Let us now consider

$$\int D[A,H,\bar{H}]\ \bar{H}_a(x)\, f(A_\mu)\, e^{\frac{i}{\hbar} S^{tot}[A,H,\bar{H}]} = 0$$

$\int d\eta\, d\eta^*\ \eta^*$ even $= 0$

Then, performing the change of variables (*) does not change the integral. Thus

$$0 = \int D[A,H,\bar{H}]\,\delta\xi \left[-\lambda F_a(x) f(A_\mu) - \bar{H}_a(x)\int d^4z\, \frac{\delta f}{\delta A(z)} (DH)(z)\right] e^{\frac{i}{\hbar}S^{tot}}$$

$$= \delta\xi \int D[A]\,\Delta[A_\mu] \underbrace{\left[-\lambda F_a(x) f(A_\mu) + i\hbar \int \frac{\delta f}{\delta A_\mu} D_\mu M^{-1}\right]}_{(\Delta)} e^{\frac{i}{\hbar}S^{tot}}$$

which is equivalent to the previous identity ($\square$) using

$$\delta F_a = \int \frac{\delta F_a}{\delta A^c_\mu}\delta A^c_\mu = \frac{1}{g}\int M_{ab}\,\theta^b \qquad \delta f(A_\mu) = \frac{1}{g}\int \frac{\delta f}{\delta A^c_\mu}(D^\mu\theta)^c$$

and acting on (Δ) with $iM\theta$ from the right.

Thus, the BRST invariance is a reformulation of the Slavnov-Taylor identities described above.

Next, we would like to express the BRST symmetry without referring to a specific choice of gauge fixing. For this we express the F^aF^a term in S^{tot} with the help of a (Nakanishi-Lautrup) auxiliary field b^a as e.m.

$$\mathcal{L}_{gf} = \left(\frac{1}{2\lambda} b^a b^a + b^a F^a\right) \quad ; \quad -\frac{b}{\lambda} + F = 0$$

Then S^{tot} is invariant under

$$(++)\left\{\begin{aligned} s(A^\mu_a)(x) &= D^\mu_{ab} H^b(x) \\ s(\bar{H}_a)(x) &= b_a(x) \\ s(H^a)(x) &= \tfrac{1}{2} C^a{}_{bc} H^b(x) H^c(x) \\ s(b_a)(x) &= 0 \end{aligned}\right.$$

In particular $\boxed{s^2 = 0}$ (exercise)

The point is then that since $s(b_a) = 0$ we can replace

$$\frac{1}{2\lambda} b^a b^a$$

by any function of b^a while preserving the BRST invariance and consequently the Slavnov-Taylor identities. Moreover, we can write

$$-\frac{1}{2\lambda} b_a b_a - b_a F_a + \bar{H}_a M^a{}_b H^b =: s(\Psi)$$

$$\Psi = -\bar{H}_a\left(\frac{1}{2\lambda} b_a + F_a\right)$$

or,

$$\boxed{S^{tot} = S + s(\Psi)} \qquad (**)$$

This makes explicit that, since the original action S is BRST invariant, the gauge fixed action S^{tot} obtained by addin an s-exact term is BRST invariant as well, due to $s^2 = 0$. Moreover, Ψ need not be of the form (**). Indeed, for any choice of Ψ the BRST invariance will survive, although not all choices of Ψ will be suitable for gauge fixing.

On the other hand, the presence of the global BRST symmetry (++) implies the existence of a conserved current

$$j^{\mu}_{BRST} \; ; \; \partial_\mu j^{\mu}_{BRST} = 0$$

which is odd, as well as a conserved charge

$$Q \equiv Q_{BRST} = \int d^3x \, j^0_{BRST}$$

with $[Q, \Phi] = i s(\Phi)$ for all functions Φ of the fields A^a_μ, H^a, $\bar{H}^a$. Here $[\,,\,]$ is the graded commutator, that is $[A, B] = AB - AB$ if A and/or B are even and $[A, B] = AB + BA$ if A and B are odd.