Fermi theory of

β decay

$\Uparrow$

(Effective theories of new phenomena)

Gravity

effective gr. distance

Newton : $(r \gg r_g)$

Action-at-distance

$$\boxed{V_{gr} = -G_N \frac{Mm}{r} = -\frac{1}{2}\frac{r_g}{r}m}$$

$$\hbar = c = 1 \qquad [t] = [L]$$

$$[p, q] = i \Rightarrow [m][L] = 1$$

$$\Downarrow$$

$$\boxed{G_N = \frac{1}{M_p^2}} \qquad \boxed{M_p = 10^{19}\ \text{GeV}}$$

Planck scale

$$V_{gr} = \Phi(r)\, m$$

$$\Phi(r) = - G_N \frac{M}{r} = -\frac{1}{2}\frac{r_g}{r}$$

$$r_g = 2\, G_N M$$

$$r_g = 1\ \text{km for Sun}$$

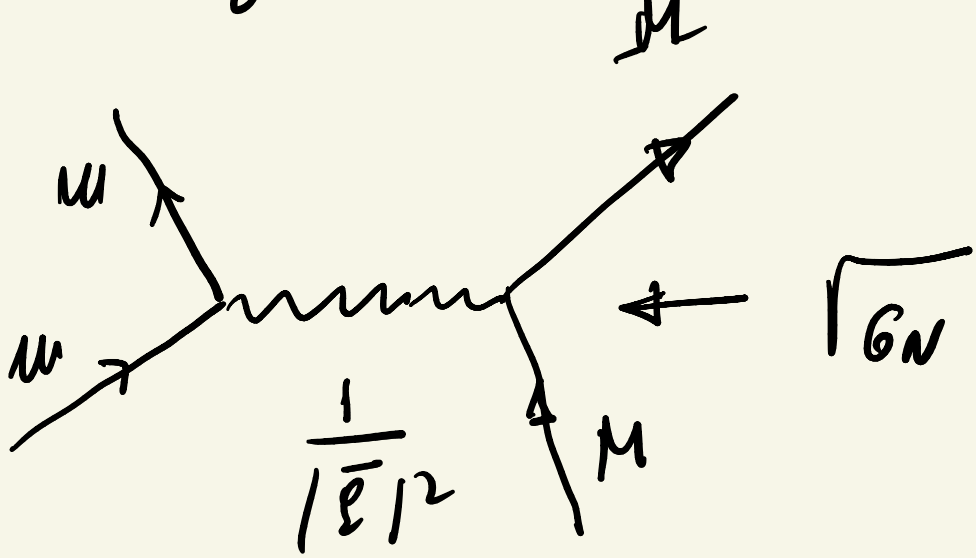
Newton: " Only a fool would believe in action at distance! There must be a messenger – but I have no means of describing it!

$$\Delta \phi = -4\pi G_N \delta^3(\vec{r})$$

$$\Delta = \frac{\partial^2}{\partial x^2} + \frac{\partial^2}{\partial y^2} + \frac{\partial^2}{\partial z^2}$$

$$\phi(r) = -\frac{1}{2} r_g \int d^3q \, \frac{e^{i\vec{q}\cdot\vec{r}}}{|\vec{q}|^2}$$

Feynman



effective theory : $r \gg r_0$

$\Downarrow$ (small momentum)

($q \ll q_0$)

$r \leq r_0 \Leftrightarrow q > q_0$

UV completion

- $r_g = 10^5 \text{cm}$ (Sun)

Mercury (perihelion)

$r \simeq 10^{13} \text{cm}$

$\Downarrow$

$$\delta \simeq \frac{\nu_g}{\nu} \simeq 10^{-8}$$ (~1915-1916)

