Rem: strictly speaking the effective action Γ is a function of Φ only since there is no Schwinger current for $\overset{*}{\Phi}$ and $\overset{*}{\phi}$ is not an external field due to $\overset{*}{\phi} = \frac{\delta\psi}{\delta\phi}$.

However, we can introduce a $\overset{*}{\Phi}$ dependence by adding a term $\int s(\phi)\overset{*}{\Phi}$ to the action with $\overset{*}{\Phi}$ interpreted as an external field. This is well defined since $f = s(\phi) \propto (S,\phi)$ is a gauge invariant operator. Indeed

$$\begin{cases} (S,f) = 0 \quad (\text{Jacobi}) \quad \text{and} \\ \Delta(f) = \Delta(S,\phi) = (\underbrace{\Delta S}_{=0},\phi) - (S,\underbrace{\Delta(\phi)}_{=0}) \quad \left(\begin{array}{c}\Delta \text{ is a} \\ \text{derivation of} \\ \text{of } S\end{array}\right) \end{cases}$$

exercise

Furthermore $(f,\phi) = 0$ since $f \neq f(\overset{*}{\phi})$

Thus $\Gamma(\Phi, \overset{*}{\Phi})$ is well defined.

Rem (*) shows that Γ is invariant under the transformation $\Phi \to \Phi + \langle s(\phi)\rangle$ but typically this transformation is different from the classical BRST transformation $\phi \to \phi + s(\phi)$ since $\langle s(\phi)\rangle \neq s\langle\phi\rangle$ unless $s(\phi)$ is linear in the fields. In the latter case they agree. In particular, $\Gamma(\underline{\Phi})$

(Δ) and $S[\phi]$ share the same <u>linear</u> symmetries.

Renormalisation of gauge theories: (Weinberg's Book)

The simple power counting argument we used for scalar field theories is, in general not sufficient to prove that a gauge theory is normaliseable. The problem is that the whole class of counter terms is not compatible with gauge invariance in the usual sense. To address this question let us expand the BV-equation for

$\Gamma(\underline{\Phi})$ in powers of $\hbar$ (or loops). Writing

$$\Gamma[\underline{\Phi},\underline{\Phi}^*] = \sum_{N=0}^{\infty} \Gamma_N(\underline{\Phi},\underline{\Phi}^*=\chi^*)$$

we have for each N (loop order)

$$\sum_{N'=0}^{N} (\Gamma_{N'};\Gamma_{N-N'}) = 0 .$$

Suppose that renormalisation has been successfull up to order $N-1$ in the sence that all divergenties in $\Gamma_{N'<N}$ were absorbed in counter terms consistent gauge inv. Then we are left with:

$$(\Gamma_0,\Gamma_N) = (S_R,\Gamma_{N,\text{finite}}+\Gamma_{N\,\infty}) = 0$$

where we have seperated the infinite part of Γ_N.

In particular,
$$(S_R,\Gamma_{N,\infty}) = 0 \qquad (\otimes)$$

The counter terms required for $\Gamma_{N,\infty}$ consist of of sums of products of the fields and their derivatives of dimension four or less. (assuming renormaliseability in the power counting sence, see ϕ^4-theory above).