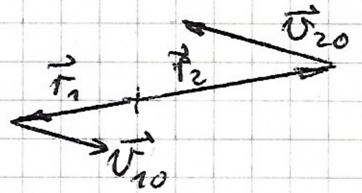
4.4. Streuung

elastischer Stoß: keine Änderung der inneren Energie



Schwerpunktsystem

$$\vec{r}_1 = \frac{m_2}{m_1+m_2}\vec{r} \qquad \vec{r}_2 = -\frac{m_1}{m_1+m_2}\vec{r}$$

$\vec{R} = 0$, $\quad \vec{r} = \vec{r}_1 - \vec{r}_2$

$$\vec{v} = \vec{v}_1 - \vec{v}_2 = \dot{\vec{r}} \qquad \underset{\uparrow}{\vec{v}_{10}} = \frac{m_2}{m_1+m_2}\vec{v} \qquad \vec{v}_{20} = -\frac{m_1}{m_1+m_2}\vec{v}$$

Index $_0$: S-System

nach dem Stoß: $'$ (vorher) (nachher)

Impulserhaltung	$\vec{p}_{10} + \vec{p}_{20} = 0$	$\Rightarrow$	$\vec{p}_{10}' + \vec{p}_{20}' = 0$
Energieerhaltung (*)	$\lvert\vec{p}_{10}\rvert = \lvert\vec{p}_{20}\rvert$	$\overset{(*)}{=}$	$\lvert\vec{p}_{10}'\rvert = \lvert\vec{p}_{20}'\rvert$
$\Rightarrow$	$v_{10} = v_{10}'$		$v_{20} = v_{20}'$

[(*): keine WW bei $t = -\infty$, $t = +\infty$ (nur Kin. Energie)]

$$\Rightarrow \quad \vec{v}_{10}' = \frac{m_2}{m_1+m_2} v\,\vec{n}_0 \qquad \vec{v}_{20}' = -\frac{m_1}{m_1+m_2} v\,\vec{n}_0$$

$$|\vec{n}_0| = 1$$

Laborsystem zunächst allg.:

Schwerpunktsgeschw. $\quad \vec{V} = \dfrac{m_1\vec{v}_1 + m_2\vec{v}_2}{m_1+m_2}$

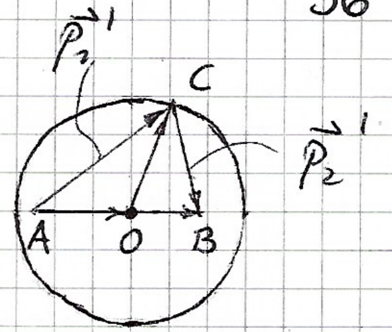
$$\vec{v}_1' = \frac{m_2}{m_1+m_2} v\,\vec{n}_0 + \vec{V} \qquad \Big|\cdot m_1$$

$$\vec{v}_2' = -\frac{m_1}{m_1+m_2} v\,\vec{n}_0 + \vec{V} \qquad \Big|\cdot m_2$$

$\Rightarrow$

$$\Rightarrow \quad \vec{p}_1{}' = \overbrace{m v \vec{n}_0}^{\vec{OC}} + \overbrace{\frac{m_1}{m_1+m_2}(\vec{p}_1+\vec{p}_2)}^{\vec{AO}}$$

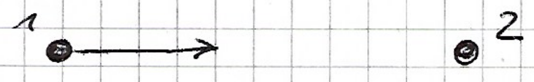
$$\vec{p}_2{}' = \underbrace{-m v \vec{n}_0}_{\vec{CO}} + \underbrace{\frac{m_2}{m_1+m_2}(\vec{p}_1+\vec{p}_2)}_{\vec{OB}}$$



$$m = \frac{m_1 m_2}{m_1+m_2} \qquad \vec{AB} = \vec{p}_1+\vec{p}_2 = \vec{p}_1{}'+\vec{p}_2{}'$$

variabel: C auf Kreis

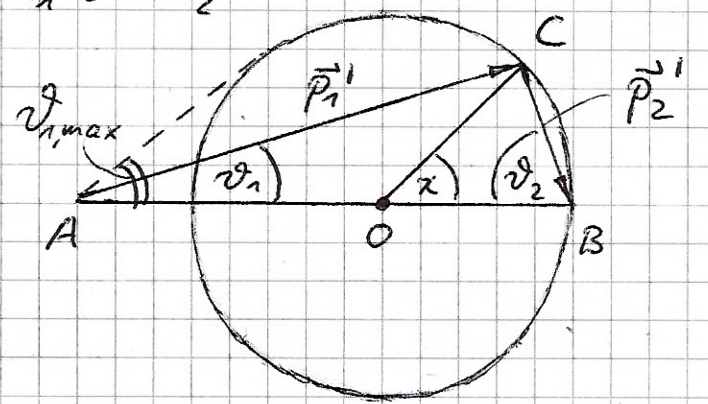
<u>Laborsystem:</u> Fall $\underline{\vec{p}_2 = 0}$



