!

$\Downarrow$

$$\boxed{G^{ew} = SU(2)}$$

Yang - Mills
Shaw '54

$\Leftarrow$

$SU(2)$ $\quad D_\mu = \partial_\mu - i g\, T_a A_\mu^a$

$\Updownarrow$

$$f = \begin{pmatrix} u \\ d \end{pmatrix} \qquad \left(\begin{pmatrix} u \\ d \end{pmatrix}, \begin{pmatrix} \nu \\ e \end{pmatrix} \right)$$

$$f \to U f \qquad U^\dagger U = 1 = U U^\dagger$$

$$\det U = 1$$

$$U = e^{iH} \qquad U U^\dagger = 1 \Rightarrow H^\dagger = H$$

$$\det U = 1 \Rightarrow \mathrm{Tr}\, H = 0$$

$$H = \theta_a T_a \qquad [T_a, T_a] = i \varepsilon_{abc} T_c$$

$$\Downarrow \qquad \boxed{\mathrm{Tr}\, T_a T_b = \frac{1}{2} \delta_{ab}}$$

$$T_a \equiv \sigma_a/2$$

$$D_\mu f \to U D_\mu f$$

$$\Downarrow \quad \boxed{\text{check}}$$

$$T_a A_\mu^a \to U T_a A_\mu^a U^+ \quad \text{(adjoint)}$$

$$\equiv A_\mu \qquad + \frac{i}{g} (\partial_\mu U) U^+ \quad \text{(gauge)}$$

$$\Updownarrow$$

$$Q_{em} A_\mu \to U_1 Q_{em} A_\mu U_1^+ + \frac{i}{e} (\partial_\mu U_1) U_1^+$$

$$U_1 \equiv e^{i \theta Q_{em}}$$

$$\Longleftrightarrow \quad A_\mu \to A_\mu - \frac{1}{e} \partial_\mu \theta(x)$$

$$T_a F_{\mu\nu}^a \to U T_a F_{\mu\nu}^a U^+ \quad SU(2)$$

$$\Updownarrow \qquad (*)$$

$$Q_{em}\, F_{\mu\nu} \to U_1\, Q_{em}\, F_{\mu\nu}\, U_1^{+}$$

$$\Updownarrow \qquad\qquad U(1)$$

$$F_{\mu\nu} \to F_{\mu\nu}$$

$$\mathcal{L}_M = -\frac{1}{4} F_{\mu\nu} F^{\mu\nu} \qquad \text{(Maxwell)}$$

$$\mathcal{L}_{YM} = -\frac{1}{4} F_{\mu\nu}^{a} F^{\mu\nu a} \qquad \text{(Yang-Mills)}$$

$$= -\frac{1}{2} \mathrm{Tr}\, \mathbb{F}_{\mu\nu} \mathbb{F}^{\mu\nu}$$

$$\mathbb{F}_{\mu\nu} \equiv F_{\mu\nu}^{a} T_a$$

$$\Downarrow$$

$$(*) \quad F_{\mu\nu}^{a} = \partial_\mu A_\nu^{a} - \partial_\nu A_\mu^{a} + g\, \epsilon_{abc} A_\mu^{b} A_\nu^{c}$$

SU(2) gauge theory

= ew theory

$$\mathcal{L}_{SU(2)} = i\bar{f}\gamma^\mu D_\mu f + \cdots$$

$$= i\bar{f}\gamma^\mu\partial_\mu f + \underbrace{g\bar{f}\gamma^\mu T_a A_\mu^a f}_{\mathcal{L}_{int} = \text{universal interaction}}$$

$$\Rightarrow \mathcal{L}_{int} = \frac{g}{2}\bar{f}\gamma^\mu \sigma_a A_\mu^a f$$

$$= \frac{g}{2}\bar{f}\gamma^\mu \begin{pmatrix} A_\mu^3 & A_\mu^1 - iA_\mu^2 \\ A_\mu^1 + iA_\mu^2 & -A_\mu^3 \end{pmatrix} f$$

$$f = \begin{pmatrix} u \\ d \end{pmatrix}$$

$$\Downarrow$$

$$\mathcal{L}_{int} = \frac{g}{2}\left(\bar{u}\gamma^\mu u - \bar{d}\gamma^\mu d\right) A_\mu^3 +$$

