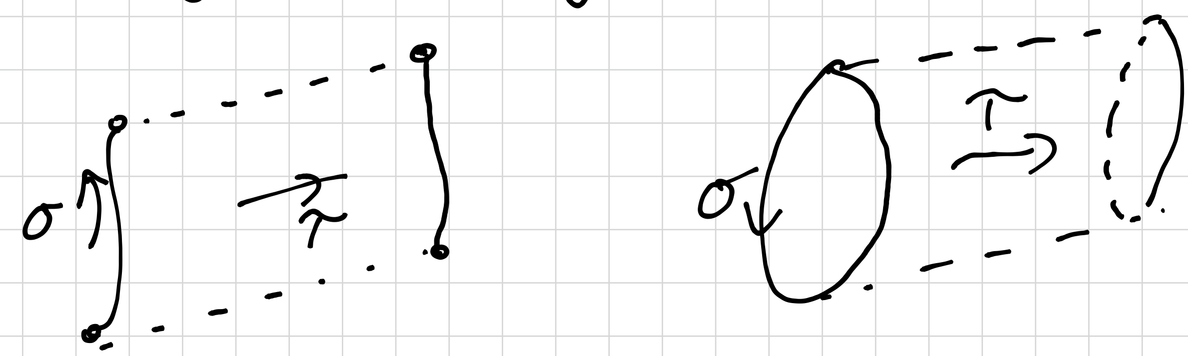
# On (open) bosonic string field theory:

In string theory one replaces the world line by a string:




open closed

Let us focus on the open string. As a consequence, the world line d.o.f of the BRST action, $\{q^\mu(\tau), b(\tau), c(\tau)\}$ become fields $\{q^\mu(\tau,\sigma), b(\tau,\sigma), c(\tau,\sigma)\}$ on the world sheet of the string, with Fourier components

$$q^\mu(\tau,\sigma) = \sum_{n\in\mathbb{Z}} q_n^\mu(\tau)\, e^{in\sigma}$$

$$c = c_n$$

$$b = b_n$$

with $Q = \sum_n c_n L_{-n}$

$$L_0 = p^\mu p_\mu + \underbrace{\sum_{m\in\mathbb{Z}\setminus\{0\}} \mathring{q}^\mu{}_m \mathring{q}_{-m\mu} + \text{const.}}_{(*)}$$

is again the Hamiltonian (or mass shell) constraint, with a mass that depends on the oscillation modes of the string through $(*)$.

$q^\mu(\tau) \equiv q^\mu{}_0(\tau)$ and $p_\mu(\tau) = \mathring{q}^\mu$

are the centre of mass coordinate and momentum of the string respectively. In addition,

$$L_n = p^\mu \mathring{q}_{n\mu} + \sum_{m\in\mathbb{Z}\setminus\{0\}} \mathring{q}^\mu{}_{n+m} \mathring{q}_{-m\mu}$$

generate the reparametisations in $\sigma$ (rather than $\tau$). The corresponding conjugate variables are denoted by

$\alpha^\mu_n = \mathring{q}^\mu_n$ and $q^\mu_n$ resp.

with equal $\tau$ commutation relations

$$[q^\mu{}_n, \alpha_{m\nu}] = i\, \delta_{m+n}\, \delta^\mu{}_\nu$$

and $\quad [b_n, c_m] = \delta_{m+n}$

a generic state $\psi \in \mathcal{H}$ is then of the form:

$$\psi = \phi^{\underline{m},\underline{n},\underline{r}}(x)\,\alpha_{-m_1} \cdots \alpha_{-m_k} b_{-n_1} \cdots b_{-n_p} c_{-r_1} \cdots c_{-r_s} \overset{\text{Fock vac.}}{|0\rangle}$$

What does $Q$ generate? In full generality we don't know. Let us look at an example:

$$\psi = c_{+1}\, \phi_\mu(x)\, \alpha^\mu_{-1}$$

$$Q|\psi\rangle = c_0 c_{+1} [P^2, \phi_\mu]\, \alpha^\mu_{-1} |0\rangle$$

$$+ c_{-1} c_1 [P_\nu, \phi_\mu] \underbrace{[\alpha^\nu_1, \alpha^\mu_{-1}]}_{\delta^{\mu\nu}} |0\rangle$$

$$\overset{!}{=} 0 \quad \Rightarrow \quad \left.\begin{array}{l} \Box \phi_\mu = 0 \\ \nabla_\mu \phi^\mu = 0 \end{array}\right\} \Rightarrow \underline{\text{photon}}$$

$$|\lambda\rangle = \lambda(x)\,|0\rangle \qquad\qquad L_{-1}$$

$$Q|\lambda\rangle = c_{+1}\, p_\mu\, \alpha^\mu_{-1}\, \lambda(x)\,|0\rangle$$

$$= -i\, c_{+1}\, \partial_\mu \lambda(x)\, \alpha^\mu_{-1} |0\rangle$$

gauge transformation.