Example (pure Yang-Mills) $\{\Phi^I\} = \{A_\mu^a, H^a, \bar{H}^a\}$
(here we do not introduce the auxiliary field in order to keep only a minimal number of fields)

$$S[\phi] = S_0[A_\mu] \; ; \; S^{tot}[\phi] = S_0[A_\mu] + \underbrace{S_{gf} + S_{FP}}_{= s(\bar{H}_a F^a(A))}$$

$$S_{BV} = S_0[A_\mu] + \int \underbrace{s(A_\mu^c)}_{=(D_\mu H)^c} A^{*c\mu} + \int \underbrace{s(H^a)}_{\frac{1}{2} f_{abc} H^b H^c} H^{*a} + \int \underbrace{s(\bar{H}^a)}_{-\lambda F_a} \bar{H}^{*a}$$

master equation:

$$\frac{\delta_R S}{\delta \Phi^{*I}} \frac{\delta_L S}{\delta \phi^I} = \overbrace{s(A_\mu^c) \frac{\delta S[A]}{\delta A_\mu^c}}^{=0 \text{ by gauge inv. of } S[A_\mu^a]} + s(H^a) \frac{\delta S[A_\mu]}{\delta H^a} \;(\to 0)$$

$$+ \underbrace{s(A_\nu^c)}_{D_\mu H^c} \underbrace{\frac{\delta_L s(A_\nu^d)}{\delta A_\mu^c}}_{i C_{dcb} H^b} A_\nu^{*d} + \underbrace{s(H^c)}_{\frac{1}{2} C_{cdb} H^d H^b} \frac{\delta_L s(A_\nu^d)}{\underbrace{\delta H^c}_{D_\mu}} A_\nu^{*d}$$

$= 0$ by $s(s(A_\mu)) = 0$

$$+ \underbrace{s(H^a)}_{C_{aef} H^e H^f} \underbrace{\frac{\delta_L s(H^b)}{\delta H^a}}_{C_{abc} H^c} H^{*b} + s(A_\mu^c) \underbrace{\frac{\delta_L s(H^b)}{\delta A_\mu^c}}_{=0} H^{*b}$$

$= 0$ by $s(s(H)) = 0$
(uses Jacobi for C_{abc})

$$+ s(A_\nu^c) \frac{\delta_L s(\bar{H}^b)}{\delta A_\nu^c} \bar{H}^{*b}$$ not needed

Thus:

$$S[A_\mu, H^a, A_\mu^*, H^{*a}] = S_0[A_\mu] + \int s(A_\mu)\, A^{*\mu} + \int s(H^a)\, H^{*a}$$

is the <u>minimal</u> solution to the master equation.

<u>Elimination of the antifield</u>: $\Phi^{*I} = \dfrac{\partial \Psi(\phi)}{\partial \phi^I}$

e.g. $\underset{\text{deg } -1}{A_\mu^{*c}} = \dfrac{\partial \Psi}{\underset{\text{deg } \phi}{\partial A_r^c}}$ ← deg −1

but there is no neg degree field in $\{A_\mu, H^a\}$!

<u>Solution</u>: add a <u>trivial pair</u>: a pair of fields with deg. d and $d+1$ that don't mix with the other fields.

One such pair is $\underset{(-1)}{\bar{H}^a}$ and $\underset{(0)}{b^a}$ with

$s(\bar{H}^a) = b^a$; $s(b^a) = 0$ together with their anti-fields $\underset{(0)}{\bar{H}^{*a}}$ and $\underset{(-1)}{b^{*a}}$

Then $S_t := \int \bar{H}^{*a} b_a$ solves the master equation separately. In addition $\bar{H}^a$ the required deg −1 field needed to construct a gauge fixing fermion. We choose

$$\Psi = -\bar{H}_a \left(\frac{1}{2\lambda} b_a + F_a \right)$$

Then, with $\Phi^{*}_{I} = \dfrac{\partial \Psi}{\partial \phi^{I}}$:

$$A^{*c}_{\mu} = -\bar{H}^{a} \frac{\partial F_a}{\partial A_\mu^{c}}$$

$$\bar{H}^{*a} = -\left(\frac{1}{2\lambda} b_a + F_a\right)$$

$$b^{*}_{a} = -\frac{1}{2\lambda} \bar{H}^{a}$$

$$H^{*a} = 0$$

upon substitution in $S = S_0 + S_t$ we get:

$$S\left[\phi^I, \Phi^{*I} = \frac{\partial \Psi}{\partial \phi^I}\right] = S_0[A_\mu] - \underbrace{\int \overbrace{s(A_\mu{}^c)}^{D_\mu H^c} \bar{H}^a \frac{\partial F_a}{\partial A_\mu^c}}_{S_{FP}} - \underbrace{\int b_a\left(\frac{1}{2\lambda} b_a + F_a\right)}_{S_{gf}}$$

Note that here, rather than starting with the gauge fixed path integral for $S[A_\mu]$, which requires the the addition of ghosts and anti-ghosts, we started with the simplest non-trivial solution of the master equation, which depends only on A_μ, H^a and their anti-fields and which does not contain a gauge fixing term. This makes sense since this is a classical theory. The anti-ghost enters only once we fix the anti-fields, whereupon the antifields are essentially identified with the anti-ghost and gauge fixing term respectively.