$$+ \frac{g}{2}\left[\bar{u}\gamma^\mu (A_\mu^1 - i A_\mu^2) d + h.c.\right]$$

$$\Downarrow$$

$$\boxed{W_\mu^{\pm} = \frac{A_\mu^1 \mp i A_\mu^2}{\sqrt{2}}}$$

$$\Downarrow$$

$$\mathcal{L}_{int} = \frac{g}{\sqrt{2}}\left(\bar{u}\gamma^\mu d\, W_\mu^+ + h.c.\right) \quad (1)$$

$$+ g\, \bar{f}\gamma^\mu T_3 f\, A_\mu^3 \quad (2)$$

$$A_\mu^3 = A_\mu$$

(photon)

$$\Downarrow$$

$$\boxed{Q_{em} = T_3}$$

prediction!

$$T_a A_\mu^a \to U T_a A_\mu^a U^+ (+\text{ gauge})$$

$$= T_a A_\mu^a + i\theta_b [T_a, T_b] A_\mu^a$$

$$(U = 1 + i\theta_b T_b)$$

Prove

$$\Rightarrow \boxed{W_\mu^+ \text{ has } Q_{em} = +1}$$

$$[T_3, T_1] = i T_2$$

$$[T_3, T_2] = -i T_1$$

$$\Rightarrow [T_3, T_1 + iT_2] = +(T_1 + iT_2)$$

$$\Downarrow$$

$$\boxed{A_1 \pm i A_2 \leftarrow Q_{em} = \pm 1}$$

$\Downarrow$

$W^\pm$: (i) correct coupling to $\partial_\mu W$

(ii) correct charge

Essence

$$\mathcal{L}_{YM} = \underbrace{i \bar{f} \gamma^\mu D_\mu f}_{\text{invariant under } SU(2)} + \cdots$$

$$D_\mu \equiv \partial_\mu - i g T_a A_\mu^a$$

$$\bar{f}\gamma^\mu D_\mu f \to \bar{f} U^\dagger \gamma^\mu (\partial_\mu - i g A'^a_\mu T_a) U f$$

$$U \equiv e^{i\theta_a(x) T_a}$$

$$\Rightarrow \quad T_a A^{a'}_\mu = U T_a A^a_\mu U^\dagger + \frac{i}{g}(\partial_\mu U) U^\dagger$$

$$\Downarrow$$

$$A_\mu = A_\mu^3$$

$$W^\pm = \frac{A_1 \mp A_2}{\sqrt{2}}$$

predictions

$$\boxed{Q_{em} = T_3}$$

charge is quantized !!!

$$Q_e = 3\,Q_d\,, \quad Q_u = -2\,Q_d$$

$$Q_p = -3\,Q_d$$

$$Q_{\Delta^{++}} = -6\,Q_d$$

$$\left(Q_p + Q_e = 0 \right)$$

$$\leq 10^{-40}\;!$$

$$\boxed{Q_{all} = n\,Q_d} \qquad n = \pm 1, \pm 2, \ldots$$

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beautiful prediction: killed

by **ugly** facts of nature!

(?)

$$Q_u^{em} = 2/3 \qquad Q_d^{em} = -1/3 \qquad \begin{pmatrix} u \\ d \end{pmatrix} \text{ quarks}$$

$$Q_\nu^{em} = 0 \qquad Q_e^{em} = -1 \qquad \begin{pmatrix} \nu \\ e \end{pmatrix} \text{ leptons}$$

u, d states

$$(Q \equiv Q_{em})$$

<u>SU(2):</u> $\quad Q_{em} = \pm 1/2$

Noble failure!

charge quantisation ✓

~~charges wrong~~

$\boxed{\text{Failures:}}$

(i) $A_\mu^3 \neq$ photon

$\Uparrow$

$Q_{em} \neq T_3$ $\not{\sigma}$

(ii) $\frac{g}{\sqrt{2}} W^+ \; \bar{u}_L \gamma^\mu d_L + h.c.$ (a)

$e A_\mu (\bar{f}_L \gamma^\mu Q f_L + \bar{f}_R \gamma^\mu Q f_R)$ (b)

$\boxed{\underline{SU(2)}! \quad g = e}$ (iii)

Q. weak = much weaker than em

$\Rightarrow g \ll e$

A. weak int. > em

$$g = \frac{e}{\sin\theta_W} \quad (g > e)$$

$$\frac{G_F}{4\sqrt{2}} = \frac{g^2}{8 M_W^2} = \frac{e^2}{8\,(M_W \sin\theta_W)^2}$$

$$\theta_W \simeq 30^\circ \quad (\text{exp.})$$

$$M_W = 80\ \text{GeV}$$

$$\Rightarrow \quad G_F \ll \frac{1}{q^2} \quad \underline{\underline{(q \ll \text{GeV})}}$$

β decay, ... experiments

weak : $M_W \gg q$ ('50s)

$\underline{SU(2)} \Rightarrow M_W = 40\,\text{GeV}$!!

