Rem.: $\Delta[A_\mu]$ depends only on equivalence classes $[A_\mu]$

Pf.: $$\frac{1}{\Delta[A_\mu^u]} = \int [Du'] \, \delta[F((A_\mu^u)^{u'})]$$

right inv. of $[Du']$
$$\xrightarrow{} \; = \int [D\, u'u] \, \delta[F(A_\mu^{uu'})]$$

change of var.
$$\xrightarrow{} \; = \int [Du''] \, \delta[F(A_\mu^{u''})]$$

$$= \frac{1}{\Delta[A_\mu]} \qquad \#$$

③ change order of integration:

$$\text{Path int.} = \int [Du] \int [DA_\mu] \, \Delta[A_\mu] \, G(A_\mu) \cdot \delta[F(A_\mu^u)] \, e^{\frac{i}{\hbar} S[A_\mu]}$$

$$= \int [Du] \int [DA_\mu^u] \, \Delta[A_\mu^u] \, G(A_\mu^u) \cdot \underbrace{\delta[F(A_\mu^u)] \, e^{\frac{i}{\hbar} S[A_\mu^u]}}_{= I \neq I[u]}$$

$$= \underbrace{\int [Du]}_{\text{number} =: N} \; I$$

We then define the Faddeev–Popov path int.
as

$$\langle G(A_\mu)\rangle = \int [DA_\mu]\, \Delta[A_\mu]\, \delta[F(A_\mu)] \, G(A_\mu)\, e^{\frac{i}{\hbar} S[A_\mu]}$$

which is well defined.

(4) Calculate $\Delta[A_\mu] \approx \det\left(\frac{\delta F(A_\mu{}^u)}{\delta u} \right)$

Expand $U = 1 - i\theta^a(x) T_a + O(\theta^2)$

Then $\Delta[A_\mu] = \det(M^a{}_b(x,y))$ with

$$M^a{}_b(x,y) = \frac{\delta F^a(A_\mu)(x)}{\delta \theta^b(y)}.$$

Rep: $\delta A_\mu^a T_a = \frac{1}{g}\partial_\mu \theta^a T_a + i\underbrace{[A_\mu{}^b T_b, \theta^c T_c]}_{= i C_{bc}{}^d A_\mu{}^b \theta^c T_d}$

Thus: $$M^a{}_b(x y) = \int d^4 z\, \frac{\delta F^a(A_\mu)(x)}{\delta A_\nu^c(z)} \frac{\delta A_\nu^c(z)}{\delta \theta^b(y)}$$

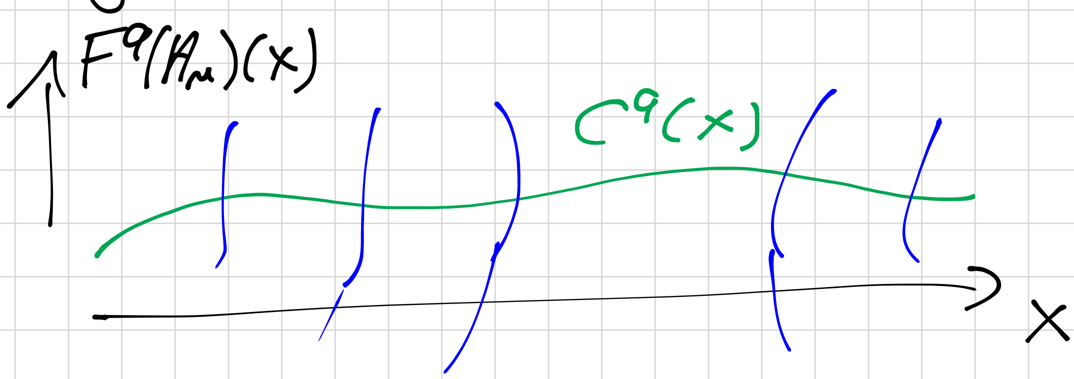
$$= \int d^4 z\, \frac{\delta F^a(A_\mu)(z)}{\delta A_\nu^c(z)} \Big(\frac{1}{g}\partial_{z^\nu} \delta^4(x-z)\delta^c{}_b - C_{ed}{}^c A_\nu^e(z) \delta^4(z-y) \delta^d_b \Big)$$

$$= \int d^4z \, \frac{\delta F^a(A_\mu)(z)}{\delta A^c_\nu(z)} \left(\underbrace{\frac{1}{g}\, \partial_{z\nu} \delta^4(z-y) + i A^e_\nu (T_e^{adj})^c{}_b}_{= \frac{1}{g} (D^{adj}_{z\nu})^c{}_b \, \delta^4(z-y)} \right)$$

