BBSM Neutrino Course

Lecture $\overline{\underline{VIII}}$

May 19, 2020

LMU

Spring 2020

It's neutrino, stupid!

More on massive gauge fields:
Proca vs Stuckelberg

Proca:

$$\text{inv.} \longrightarrow \cancel{A_\mu \to A_\mu + \partial_\mu \alpha}$$

$$\mathcal{L}_P = -\frac{1}{4} F_{\mu\nu} F^{\mu\nu} + \frac{1}{2} m_A^2 A_\mu A^\mu$$

$$- j_\nu A_\nu$$

$$\Rightarrow \boxed{\partial_\mu F^{\mu\nu} + m_A^2 A^\nu = j^\nu}$$

$$m_A = 0 \Rightarrow \text{Maxwell} \Rightarrow \partial^\mu j_\mu = 0$$

$$\partial_\nu \partial_\mu F^{\mu\nu} = 0$$

$$\Rightarrow \boxed{m_A^2 \partial^\nu A_\nu = \partial^\nu j_\nu}$$

$\boxed{\bullet\ j_\nu = 0}$ $\quad \partial^\nu A_\nu = 0 \Rightarrow 3$ d.o.f.

$(s = 1)$

<u>at rest</u> $\quad \varepsilon_0 = 0 \qquad (A_\mu = \epsilon_\mu e^{-ipx})$

$$\mathcal{L}_P = \frac{1}{2} A_\mu \left[(\Box + m_A^2) \delta^{\mu\nu} - \partial^\mu \partial^\nu \right] A_\nu$$

$$\Downarrow$$

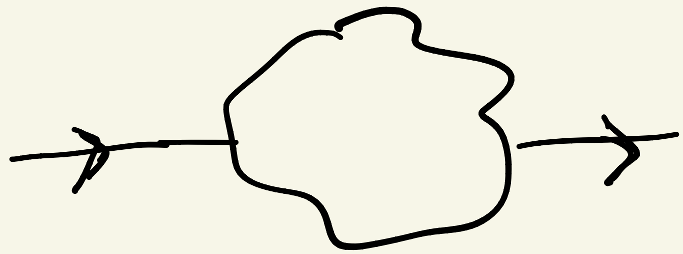
$\partial_\mu A^\mu = 0$ $\quad P$

$$\Delta_{\mu\nu} = -i \, \frac{g_{\mu\nu} - \frac{p_\mu p_\nu}{m_A^2}}{p^2 - m_A^2}$$

$$= \boxed{\frac{i \sum_{i=1}^{3} \epsilon_\mu^i \epsilon_\nu^{*i}}{p^2 - m_A^2}}$$

$m_A \to 0$ | $M_W = 80$ GeV $= 80\, m_p$

$E \to \infty$



$$\int_0^\infty d^4k \;\; \square = \text{finite}$$

$$\downarrow$$

$$0\,, \quad k \to \infty$$

$$\boxed{\Delta_{\mu\nu}\,(\text{Proca}) \xrightarrow[p \to \infty]{} \frac{1}{m_A^2}}$$

$$\mathcal{L}_M = -\frac{1}{4} F_{\mu\nu} F^{\mu\nu}$$

2 d.o.f.

transverse $\quad h = \pm 1$

$$h \equiv \vec{S}\cdot\hat{p}$$

<u>QED</u> $\Longleftrightarrow$ gauge invariance

(gauge redundancy)

$$\boxed{\text{2 physical d.o.f.}} \longleftrightarrow \boxed{4\ A_\mu}$$

$$\Downarrow$$

$$\boxed{A_\mu \to A_\mu + \partial_\mu \alpha(x)}$$

$$2 = 4 - 2$$

<u>Proca</u> $\underbrace{3\ \text{d.o.f.}}_{\text{physical}} \longleftrightarrow A_\mu$

$$3 = 4 - 1$$

$$\Downarrow$$

5 d.o.f.

$\boxed{\text{Stueckelberg '50}}$

$A_\mu, \; G$

$$\mathcal{L}_P \to \mathcal{L}_S = -\frac{1}{4} F_{\mu\nu} F^{\mu\nu} + \frac{1}{2} m_A^2 \left(A_\mu - \frac{1}{m_A} \partial_\mu G \right)^2$$

$\downarrow$

$\partial_\mu G$

$$\boxed{\begin{aligned} A_\mu &\to A_\mu + \partial_\mu \alpha(x) \\ G &\to G + m\,\alpha(x) \end{aligned}}$$

(1)

$$\frac{1}{2} m_A^2 A_\mu A^\mu - m_A A^\mu \partial_\mu G + \frac{1}{2} (\partial_\mu G)^2$$

$$= -\shortparallel- \quad + \boxed{m_A (\partial^\mu A_\mu) G} \; (*) + -\shortparallel-$$

reminder

get rid!