$$\Rightarrow \quad \vec{OB} = \frac{m_2}{m_1+m_2} m_1 \vec{v}_1 = m\vec{v} \qquad \vec{OC} = m v \vec{n}_0$$

$$\Rightarrow \quad |\vec{OB}| = |\vec{OC}|, \qquad \frac{|\vec{AO}|}{|\vec{OB}|} = \frac{m_1}{m_2}, \qquad \vec{AB} = \vec{p}_1$$

für $m_1 > m_2$

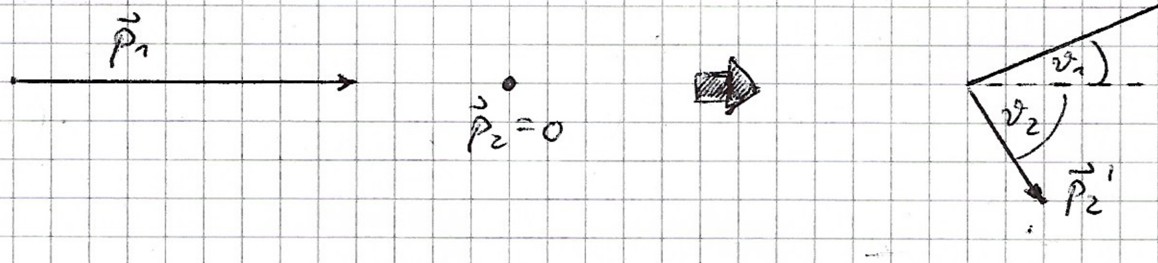


Ablenkwinkel:

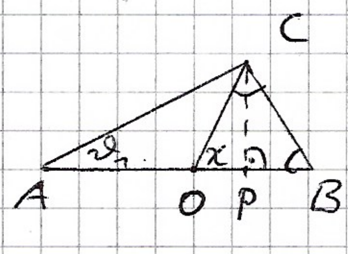
χ Teilchen 1, S-System

ϑ_1 Teilchen 1, L-System

ϑ_2 Teilchen 2, L-System



$$\tan\vartheta_1 = \frac{|\vec{CP}|}{|\vec{AO}|+|\vec{OP}|} = \frac{m_2 \sin\chi}{m_1 + m_2\cos\chi}$$



$$|\vec{OC}| = |\vec{OB}| \sim m_2 \qquad |\vec{AO}| \sim m_1$$

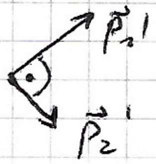
$$\vartheta_2 = \frac{\pi - \chi}{2}$$

$\underline{m_1 > m_2}$: $\vartheta_1 + \vartheta_2 < \frac{\pi}{2}$

$$\vartheta_1 \leq \vartheta_{1max} \qquad \sin \vartheta_{1max} = \frac{m_2}{m_1}$$

$\underline{m_1 < m_2}$: $\vartheta_1 + \vartheta_2 > \frac{\pi}{2}$ $\qquad \vartheta_1$ beliebig

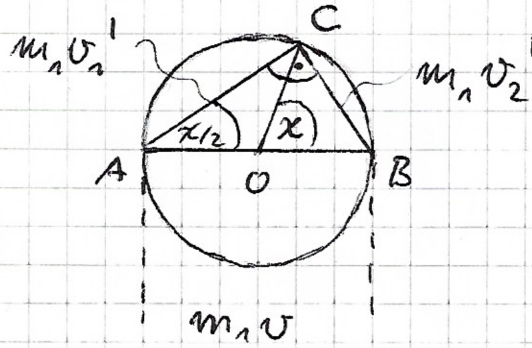
$\underline{m_1 = m_2}$: $\vartheta_1 + \vartheta_2 = \frac{\pi}{2}$



$|\vec{AO}| = |\vec{OB}|$

mit $\vartheta_2 = \frac{\pi - \chi}{2} \Rightarrow \vartheta_1 = \frac{\chi}{2}$

$\Rightarrow v_1' = v \cos \frac{\chi}{2} \qquad v_2' = v \sin \frac{\chi}{2}$



Grenzfälle in χ $\quad$ (m_1, m_2 beliebig)