$\underline{SU(2)_L}$

$W^\pm_\mu \leftarrow LH \Rightarrow \underbrace{\begin{pmatrix} u \\ d \end{pmatrix}_L , \begin{pmatrix} \nu \\ e \end{pmatrix}_L}$

only $f_L \leftrightarrow W_\mu$

$\Rightarrow \quad u_R, \; d_R, \; e_R \; = \;$ singlets

$$\boxed{\begin{aligned} T_a f_R &= 0 \\ T_a f_L &= \frac{\sigma_a}{2} f_L \end{aligned}}$$

$$\boxed{\Rightarrow \quad A_\mu^3 \neq \text{photon}}$$

$$\Rightarrow \quad Q_{em}\, f_R = 0$$

$$\Downarrow$$

$$Q_{em} \neq T_3$$

quantised

$$\Downarrow$$

$$\boxed{Q_{em} = T_3 + \frac{Y}{2}} = \boxed{\text{hypercharge}}$$

$$Y = 2\left[Q_{em} - T_3\right] \qquad \text{'1961}$$

$$\Leftrightarrow \quad G_{ew}^{\min} = \underset{(A_\mu^a)}{SU(2)_L} \times \boxed{U(1)_Y} \quad (B_\mu)$$

$$S.M. =$$

the rest is (almost) history

Standard Model

(+ Higgs boson)

$$\Downarrow \quad [Y, T_a] = 0$$

$$\begin{aligned} Y\, u_L &= Y\, d_L \\ Y\, e_L &= Y\, \nu_L \end{aligned} \Rightarrow Y \neq Q_{em}$$

$$\Downarrow$$

$$A_\mu = \sin\theta\, A^3_\mu + \cos\theta\, B_\mu$$

$$(Z \perp A)$$

$$Z_\mu = \cos\theta\, A^3_\mu - \sin\theta\, B_\mu$$

Crucial prediction:

$$\boxed{m_\nu = 0}$$

$\underline{\text{SM}}$ $\begin{pmatrix} u \\ d \end{pmatrix}_L$ $u_R,\ d_R$

$\begin{pmatrix} \boxed{\nu} \\ e \end{pmatrix}_L$ e_R $\cancel{\nu_R}$

$\Downarrow$

- $\cancel{\bar{\nu}_L \nu_R}$ not there

- $\nu_L^T C \nu_L = u_L^T i\sigma_2 u_L$

$\nu_L = \begin{pmatrix} u_L \\ 0 \end{pmatrix}$ $u_L = \begin{pmatrix} \uparrow \\ \downarrow \end{pmatrix}$

$u_L^T i\sigma_2 u_L = \uparrow\downarrow - \downarrow\uparrow \ (s=0)$

$\boxed{\text{Lorentz invariant}}$

$$\nu_L^T C \nu_L \Leftarrow \text{breaks } SU(2)$$

$$T_3: \ \frac{1}{2} + \frac{1}{2} - 1 \neq 0$$

$$\mathcal{L}_{SU(2)} = i\bar{f}\gamma^\mu D_\mu f = i\bar{f}_L \gamma^\mu D_\mu f_L +$$

$$+ i\bar{f}_R \gamma^\mu D_\mu f_R$$

$$W^\pm \equiv \frac{A_1 \mp i A_2}{\sqrt{2}} \qquad \Downarrow \quad \boxed{\text{exp}}$$

$$\frac{g}{\sqrt{2}} W_\mu^+ \left(\bar{u}_L \gamma^\mu d_L + 0 \right)$$

$$T_a f_L = \frac{\sigma_a}{2} f_L$$

$$T_a f_R = f_R$$

def. of theory :

$$D_\mu = \partial_\mu - i g T_a A_\mu^a$$

$$T_a^L = \, ? \qquad \underline{\cancel{P}}$$

$$T_a^R = \, ?$$

Lee, Yang '56

- $\cancel{P}$ in nature

- at fundamental level : P good!