In closing our discussion of the BRST quantisation, let us return to the geometric interpretation of what we have done. Starting with the ill-defined path integral

$$\int D[A]\ f(A_\mu)\ e^{\frac{i}{\hbar} S[A_\mu]}$$

which formally integrates over cell vector potentials including orbits over gauge equivalent configurations.

we factorised the pure gauge contributions $\int D[g]$

with the help of the Faddeev-Popov trick and then replaced

$$\int D[g] \quad \text{by} \quad \int D[H,\bar{H}]$$

which has the advantage of

$$\int D[A,\bar{H},H]\ \delta[F^a]\ e^{\frac{i}{\hbar} S(A_\mu) - \int \bar{H} M H}$$

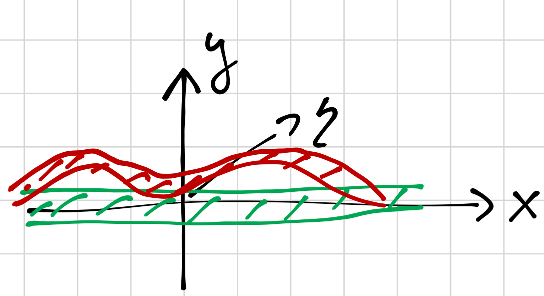
being finite and furthermore independent on the choice of gauge fixing

Illustration:

$$\int dx\, e^{-x^2} = \int dx\,dy\ \delta(f(y))\,|f'(0)|\, e^{-x^2}$$

$f^{-1}(0)$

$$= \int dx\,dy\,d\eta^* d\eta\ \delta(f(y))\, e^{-x^2+\eta^*|f'(0)|\eta}$$



more generally, $f(y) \to f(y,x)$

n. b: only tangent space at zero of f is relevant

Rep: In the FP-path integral ghost fields where first introduced as a technical tool to exponentiate the FP-determinant. Then, using the (odd) BRST invariance of the so-obtained extended action we were able to reformulate the gauge invariance of physical observables as well as the Slavnov-Taylor identities of the gauge variant correlation functions as a chomological problem in the extended field space.

In the BV-formulation one enlarges the field space already at the classical level by introducing a graded space of fields $\mathcal{F}$, in a way that the BRST invariance is manifest before choosing a gauge fixing. For this one introduces yet more extra fields (usually called anti-fields) that will be determined in terms of the actuel fields only after gauge fixing. Concretely, let $\{\phi^I\}$ Denote the fields encountered in the FP-path integral, that

is $\{\phi^I\} = \{ A^c_\mu, \psi^i_\alpha, H^a, b_a, \bar{H}_a \}$ $\qquad \deg(\phi^I) = gh(\phi^I)$

where I Is a multi index ~~running over~~ .Then

$\{\phi^{I*}\}$

Are the corresponding anti fields with $\deg(\phi^{*I}) = -gh(\phi^I) - 1$.

The first problem is then to choose an appropriate invariant action of deg $\emptyset$.

For a given gauge-invariant action $S[\phi]$ (boundary condition at $\phi^* = 0$)

The simplest choice is $\quad S'[\phi,\phi^*] = S[\phi] + \int s(\phi^I)\,\phi^{*I}\, d^4x$

This action satisfies the master eqn.

$$\sum_I \int \frac{\delta_R S'}{\delta \phi^{*I}(z)} \frac{\delta_L S'}{\delta \phi^I(z)}\, d^4z = 0$$

Indeed, at zero-th order in ϕ^* we have

$$\int s(A^a_\mu)(z)\, \frac{\delta S}{\delta A^a_\mu(z)}\, d^4z = 0$$

gauge inv. of $S[A]$

at 1st order in ϕ^{*I}:

$$\int d^4z \, s(\phi^I)(z) \frac{\delta_L S'}{\delta \phi^I(z)} = \int s(\phi^I)(z) \int \frac{\delta s(\phi^J)(y)}{\delta \phi^I(z)} \phi^{*J}(x) d^4y \, d^4z$$

$$= \sum_I s^2(\phi^J) \phi^{*J} \quad = 0.$$

by the nilpotency of s.

In order to recover the FP-action we introduce a fermionic gauge fixing functional (or <u>gauge-fixing fermion</u>)

$$\Psi[\phi^I] \text{ together with } \phi^{*I} \stackrel{!}{=} \frac{\delta \Psi[\phi]}{\delta \phi^I}$$

Then,

$$S'[\phi] = S[A_\mu] + \int s(\phi^I) \frac{\delta \Psi[\phi]}{\delta \phi^I}$$

$$= S[A_\mu] + s\Psi[\phi] = S^{tot}$$

with path integral

$$\int D[\phi, \phi^*] \, \delta\left(\phi^{*I} - \frac{\delta \Psi}{\delta \phi^I}\right) e^{\frac{i}{\hbar} S^{tot}}$$

More generally, we can assume that S' is a non-linear functional of the antifields ϕ^{*I} with an expansion of the form

$$S'[\phi, \phi^*] = S[A_\mu] + \int \underset{(1)}{H^a} \underset{(0)}{f_a^{\,r}[A]} \underset{(-1)}{A^{*r}}$$

$$+ \frac{1}{2} \int \underset{(1)\,(1)}{H^a H^b} \underset{(0)}{f^c_{ab}[A]} \underset{(-2)}{H^{*c}}$$

$$+ \frac{1}{2} \int H^a H^b f^{rs}_{ab}[A] A^{*r} A^{*s}$$

$$+ \ldots$$

Then the master equation $0 \overset{!}{=} \int \frac{\delta_R S'}{\delta \phi^{*I}} \frac{\delta_L S'}{\delta \phi^I}$ gives

$O(\phi^{*0})$: $\int \underbrace{H^a f^I_a[A]}_{\delta A^I = s(A^I)} \frac{\delta S[A]}{\delta A^I} = 0$ $\Rightarrow$ gauge invariance of $S[A_\mu]$ with $\theta^a \rightsquigarrow H^a$

$$O(\phi^*): \left.\begin{array}{l} \int H^a f_a^{\,r}(A) H^b \dfrac{\delta f^s_b(A)}{\delta A^r} A^{*s} \\ + \int \frac{1}{2} H^a H^b f^c_{ab}[A] f^s_c(A) A^{*s} \end{array}\right\} \Leftrightarrow [\delta_\alpha, \delta_\beta] = \delta_{[\alpha,\beta]}$$

$$+ \int H^a H^b f^{rs}_{ab}[A] A^{*r} \frac{\delta S}{\delta A^s} \Big\} = 0 \quad \text{for } f^{rs}_{ab} = 0 \text{ or } \frac{\delta S}{\delta A^s} = 0 \text{ e.m.}$$