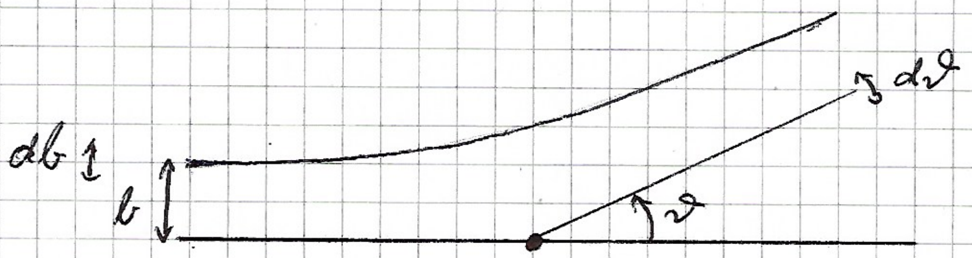
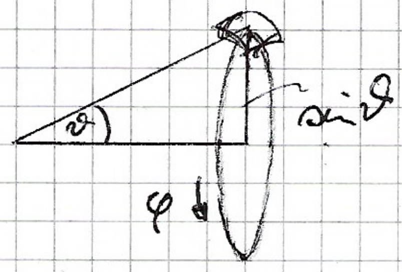
Wirkungsquerschnitt σ





Raumwinkel $d^2\Omega = \sin\vartheta \, d\vartheta \, d\varphi$

$\hat{=}$ Flächenelement auf Einheitskugel

Isotropie $\rightarrow$ $d\Omega = 2\pi \sin\vartheta \, d\vartheta$

voller Raumwinkel $\Omega = \int_0^\pi 2\pi \sin\vartheta \, d\vartheta = -2\pi \cos\vartheta \Big|_0^\pi = 4\pi$

j: Zahl einlaufender Teilchen pro Zeit und Fläche (Intensität)

dN: Zahl in $d\Omega$ gestreuter Teilchen pro Zeit

$\rightarrow$ $d\sigma := \frac{dN}{j}$ (differentieller) Wirkungsquerschnitt

[S-System: $\vartheta = \chi$]

Annahme $b = b(\chi)$ monoton: $[b, b+db] \rightarrow [\chi, \chi + d\chi]$

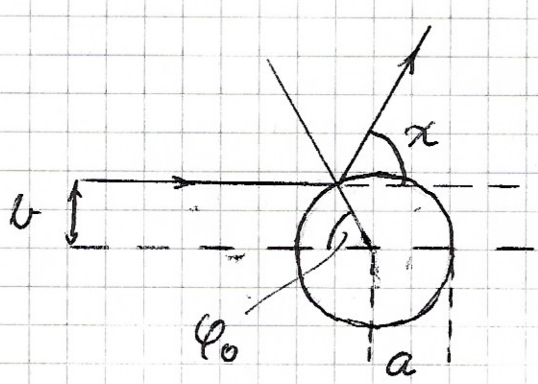
$$\Rightarrow \quad dN = 2\pi b \, db \, j$$

$$\Rightarrow \quad d\sigma = 2\pi b \, db = 2\pi \, b(\chi) \left| \frac{db(\chi)}{d\chi} \right| d\chi =$$

$$= \frac{b(\chi)}{\sin\chi} \left| \frac{db(\chi)}{d\chi} \right| d\Omega$$

Beispiel: Streuung an harter Kugel

Zentralpotential $\qquad U(r) = \begin{cases} \infty & r < a \\ 0 & r > a \end{cases}$



$$b = a \sin\varphi_0 =$$

$$= a \sin\frac{\pi - \chi}{2} = a \cos\frac{\chi}{2}$$

$$\frac{db}{d\chi} = -\frac{a}{2}\sin\frac{\chi}{2}$$

$$\Rightarrow d\sigma = \frac{b(\chi)}{\sin\chi}\left|\frac{db}{d\chi}\right| d\Omega = \frac{a\cos\frac{\chi}{2}}{\sin\chi}\,\frac{a}{2}\sin\frac{\chi}{2}\,d\Omega = \frac{a^2}{4}\,d\Omega$$

differentieller W.q. $\quad \frac{d\sigma}{d\Omega} = \frac{a^2}{4} \quad$ isotrop im S-System

totaler W.q. $\quad \sigma = \int \frac{d\sigma}{d\Omega}\,d\Omega = \frac{a^2}{4}\,4\pi = \pi a^2$

→ Querschnittsfläche der Kugel

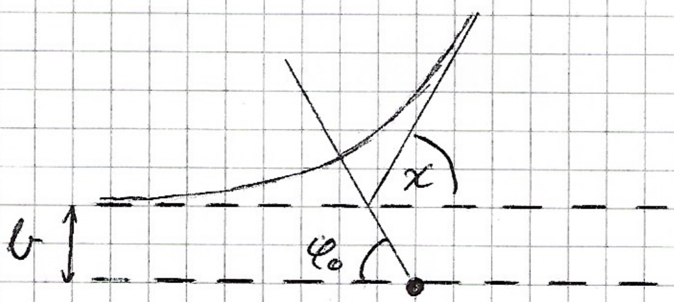
W.q. im Laborsystem ?

