A toy model for string field theory:

Let us consider a relativistic point particle with action

$$I[q^\mu, e] = \frac{1}{2}\int \left(\frac{\mathring{q}^\mu \mathring{q}_\mu}{e} - m^2 e\right) d\tau$$

where τ is some parameter to parametrise the curve and $e(\tau)$ is the "Einbein" which transforms under reparametrisation $\tau' = \tau'(\tau)$ s.t.

$$e(\tau)\,d\tau = e'(\tau')\,d\tau'$$

Finally $\mathring{q}^\mu(\tau) = \frac{d}{d\tau} q^\mu(\tau)$. The action $I[q^\mu, e]$ is invariant under the infinitesimal transformation $\tau' = \tau + \frac{\varepsilon(\tau)}{e(\tau)}$

$$\delta q^\mu(\tau) = \frac{\mathring{q}^\mu}{e}\varepsilon(\tau) \quad ; \quad \delta e = \mathring{\varepsilon}(\tau) \qquad (*)$$

We can eliminate the Einbein e by its equation of motion giving

$$-\frac{1}{e^2}\mathring{q}^\mu \mathring{q}_\mu - m^2 = 0$$

and after substituting back into I we get the familiar action:

$$I[q^\mu] = m\int \sqrt{-\mathring{q}^\mu \mathring{q}_\mu}\; d\tau$$

In order to quantise $I[q^\mu, e]$ we consider the Evolution Kernel

$$K(q_f, q_i; \tau_f, \tau_i) = \int\limits_{\substack{q(\tau_f)=q_f \\ q(\tau_i)=q_i}} [D q^\mu e] \, e^{\frac{i}{\hbar} I[q^\mu, e]}$$

which, due to the redundancy (*) needs to be gauge fixed. Applying the by now familiar BRST - Faddeev - Popov procedure we will end up with

$$K(\dots) = \int\limits_{\dots} [D q, e, \pi, b, c] \, e^{\frac{i}{\hbar} I^{tot}}$$

where

$$I^{tot}[\dots] = \int \frac{1}{2e} \dot{q}^\mu \dot{q}_\mu - \frac{m^2 e}{2} + i b \dot{c} + \bar{\pi}(e - \hat{e})$$

where we have renamed:
$H^a \rightsquigarrow c$
$\bar{H}^a \rightsquigarrow ib$
$b^a \rightsquigarrow \bar{\pi}$

and $F(e) = (e - \hat{e})$ is the gauge fixing condition. The BRST invariance of I^{tot} is given by: $\delta e = -i\dot{c}$, $\delta q^\mu = -i \dot{q}^\mu \frac{c}{e}$

$$\delta b = \bar{\pi} \, ; \, \delta c = \delta \bar{\pi} = 0$$

We can also obtain an "effective action" $\tilde{I}$ by integrating over e and π. This gives

$$\tilde{I}[q^\mu, b, c] = \int \left(\frac{1}{2} \dot{q}^\mu \dot{q}_\mu - \frac{1}{2} m^2 + i b \dot{c} \right) d\tau$$

which enjoys a residual symmetry (with

$\pi = \frac{1}{2}(\dot{q}^\nu \dot{q}_\nu + m^2)$ by its e.o.m. and $e = \hat{e} = 1$)

$$\delta q^\mu = -i \dot{q}^\mu c \quad ; \quad \delta b = \frac{1}{2}(\dot{q}^\nu \dot{q}_\nu + m^2) \quad (**)$$

In order to identify the Noether charge we need to first describe the canonical structure of $\tilde{I}[q_\mu, b, c]$. We first identify the canonical momenta

$$p_\mu = \frac{\delta \tilde{I}}{\delta \dot{q}^\mu} \quad ; \quad \pi_c = \frac{\delta_R \tilde{I}}{\delta \dot{c}} = i b$$

with canonical commutation relations:

$$[\hat{q}_\mu, \hat{p}^\nu] = i \delta_\mu{}^\nu \quad (\hbar = 1)$$

$$[\hat{b}, \hat{c}] = 1$$

Then the conserved Noether charge

$$Q = c H \quad ; \quad H = (p^\mu p_\mu + m^2)$$

generates the BRST transf $(**)$ upon using the e.o.m. $\dot{q}^\mu = p^\mu$.