Einstein

$$\Phi \longleftrightarrow g_{\mu\nu} \text{ (graviton)}$$

$$m \longleftrightarrow T_{\mu\nu} = \text{energy-momentum}$$

$$(m \longleftrightarrow E)$$

- $\Downarrow$ fast moving bodies (light)
- strong field

$$V_{em}(r) = \frac{\alpha\, q_1 q_2}{r} \qquad \alpha \equiv \text{"}\frac{e^2}{4\pi}\text{"} = \text{"}\frac{1}{137}\text{"}$$

$$V_{gr}(r) = -\,G_N \frac{M m}{r}$$

Q. Why gravity matters? Why matter gravitates?

$$M_\odot \simeq 10^{60}\,\text{GeV}$$

$$m_{earth} \simeq 10^{50}\,\text{GeV}$$

$$q_\odot = q_{earth} = 0$$

A1. Celestial objects are neutral

$$\left(Q_e + Q_e \leq 10^{-40},\ Q_n \leq 10^{-40}\right)$$

A2. Very massive

<u>proton</u> $V_{em} \simeq \alpha/r \simeq 10^{-2}/r$

$|V_{gr}| \simeq 10^{-38}/r$

$|V_{gr}| \lesssim 10^{-36}\,|V_{em}|$

$\boxed{\text{gravity is irrelevant}}$

$$\alpha_{gr} \simeq \frac{E^2}{M_p^2} = \frac{L_p^2}{r^2}$$

$L_p = M_p^{-1} \simeq 10^{-33}\,\text{cm}$

$rE = O(1)$

$\boxed{\cdot\ r = \text{cm} \Rightarrow \alpha_{gr} \simeq 10^{-66}}$

$$\boxed{\cdot\ r = 10^{13}\,\mathrm{cm} \Rightarrow \alpha_{gr} \simeq 10^{-92}}$$

$\cdot$ LHC : $E \simeq 10\,\mathrm{TeV} \simeq 10^{4}\,\mathrm{GeV}$

$$\boxed{\alpha_{gr} = 10^{-30}}$$

$\cdot$ Fermi 1934

theory of weak interaction

$$n \to p + e + \bar{\nu}_e$$

$$q \longrightarrow J^{\mu}\ (q,\ J_i)$$

$$V_{em} = \frac{\alpha\, q_1 q_2}{r} = \alpha q_1 q_2 \int e^{i\vec{q}\cdot\vec{r}} / |\vec{q}|^2$$

$$\mathcal{H}_{eff}^{em} = \alpha\, J_\mu^{em} \bar{J}^\mu_{em} \frac{1}{q^2}$$

$$q^2 \equiv q_0^2 - \vec{q}^{\,2}$$

$$\mathcal{L} = e A_\mu J^\mu_{em} - \frac{1}{4} F_{\mu\nu} F^{\mu\nu} \quad (1)$$

$$\partial^\mu j_\mu^{em} = 0 \quad (2)$$

$$A_\mu \to A_\mu - \frac{1}{e}\partial_\mu \theta(x) \quad (3)$$

$\partial^\mu A_\mu = 0$ gauge

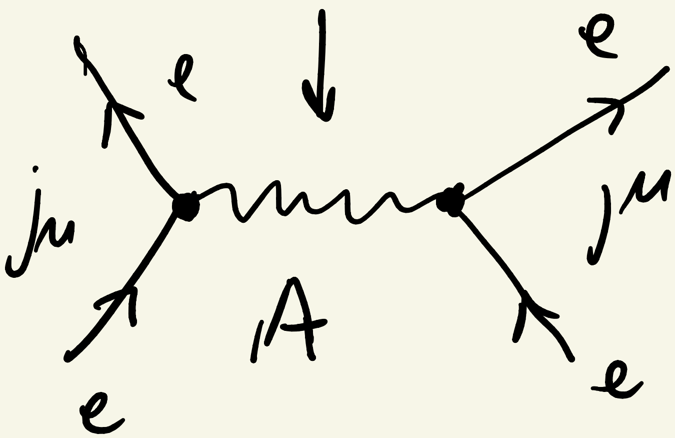
$$\Rightarrow \quad \Box A_\mu = e\, j^\mu_{em}$$