⑤ Implementation of $\delta[F(A_\mu)] = \prod_{x \in \mathbb{R}^4} \delta(F(A_\mu)(x))$

ⓐ gauge inv. $\Rightarrow$ $\delta[F^a(A_\mu)(x)] \sim \delta[F^a(A_\mu)(x) - C^a(x)]$

function w/ values in $\mathfrak{g}$



ⓑ Average over $C^a(x)$:

weight fn'l

$$\delta[F^a(A_\mu)(x) - C^a(x)] \rightsquigarrow \frac{1}{\tilde{N}} \int [DC] \, \delta(F^a - C^a) \, G[C^a]$$

Normalisation

E.g. $G(C) = e^{\frac{i}{2\xi}\int \langle C, C\rangle d^4x}$

ξ ← parameter; $\langle\,,\rangle$ ← Killing form

(gaußian averaging)

Integrate over $C^a(x) \Rightarrow C^a \to F^a(A_\mu)$

This is then equivalent to

$$\mathcal{L}(A_\mu, \partial A_\mu) \rightsquigarrow \mathcal{L}(A_\mu, \partial A_\mu) + \underbrace{\frac{\hbar}{2\xi}\langle F(A_\mu), F(A_\mu)\rangle}_{= \mathcal{L}_{gf}}$$

Rem: (i) $\mathcal{L} + \mathcal{L}_{gf}$ is no longer gauge inv. but $\int [DA]\, e^{\frac{i}{\hbar}\mathcal{L} + \mathcal{L}_{gf}}$ is well defined.

(ii) for $\xi \to 0$ we recover $\delta[F(A_\mu)]$.

⑥ Exponentiate $\Delta[A_\mu]$:

$$\Delta[A_\mu] = \det(M^a{}_b(x,y)) =$$

$$\int [DH\bar{H}]\, e^{\iint \bar{H}_a(x) M^a{}_b(x,y) H^b(y)\, d^4x\, d^4y}$$

anti-comm. bosons

$H^a(x) = \sum_n \eta_n H_n^a(x)$ ← Grassmann valued

$\overset{(-)}{H}{}^a$: Faddeev Popov ghosts all together:

$$\langle O(A_\mu)\rangle = \frac{1}{Z}\int [DA_\mu, H, \bar{H}]\, O(A_\mu)\,\cdot$$

$$\cdot\, e^{\frac{i}{\hbar}\int \mathcal{L}(A_\mu, \partial A_\nu) + \frac{\hbar}{2\xi}\langle F, F\rangle + \underbrace{\iint \bar{H}_a M^a{}_b H^b}}$$

$\Rightarrow$ looks non-local but is actually local for a suitable $F^a(A_\mu)$

Example: Yang-Mills theory in Lorentz gauge:

take: $\boxed{F^a(A_\mu) = \partial_\mu A^{\mu a}}$

$$\frac{\delta F^a(A_\mu)(x)}{\delta A^c_\nu(z)} = \partial_{x^\mu}\delta^4(x-z)\,\delta^a{}_c$$

$$M^a{}_b(x,y) = \partial_{x^\mu}\delta^4(x-z)\,\delta^a{}_c\,\eta^{\mu\nu}\frac{1}{g}\left(D^{adj}_{z^\nu}\right)^c{}_b\,\delta^4(z-y)$$

Then:

$$\iint \bar{H}_a(x)\, M^a{}_b(x,y)\, H^b(y)\, d^4x\, d^4y =$$

$$= \frac{1}{g}\int \bar{H}_a(x)\,\partial_{x^\mu}\eta^{\mu\nu}\left(D^{adj}_{x^\nu}\right)^a{}_b H^b(x)\, d^4x$$