

“QCD AND STANDARD MODEL”  
**Problem Set 3**

## 1. An example of UV completion of gauge theories : the Abelian Higgs mechanism for Proca fields

Let us consider the so-called Proca Lagrangian

$$\mathcal{L}[\tilde{A}_\mu] = -\frac{1}{4}\tilde{F}_{\mu\nu}\tilde{F}^{\mu\nu} + \frac{m^2}{2}\tilde{A}_\mu\tilde{A}^\mu + \frac{\xi^4}{4}\left(\tilde{A}_\mu\tilde{A}^\mu\right)^2, \quad \tilde{F}_{\mu\nu} \equiv 2\partial_{[\mu}\tilde{A}_{\nu]} , \quad (1)$$

and provide a UV completion by integrating in an additional degree of freedom.

- a) Check that the Stückelberg decomposition<sup>1</sup>  $\tilde{A}_\mu = A_\mu + \frac{1}{m}\partial_\mu\theta$  implies a redundancy of the form

$$\begin{cases} A_\mu & \rightarrow A'_\mu = A_\mu + \frac{1}{m}\partial_\mu\chi \\ \theta & \rightarrow \theta' = \theta - \chi \end{cases} \quad (2)$$

Consider the free theory (*i.e.*  $\xi = 0$ ), for the sake of simplicity, and compute the number of propagating degrees of freedom of a massive vector field.

- b) Compute the propagator of the theory in momentum space. Sandwich it between two sources  $j_\mu$  and investigate the behavior when  $m \rightarrow 0$ .
- c) Compute the 2→2 scattering amplitude at tree level. Show that at large momenta unitarity is lost.  
**(Hint :** It is useful to rewrite the Lagrangian in terms of  $A_\mu$  and  $\theta$  instead of  $\tilde{A}_\mu$ .)
- d) Unitarity is restored by integrating in a new degree of freedom  $H = \rho(x)e^{i\theta(x)/v}$  and realising the Abelian Higgs model

$$\mathcal{L}[A_\mu, H] = -\frac{1}{4}F_{\mu\nu}F^{\mu\nu} + (D_\mu H)^\dagger D^\mu H - \frac{\lambda^2}{2}(H^\dagger H - v^2)^2, \quad (3)$$

where  $F_{\mu\nu} \equiv 2\partial_{[\mu}A_{\nu]}$  and  $D_\mu = \partial_\mu + igA_\mu$ . Check that the conditions, similar to Eq. (2),

$$\begin{cases} A_\mu & \rightarrow A'_\mu = A_\mu + \frac{1}{g}\partial_\mu\alpha \\ H & \rightarrow H' = e^{-i\alpha}H \end{cases} \quad (4)$$

still leave the Lagrangian invariant. Moreover, assume that the Higgs has a VEV, expand the theory around the vacuum according to the prescription  $\rho(x) = \frac{1}{\sqrt{2}}(v + h(x))$  and identify the massive modes.

- e) Assuming that the Higgs is heavier than the vector, that is  $m \lesssim E \ll m_H$ , one can integrate the former out. Show that in this limit one recovers the Proca Lagrangian. Identify the coupling  $\xi$  in terms of the parameters in Eq. (3).

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1. Here,  $A_\mu$  is the transverse part of the Proca field  $\tilde{A}_\mu$ .