$$\Downarrow \qquad (M_A = 0)$$

$$A_\mu = \frac{e}{\Box}\, j^\mu_{em}$$

$$\to \underbrace{e\, \frac{1}{q^2}\, j^\mu_{em}(q)}_{\text{momentum space}}$$

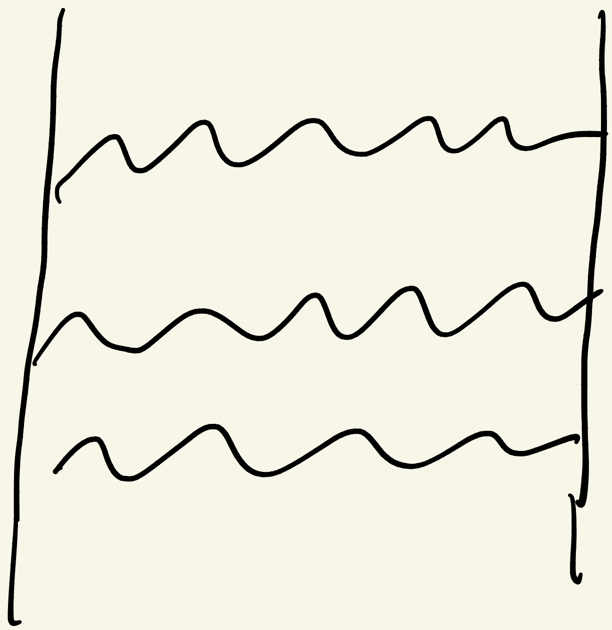
$(M_A = 0)$ $\frac{1}{q^2}$



pert. theory in $\alpha = \frac{e^2}{4\pi}$

$$j_\mu = \bar{\Psi}\, \gamma_\mu\, Q_{em}\, \Psi$$

$$Q_{em}\, \Psi = q\, \Psi$$



$(\psi \equiv f)$

$$\mathcal{L}_{QED} = i\,\bar{\psi}\gamma^\mu D_\mu \psi - m\bar{\psi}\psi$$
$$- \frac{1}{4}F_{\mu\nu}F^{\mu\nu}$$

$(Q \equiv Q_{em})$

$$D_\mu = \partial_\mu - ie\,A_\mu\,Q_{em}$$

$$\Downarrow$$

(1), (2), (3) equation

$$\Updownarrow$$

$$\psi \to e^{i\,\theta(x)\,Q_{em}}\,\psi$$

$$\theta(x) \equiv \theta = \text{const.} \Rightarrow \partial_\mu j^\mu_{em} = 0$$

$$j^\mu_{em} = \bar{\psi}\gamma^\mu Q_{em}\psi$$

Noether

$\uparrow$

Dirac equation

$$n \to p \qquad e \to \nu$$

$$e\,A_\mu\,j^\mu_{em} \longleftrightarrow g\,W^+_\mu\,j^\mu_{\bar{w}}$$

$$j^\mu_w = \bar{p}\,\gamma^\mu n + \bar{\nu}\,\gamma^\mu e$$

messenger $W^\pm$ = charged

$M_W \neq 0$

<u>not</u> a long range force

$(r \ll 10^{-14}\,\mathrm{cm})$

limited, effective theory

(Newton)

$\Downarrow$

$$\mathcal{H}_{\text{eff}}^{(w)} = \frac{1}{\Lambda_F^2} J_w^{\mu} \, J_{\mu}^{w} \qquad (q \simeq \text{MeV})$$

strength of interaction

dimensional analysis

$$\boxed{\Lambda_F \simeq 300\ \text{GeV}} \qquad (\sim M_W \simeq 100\,\text{GeV})$$

$$\mathcal{H}^F_{eff} = \frac{G_F}{\sqrt{2}} J^\mu_W \bar{J}^W_\mu$$

$$G_F \simeq 10^{-5}\,\mathrm{GeV}^{-2}$$

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lazy person's approach:

$$S = \int d^4x\, \mathcal{L} \qquad [S] = 0$$

no dimension $\Rightarrow$

$$d[\mathcal{L}] = 4 \text{ in } [m]$$

$$[m][L] = 1$$

$$\boxed{\begin{aligned} F_{\mu\nu} &= \partial_\mu A_\nu - \partial_\nu A_\mu \\ \Rightarrow\ d(A) &= 1 \text{ in } [m] \end{aligned}}$$

