Structure of $\Gamma_{N,00}$:

- dim $(A_\mu, \psi_\alpha, H, \bar{H}, b) = (1, \frac{3}{2}, 1, 1, 2)$
  gh $\phantom{(A_\mu, \psi_\alpha, H, \bar{H}, b) =} (0, 0, 1, -1, 0)$

$$\left.\begin{array}{ll} \Rightarrow \text{dim}(A^*_\mu, \psi^*_\alpha, H^*, \bar{H}^*, -) = (2, \frac{3}{2}, 2, 2, \quad) & (1) \\ \text{gh} \qquad\qquad\qquad\qquad\qquad\qquad (-1, -1, -2, 0, \quad) & (2) \end{array}\right\} (*)$$

(1) $\Rightarrow$ $\Gamma_{N,00}$ at most quadratic in Φ^*

(2) $\Rightarrow$ " " " linear in Φ^* except $\bar{H}^*$ but since $s(\bar{H}) = b$ (linear)

(⊛) $\Rightarrow$ $\Gamma_{N,00} = -b\bar{H}^* + \bar{H}^*$ indep.

$\Longrightarrow$ $\Gamma_{N,00}$ is at most linear in Φ^*. To continue we write

$$\Gamma_{N,00} = \Gamma_{N,00}(\Phi) + \int \Delta^I_{00}(\phi)\, \Phi^{*I}$$

rep. $\quad S_R[\phi, \phi^*] = S_R[\phi] + \int s(\phi^I)\, \phi^{*I}$

<u>Def:</u> $\quad \Gamma_N^{(\varepsilon)}[\phi] \equiv S_R[\phi] + \varepsilon\, \Gamma_{N,00}[\phi] \; ; \; \varepsilon^2 = 0$ "∞^2"

(⊗) $\Rightarrow$ $\Gamma_N^{(\varepsilon)}$ is invariant under

$$\phi^I \mapsto \phi^I + \delta\xi\, \underbrace{\Delta_N^{(\varepsilon)}}_{=\, s^I(\phi) + \varepsilon\, \Delta^I_{00}(\phi)}$$

(*) there is some ambiguity in assigning dimensions. An alternative choice is

$$\begin{cases} \text{dim}(A_\mu, \psi_\alpha, H, \bar{H}, b) = (1, \frac{3}{2}, 0, 2, 2) \\ \text{gh} \qquad\qquad\qquad\qquad\qquad (0, 0, 1, -1, 0) \\ \text{dim}(A^*_\mu, \psi^*_\alpha, H^*, \bar{H}^*, -) = (2, \frac{3}{2}, 4, 2, \quad) \\ \text{gh} \qquad\qquad\qquad\qquad\qquad (-1, -1, -2, 0, \quad) \end{cases}$$

However this does not change the result.

Dimensional analysis, Poincaré invariance, and nilpotence imply (Weinberg)

$\Delta^{I(\varepsilon)}$ has the same Lorentz tf. rules, dim. and gh.# as s $\Rightarrow$

$$\begin{cases} \delta\psi = i Z \delta\xi \, H^a t^a \psi \\ \delta A_\mu^a = Z\delta\xi \left[N B^{ac} \partial_\mu H^c + D_{abc} A_\mu^b H^c \right] \\ \delta H^a = -\frac{1}{2} Z\delta\xi \, E_{abc} H^b H^c \\ \delta \bar{H}_a = -\delta\xi \, b_a \\ \delta b_a = 0 \end{cases}$$

linear tf. are unchanged

B^{ac}, D_{abc}, E_{abc}, Z, N constants.

Nilpotency of $\Delta^{(\varepsilon)}$:
- on $H^a \Rightarrow$ Jacobi for $E \Rightarrow$ Lie alg
- on $A_\mu \Rightarrow D \propto E$ and B invariant tensor ($\Rightarrow \delta^{ab}$)
- on $\psi \Rightarrow t^a \propto T^a$

If we write $\Gamma_N^{(\varepsilon)} = \int d^4x \, h_N^{(\varepsilon)}$ then we can restrict the form of $h_N^{(\varepsilon)}$ by demanding that

① $\Gamma^{(\varepsilon)}$ is invariant under all <u>linear</u> symmetries of S_R

② $h_N^{(\varepsilon)}$ is of dimension 4

③ $\Gamma_N^{(\varepsilon)}$ is $\Delta_N^{(\varepsilon)}$ invariant ("effective" BRST tf.)

