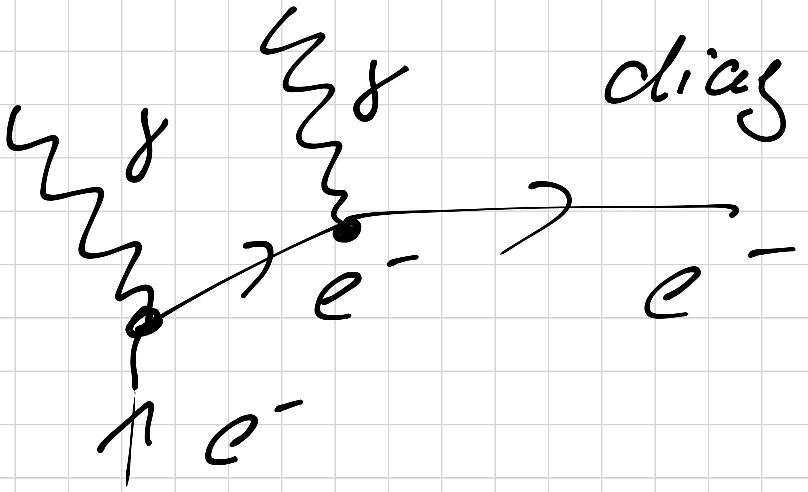
Quantization of gauge theories

Prerequisites:

- Canonical quantisation
- QED (Spinors, γ-matrices
  Dirac eqn. (Feynman diag.)



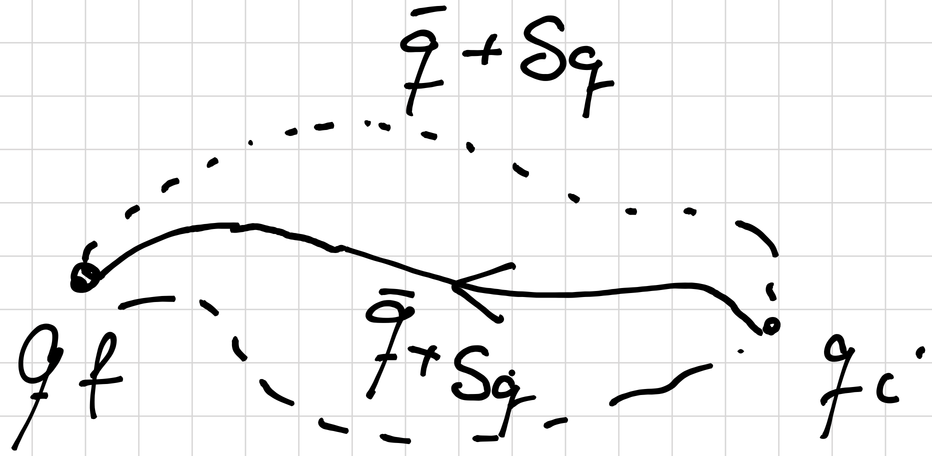
Prerequisites:

- QM- path integral:

$$\langle f| e^{-\frac{i}{\hbar}Ht} |i\rangle = N \int D[q]\, e^{\frac{i}{\hbar} S[q(t)]}$$

$\bar{q}(t)$: class. traj.

$=$



PI for gauge theories:

$q \sim q + \delta q$ eg: $A_\mu(\underline{x},t) + A_\mu(\underline{x},t) + \partial_\mu \lambda(\underline{x},t)$

vect. pot. in ED.

pure gauge

$$S[q+\delta q] = S[q]$$

$$\int D[q]\, e^{\frac{i}{\hbar} S[q]}$$ not well-defined!

$q + \delta q$ is not suppressed.

→ <u>gauge fixing</u>:

QED: can choose Coulomb gauge $\boxed{\underline{\nabla} \cdot \underline{A} = 0}$

→ solve this constraint and subsitute in PI

- often inconvenient
- not covariant
- conservation laws not explicit

→ gauge fixing:

① Faddeev-Popov trick
- ↳ BRST quantization
  - ↳ BV action

Contents:

0) Motivation and construction of gauge theories

1) Path integrals in field theory

2) Feynman-deWitt-Faddeev-Popov quantisation

3) Ward- Takahashi-Slavnov-Taylor

4) BRST quantisation

gauge fixing of PI:

5) BV-formulation <u>Optional</u>

6) Applications:
    - Renormalisation and anomalies
    - “baby” string field theory
    - construnctive field theory

Literature:

S. Pokorski, gauge field theories, Cambridge gauge th. BRST

S. Weinberg, the quantum theory of fields, volume 2 BV

Itzyskon-Zuber, quantum field theory QFT

J. Zinn-Justin, Quantum Field Theory and Critical Phenomena

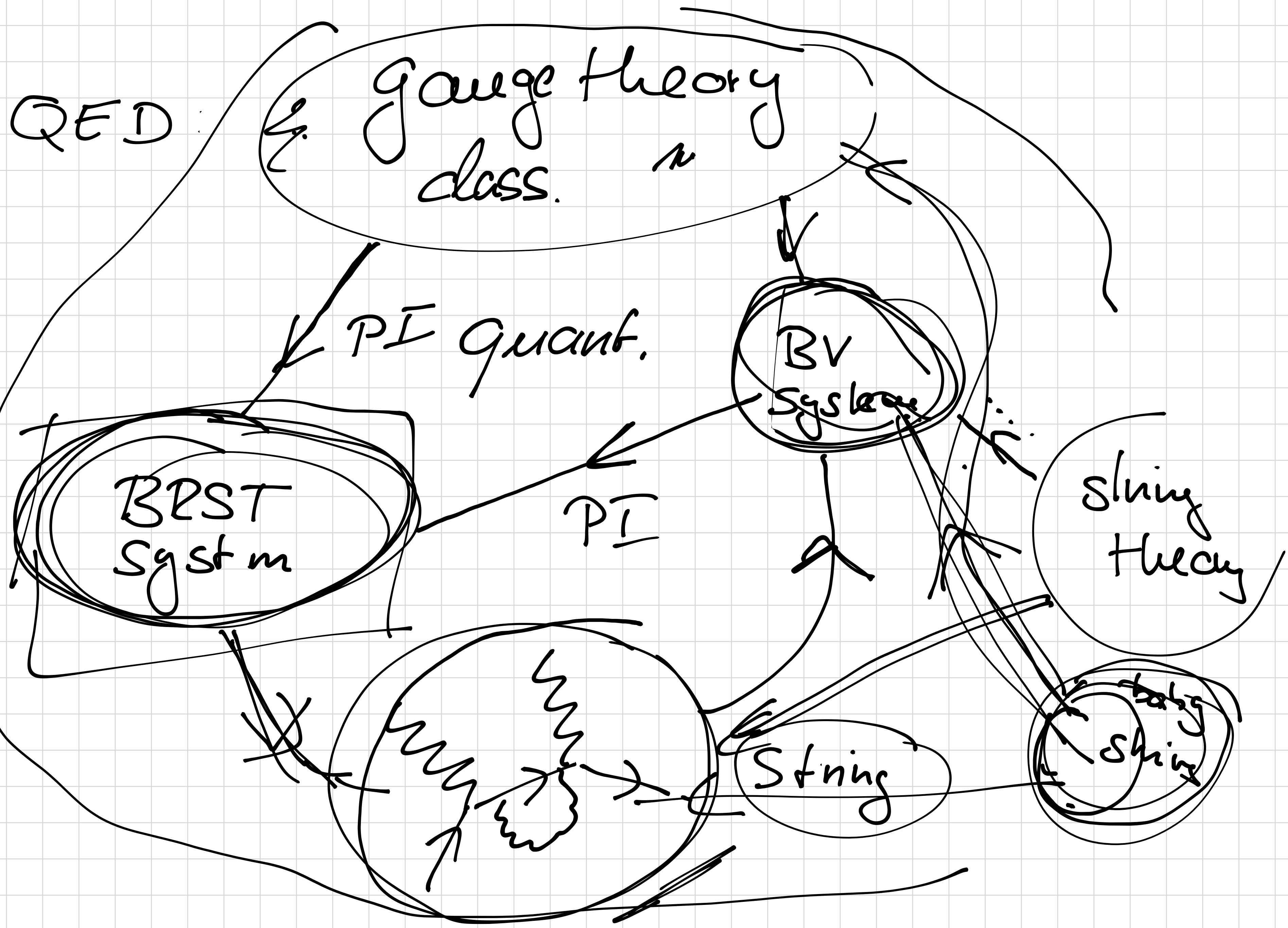
A. Zee, Quantum Field Theory in a Nutschell

very good for beginners

check library for E-books !

Organisation:

- Lecture starts at 2:15 and 10:15 (c.t)
- Need to register for Friday lecture separately! ✓
- I will send a link for recorded lectures. You will need an LMU account!
- If you are not sure about prerequisites, ask me in the break.
- visit the home page for more info.
- Exam: later (we don't know yet)
- Tutorials: see poll. Fri 12 - 2pm.


QED :
Gauge theory class.
PI quant.
BV System
BRST System
PI
string theory
String
baby string

0) Construction of gauge th's

Generalities

- global symmetries + Lagrangian

  $\rightsquigarrow$ Noether charge $\underline{Q}$

ex: 1) Phase of QM wave fn $\psi(\underline{x},t)$

Notation: $x = (\underline{x}, t)$

internal symm. $\boxed{\psi(x) \mapsto e^{i\overset{\in \mathbb{R}}{\alpha}} \psi(x)}$

conservation law is particle number

lecture.

Register for Friday lecture as well

Rem: in a Lorentz inv. descr.

$$\underline{\underline{Q}} = \int d^3x \; j_0(\underline{x},t) \; ; \; \boxed{\partial_\mu j^\mu(x) = 0}$$

locally conserved current.

$$L_{int} = \boxed{A_\mu(x) j^\mu(\underline{x},t)} \sim (A_\mu + \underline{\underline{\partial_\mu \alpha}})\, \underline{\underline{j^\mu}}$$

↑ interaction.