$\Downarrow$

$$d(j_\mu) = 3 \text{ in } [m]$$

$\boxed{\text{\underline{digression:}} \quad g_{\mu\nu} = \text{messenger ??}}$

<u>em:</u> $\quad A_\mu j^\mu_{em}$

<u>gravity:</u> (a) $\Phi(\text{mass}) = \Phi\,\rho(\text{density})$

Newtonian gravity

(b) Einsteinian

source $= T_{\mu\nu}$

$$\boxed{A_\mu j^\nu_{em} \Leftrightarrow T_{\mu\nu}\ g^{\mu\nu}}$$

M messenger

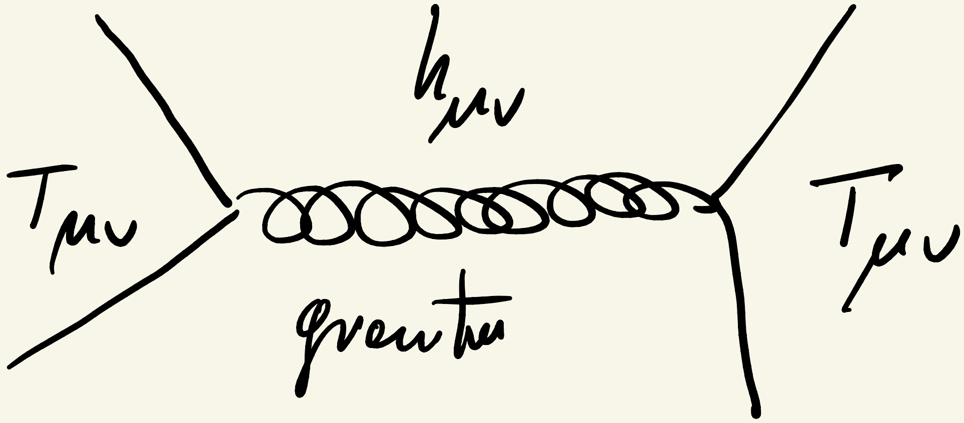
$$g^{\mu\nu} = \eta^{\mu\nu} + h^{\mu\nu}/M_{pe}$$

Lorentz $= \mathrm{diag}(1, -1, -1, -1)$

$$\boxed{R_{\mu\nu} - \frac{1}{2} g_{\mu\nu} R = 8\pi G_N T_{\mu\nu}}$$

curvature $\quad R_{\mu} = f(g_{\mu\nu})$

$$R = g_{\mu\nu} R^{\mu\nu}$$



$$\boxed{\Box h_{\mu\nu} \cong G_N T_{\mu\nu}}$$

(linearized version)

BH

$$\boxed{r_{sun}\,(BH) = 1\,\text{km}}$$

S_{BH}, T_{BH} $\qquad S_{BH} \propto \text{Area}$

Prof. Dvali

(QM of many gravitons)

$\boxed{\text{Newton – Fermi approach}}$

$$H^F_{eff} = G_F \, J^\mu_w \, \bar{J}^w_\mu$$

$$(S \cdot S , \; T^w_{\mu\nu} T^{\mu\nu}_w)$$

Fermi: $J^\mu_w = \bar{\nu} \gamma^\mu e$

Grav: $\bar{\nu} e$ (messenger = scalar)

$\bar{\nu}\gamma^\mu\gamma^\nu e$ (tensor = message)