Rep: $S_R^{tot} = S_R[A_\mu] + s(\Psi)$ ← gauge fixing.

$$= S_R[A_\mu] - \frac{1}{2\lambda} b_a b_a - b_a F_a + \bar{H}_a \underbrace{M^a{}_b}_{\partial^\mu D_\mu} H^b$$

$$= S_R[A_\mu] - \int \frac{1}{2\lambda_R} b_a b_a + b_a F_a + \partial_\mu \bar{H}_a \partial^\mu H^a$$

$$- f_{abc} \partial_\mu \bar{H}_a A^\mu_b H_c$$

$F_a = \partial_\mu A^\mu_a$

① linear symm:

- Lorentz
- global gauge

$$\begin{cases} \delta A_\mu^a := f_{abc} \, \varepsilon^c A_\nu^b \\ \varepsilon H_a = f_{abc} \, \varepsilon^c H_b \\ \delta \bar{H}_a = f_{abc} \, \varepsilon^c \bar{H}_b \\ \delta b_a = f_{abc} \, \varepsilon^c b_b \end{cases}$$

this just expresses the fact that these fields transform in the adjoint rep.

- anti-ghost translation: $\bar{H}_a \to \bar{H}_a + \text{const}$

② The general expression for $h_N^{(\varepsilon)}$ invariant under these linear tf. of dim 4 is then

$$h_N^{(\varepsilon)} = h_N^{(\varepsilon)}(A_\mu) - \frac{1}{2\lambda'} b_a b_a - c\, b_a F_a - Z_\eta \partial_\mu \bar{H}_a \partial^\mu H^a$$
$$+ e_{abc} b_a A^\mu_b A_{\mu c} + \epsilon'_{abc} \partial_\mu \bar{H}_a A^\mu_b H_c$$

with $\lambda', c, Z_\eta, e_{abc}, \epsilon'_{abc}$ a priori unknown.

n.b. $h_N^{(\varepsilon)}(A_\mu)$ contains all the normaliseable terms, including e. g. $m^2 A_\mu^a A^\mu_a$.
as such a term is inv. under the global tf. ①.

③ If we impose $\Delta_N^{(\varepsilon)}$ - invariance we find in particular, $m^2 = 0$ (Rep: $\Rightarrow h_N^{(\varepsilon)}(A)$ has to be gauge inv.)

$$\xi \partial_\mu b_a \partial^\mu H^a \in \delta h_N^{(\varepsilon)} \Rightarrow c = Z_\eta / ZN$$

$$\xi \partial_\mu b_a H_b A^\mu_c \in \delta h_N^{(\varepsilon)} \Rightarrow \epsilon'_{abc} = -\frac{Z_\eta}{N} \epsilon_{abc}$$

$$\xi b_a \partial_\mu H_b A^\mu_c \in \delta h_N^{(\varepsilon)} \Rightarrow e_{abc} = 0$$

Consequently $h_N^{(\varepsilon)}$ has the same structure as S_R.

$\Rightarrow \Gamma_{N,00}$ can be removed by renormalisation.

n.b. In the absence of antighost invariance, as it happen for instance for

$$F_a = \partial_\mu A^\mu_a + \alpha_{abc} A^\mu_b A_{\mu c}$$

inv. tensor, symm in b and c (exists for $SU(N\geq 3)$)

the BV-action will not have anti-ghost translation invariance.Consequently. we cannot exclude terms like

$$\underbrace{b_{abc}}_{\text{some invariant antisymm tensor (e.g. } c_{abc})} s'(\bar{H}_a \bar{H}_b H_c) = -b_{abc}\left(2 b_a H_b H_c + \frac{1}{2} Z \epsilon'_{cde} \bar{H}_a \bar{H}_b H_d H_e\right)$$

such terms cannot arise as counter terms in Faddeev-Popov quantisation, since they involve a four- ghost interaction. Note that such terms do actually arise at 1-loop with this gauge fixing . They do also arise in the « background field method » where we replace $\partial_\mu A^\mu$ by $\bar{D}_\mu A^\mu$ where $\bar{D}_\mu = \partial_\mu + i\bar{A}_\mu$ is the background field covariant derivative. If we want to work with such a gauge we need to start with a Lagrangian with a generalised BRST invariance (see lecture) where we add to S_0 [A] a term s(Psi) and we use that we are free in choosing any gauge fixing fermion so long it gauge-fixes the functional integral. In this way we can introduce a 4- ghost interaction in the classical generalised BRST action.

Rem: A convenient choice in the background field method is to work with the axial gauge where the ghosts decouple and thus we can choose Ψ as we like.