→ $A_\mu(\underline{x},t)$ is partly redundant.

forced upon us by insisting on locality.

∴ Consistent with gauge invariance
in Electro-magnetism

<u>Rep:</u>

$$F_{\mu\nu} = \underline{\partial_\mu A_\nu} - \underline{\partial_\nu A_\mu} = \left(\begin{matrix} 0 & E_1 & E_2 & E_3 \\ & 0 & B_2 \cdot & B_2 \\ & & 0 & B_1 \\ & & & . \end{matrix} \right.$$

A_μ partly redundant.

$$L_{int} = A_\mu \underset{\overset{\nwarrow}{\underline{\bar{\Psi}}\,\gamma^\mu\,\underline{\Psi}}}{j^\mu}$$

ex 2: rotation invariance

→ conservation of ang. mom.

Space-time symm.

In a Lorentz inv. descr. angular mom.

is a moment of $\underbrace{T_{\mu\nu}}_{\text{stress tensor}} = T_{\nu\mu}$

$\partial^\mu T_{\mu\nu} = 0$ conserved.

$$\mathcal{L}_{int} = \underset{\nwarrow \text{ spacetime metric}}{g_{\mu\nu}} T^{\mu\nu} \sim (g_{\mu\nu} + \xi_{\mu,\nu}) T^{\mu\nu}$$

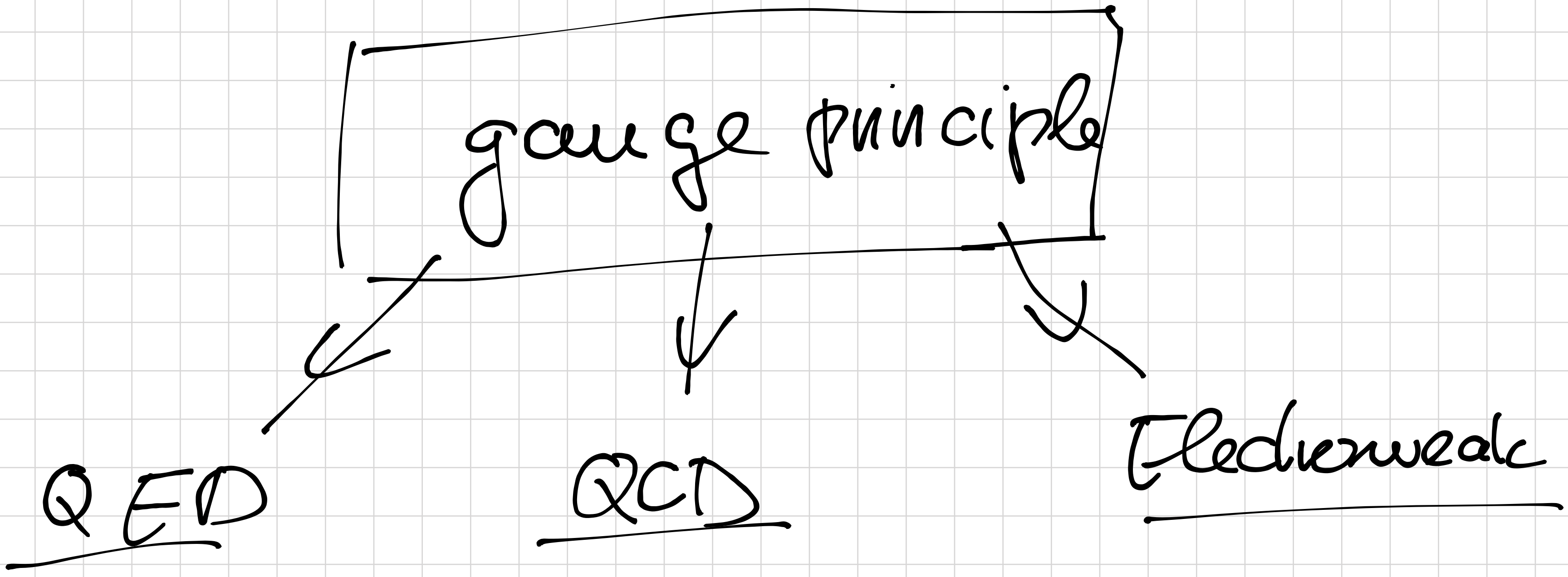
$$\xi_{\mu,\nu} \equiv \partial_{x^\nu} \xi_\mu(x)$$

this does not give rise to a gauge theory in the usual sense.

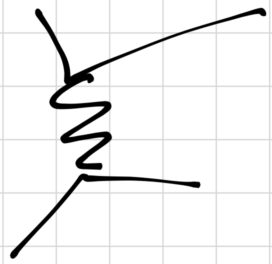
Generally global symmetries allways seem to be part of a local invariance when gravity is included.

arguments for this come from

- Black Holes
- String theory (does not have global symm)



seem to describe nature in terms of gauge theory.



except perhaps Yukawa couplings ← Higgs