$\Downarrow$ 1934 – 1957

Untangling Hell

$\boxed{1956}$

Cowan, Reines

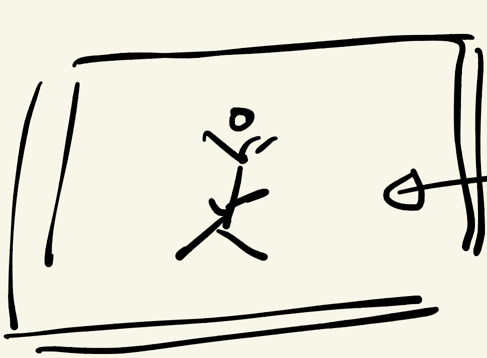
(i) • ν = discovered

(ii) • $\not{P}$ = maximal

"good old times"

$\boxed{\text{reactor}}$

(i)



one person

$V = 10^5\ cm^3$

$$\bar{\nu} + p \to \bar{e} + n$$

water

$$\Phi = 10^{13} / \mathrm{cm}^2\,\mathrm{sec}$$

$$N(\text{events}) = \sigma \cdot \Phi \cdot (\text{density}) \cdot V$$

$$(\text{density}) = 10^{24}/\mathrm{cm}^3 \qquad V = 10^5\,\mathrm{cm}^3$$

$$\sigma_W = G_F^2\, q^2$$

lazy person

$$q = \mathrm{MeV} \qquad G_F = 10^{-5}\,\mathrm{GeV}^{-2}$$

$$\sigma_W(\mathrm{MeV}) \simeq 10^{-10} \cdot 10^{-6}\,\mathrm{GeV}^{-2}$$

$$\mathrm{GeV}^{-1} \simeq 10^{-14}\,\mathrm{cm} \quad (\sim \pi\, 10^{-14})$$

$$\simeq 10^{-44}\,\mathrm{cm}^2$$

$$\sigma_{em}(\mathrm{MeV}) = \frac{\alpha}{q^2} = (10^{-6}\,\mathrm{GeV}^{-2} \times 10^{-2}) = 10^{-22}\,\mathrm{cm}$$

$$\boxed{\frac{\sigma_w}{\sigma_{em}}(\mathrm{MeV}) \simeq 10^{-22}}$$

$$\Downarrow$$

$$\boxed{N \simeq 30/\mathrm{hour}}$$

(ii) $\boxed{\text{Ø experiment}}$

$$\downarrow$$

$$Co^{60} \to Ni^{61} + e + \bar{\nu}_e \quad (m_e = 0)$$

$\Uparrow$ (z axis)

$\cancel{\uparrow e}$ $\quad \downarrow \; (e \Rightarrow e_L)$

$\cancel{\downarrow \bar{\nu}}$ $\quad \uparrow \; (\bar{\nu} = (\bar{\nu})_R)$

Maximal asymmetry

$\Downarrow$ Marshak, Sudarshan

Feynman, Gell-Mann '57 - '58

$\Downarrow$ "V−A theory"

$$\boxed{J^\mu_W = \bar{\nu}_L \gamma^\mu e_L + \bar{\nu}_{\mu L} \gamma^\mu \mu_L}$$

$\Uparrow$

$$\mathcal{L}^{F}_{eff} = \frac{4G_F}{\sqrt{2}} J^\mu_w J_\mu^w$$

$$\psi_L = \begin{pmatrix} u \\ 0 \end{pmatrix}$$

$$\chi_R = \begin{pmatrix} 0 \\ v \end{pmatrix}$$

$$\Downarrow$$

$$(m=0) \quad h\,\psi_L = -\tfrac{1}{2}\psi_L$$

$$h \equiv \vec{S}\cdot\hat{p}$$

$$h\,\chi_R = +\tfrac{1}{2}\chi_R$$

$$\bar{f} \longrightarrow f^c \equiv C\bar{f}^{\,T} \quad (C \equiv i\gamma_2\gamma_0)$$

$$f \to \Lambda f \Rightarrow f^c \to \Lambda f^c$$

Weinberg

"V-A was the key"