$$\frac{d\sigma}{d\Omega_1} = \frac{d\sigma}{d\Omega}\cdot\frac{d\Omega}{d\Omega_1}$$

$$d\Omega = 2\pi \sin\chi\, d\chi \qquad d\Omega_1 = 2\pi \sin\vartheta_1\, d\vartheta_1$$

$$\tan\vartheta_1 = \frac{m_2\sin\chi}{m_1 + m_2\cos\chi} = \frac{\sin\chi}{\mu + \cos\chi} \qquad \mu = \frac{m_1}{m_2}$$

Rutherford - Streuung



Coulomb-Feld

reduziertes 1-Körper-Problem

$\leftrightarrow$ S-System

$U = -\frac{\alpha}{r}$ $\quad \alpha < 0$ (gleiches Resultat für $\alpha > 0$)

Bahn: $\frac{p}{r} = -1 + e\cos\varphi$

$\varphi = 0 \leftrightarrow r = r_{min}$

$\varphi = \varphi_0 \leftrightarrow r = \infty$

$\Rightarrow$

$$\cos\varphi_0 = \frac{1}{e} = \frac{1}{\sqrt{1 + \frac{2El^2}{m\alpha^2}}} = \frac{\frac{|\alpha|}{m v_\infty^2 b}}{\sqrt{1 + \left(\frac{\alpha}{m v_\infty^2 b}\right)^2}}$$

$E = \frac{m}{2} v_\infty^2$, $\quad l = m v_\infty b$

$$\Rightarrow \tan^2\varphi_0 = \frac{1-\cos^2\varphi_0}{\cos^2\varphi_0} = \frac{2El^2}{m\alpha^2} = \frac{m^2 v_\infty^4}{\alpha^2} b^2 \; ; \quad \varphi_0 = \frac{\pi - \chi}{2}$$

$$\tan\frac{\pi-\chi}{2} = \cot\frac{\chi}{2} \quad \Rightarrow \quad \boxed{b = \frac{|\alpha|}{m v_\infty^2}\cot\frac{\chi}{2}}$$

$\chi \to 0 : b \to \infty$; $\quad \chi \to \pi : b \to 0$

S-System: $\frac{d\sigma}{d\Omega} = \frac{b(\chi)}{\sin\chi}\left|\frac{db}{d\chi}\right|$, $\quad \frac{d\cot\frac{\chi}{2}}{d\chi} = -\frac{1}{2}\frac{1}{\sin^2\frac{\chi}{2}}$

$$\frac{\cot\frac{\chi}{2}}{\sin\chi} = \frac{\cos\frac{\chi}{2}}{\sin\frac{\chi}{2}\cdot 2\sin\frac{\chi}{2}\cos\frac{\chi}{2}} = \frac{1}{2\sin^2\frac{\chi}{2}}$$

$$\Rightarrow \frac{d\sigma}{d\Omega} = \frac{\alpha^2}{4m^2v_\infty^4}\frac{1}{\sin^4\frac{\chi}{2}}$$

$m_2 \gg m_1 \Rightarrow m = m_1, \quad \chi = \vartheta_1$

Target — Projektil

$\Rightarrow$

$$\boxed{\frac{d\sigma}{d\Omega_1} = \frac{\alpha^2}{16 E_1^2} \frac{1}{\sin^4 \frac{\vartheta_1}{2}}}$$

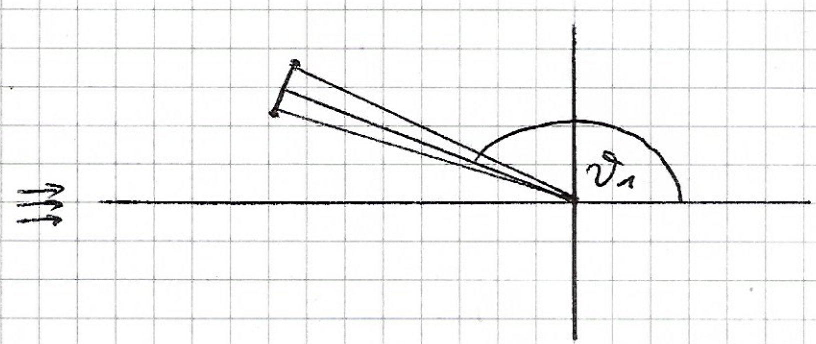
L-System, mit Näherung $m_2 \gg m_1$, $E_1 = \frac{m_1}{2} v_\infty^2$

gültig für $\alpha \gtrless 0$

$\sigma = \int \frac{d\sigma}{d\Omega_1} d\Omega_1$ divergiert: $U(r) \sim \frac{1}{r}$ Reichweite ∞

endlicher W.q. $\frac{d\sigma}{d\Omega_1} = \frac{\alpha^2}{16 E_1^2}$ für $\vartheta_1 = \pi$

Rutherford-Versuch $\quad {}^4He \rightarrow$ Gold



$$\frac{d\sigma}{d\Omega} = \frac{1}{j} \frac{dN}{d\Omega}$$