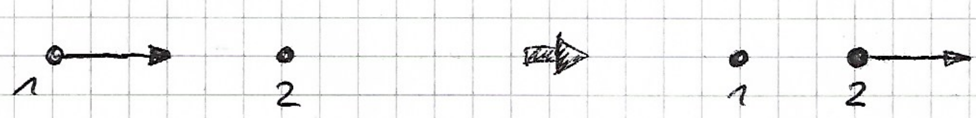
$\underline{\chi = 0}$: $\vec{p}_1{}' = \vec{p}_1$, $\vec{p}_2{}' = \vec{p}_2 = 0$ (↑ Diagramm) → kein Stoß

$\underline{\chi = \pi}$: → zentraler Stoß (Rückwärtsstreuung im S-System)

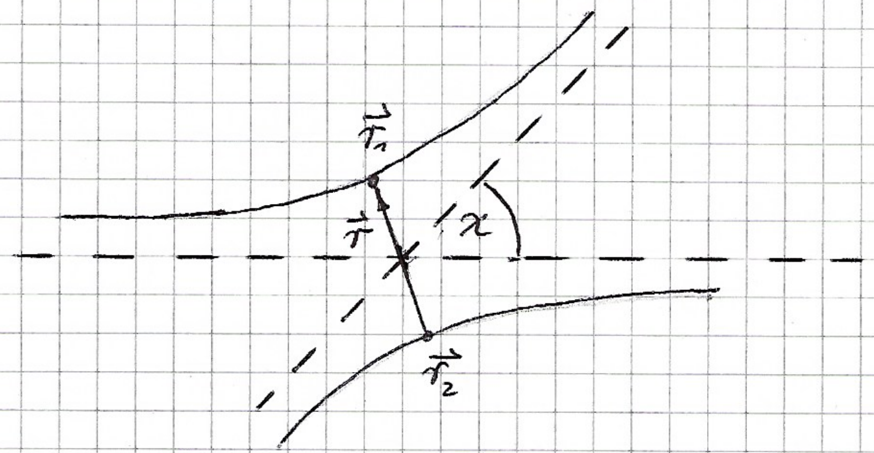
$$\vec{p}_1{}' = - m\vec{v} + \frac{m_1}{m_1 + m_2} m_1 \vec{v} = \frac{m_1 - m_2}{m_1 + m_2} m_1 \vec{v}$$

$$\Rightarrow \vec{v}_1{}' = \frac{m_1 - m_2}{m_1 + m_2} \vec{v} \quad ; \qquad \vec{v}_2{}' = \frac{2 m_1}{m_1 + m_2} \vec{v}$$

$\chi = \pi, \; m_1 = m_2 \Rightarrow v_1' = 0 \quad , \quad v_2' = v = v_1$



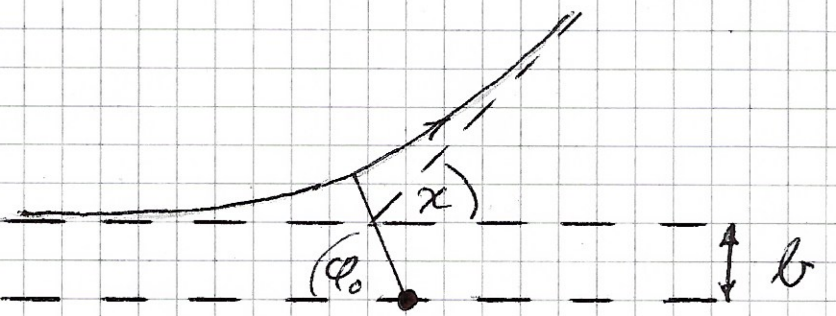
Berechnung von χ



$\underline{\underline{\chi}}$ im S-System $\hat{=}$ Ablenkwinkel $\underline{\underline{\chi}}$ im reduzierten Einkörperproble[m]

$\vec{r}$ rotiert um $\chi' = \pi - \chi \equiv$ Streuung um χ

zunächst: reduziertes Einkörperproblem:



$$\chi = |\pi - 2\varphi_0|$$

Potential

anziehend $\quad \chi = \pi - 2\varphi_0$

<u>abstoßend</u> $\quad \chi = 2\varphi_0 - \pi$

<u>b Stoßparameter</u>

$$\varphi_0 = \int_{r_{min}}^{\infty} \frac{\frac{l}{r^2}\,dr}{\sqrt{2m(E-U(r)) - \frac{l^2}{r^2}}}$$

$E,\ l \ \leftrightarrow\ v_\infty,\ b$: $\qquad E = \frac{m v_\infty^2}{2}$, $l = m b v_\infty$

$$\Rightarrow \quad \varphi_0 = \int_{r_{min}}^{\infty} \frac{\frac{b}{r^2}\,dr}{\sqrt{1 - \frac{b^2}{r^2} - \frac{U(r)}{E}}}$$

$$\Rightarrow \chi = |\pi - 2\varphi_0| = \chi(b) \quad \Rightarrow \quad b = b(\chi)$$