QED $(m_A = 0)$ $\mathcal{L} = -\frac{1}{4} F^2$

$$= \frac{1}{2} A^\mu \underbrace{(\Box g^{\mu\nu} - \partial^\mu \partial^\nu)}_{\boxed{\text{NO inverse}}} A_\nu$$

momentum

$$[-k^2 g^{\mu\nu} + k^\mu k^\nu]$$

$\boxed{\text{no propagator}}$

$$\boxed{+ \mathcal{L}_{gf}^{\text{Maxwell}} = \frac{1}{2\xi} (\partial^\mu A_\mu)^2}$$

QED

$\Downarrow$

$$D_{\mu\nu}(\text{Maxwell}) = -i \frac{g_{\mu\nu} + (\xi - 1)\frac{k_\mu k_\nu}{k^2}}{k^2}$$

$m_A = 0$

$\xi = 1$ Feynman

$$\boxed{\text{Physics} \neq \xi \text{ depend}}$$

$\downarrow$

<u>generalize to Stueckelberg</u>

$$\mathcal{L}_{g.f}(\xi) = \frac{1}{2\xi}\left(\partial_\mu A^\mu - \xi m_A \sigma\right)^2$$

$$= \underbrace{\frac{1}{2\xi}\left(\partial_\mu A^\mu\right)^2}_{\boxed{\text{QED}}} \boxed{- m_A \sigma \partial^\mu A_\mu} +$$

(*) vs (1)

$$\boxed{+ \frac{1}{2}\xi^2 m_A^2 \sigma^2}$$

(2)

$\Downarrow$

$$\mathcal{L}_S + \mathcal{L}_{gf} = -\frac{1}{4} F_{\mu\nu} F^{\mu\nu} + \boxed{\frac{1}{2\xi}\left(\partial^\mu A_\mu\right)^2}$$

$$+ \frac{1}{2} m_A^2 A_\mu A^\mu$$

$$+ \frac{1}{2}\left(\partial_\mu \sigma\right)^2 - \frac{1}{2}\boxed{\xi} m_A^2 \sigma^2$$

$$\boxed{m_G = \xi m_A \qquad \underline{\text{NOT}} \text{ physical} \qquad \xi \to \infty \Rightarrow G \text{ decouples}}$$

Propagators

$$G: \quad \frac{i}{k^2 - m_G^2} = \frac{i}{k^2 - \xi m_A^2}$$

$$\xi \to \infty \Rightarrow \underline{\text{goes to zero!}}$$

$$A: \quad \boxed{\Delta_{\mu\nu}(A) = -i \, \frac{g_{\mu\nu} + (\xi - 1) \frac{k_\mu k_\nu}{k^2 - \xi m_A^2}}{k^2 - m_A^2}}$$

$$\Downarrow$$

- $\boxed{\xi = \text{finite}} \Rightarrow \Delta_{\mu\nu} \xrightarrow[k\to\infty]{} \frac{1}{k^2}$

$\boxed{\text{well-behaved!}}$

- Proca : $\xi \to \infty$ after the computation

$\parallel$

$\boxed{\text{unitary gauge}}$ $(\xi = \infty)$

$\hookrightarrow \boxed{\Delta_{\mu\nu} \to \Delta_{\mu\nu}\,(\text{Proca})}$

- $$\Delta_{\mu\nu}(A) = \Delta_{\mu\nu}(\text{Proca}) + \underbrace{\text{rest}}_{}\overset{\overset{C(\xi)}{\shortparallel}}{} k_\mu k_\nu$$

$$\cancel{g_{\mu\nu}} + (\xi - 1)\frac{k_\mu k_\nu}{k^2 - \xi m_A^2} = \cancel{g_{\mu\nu} - \frac{k_\mu k_\nu}{m_A^2}}$$

$$+ \underset{\overset{\shortparallel}{C(\xi)}}{\boxed{\text{rest}}}\, k_\mu k_\nu$$

$$\Rightarrow \quad C(\xi) = \frac{\xi - 1}{k^2 - \xi m_A^2} + \frac{1}{m_A^2}$$

$$= \frac{(\cancel{\xi} - 1) m_A^2 + k^2 \cancel{- \xi m_A^2}}{(k^2 - \xi m_A^2) m_A^2}$$

$$= \frac{k^2 - m_A^2}{(k^2 - \xi m_A^2) m_A^2}$$

$$\Downarrow$$

$$\boxed{\Delta_{\mu\nu}(A) = \Delta_{\mu\nu}(\mathrm{Proca}) + \boxed{(-i) \frac{k_\mu k_\nu}{m_A^2 (k^2 - \xi m_A^2)}}}$$

$$\boxed{D(G) = \frac{i}{k^2 - \xi m_A^2}}$$

cancel

Stückelberg '50s

't Hooft '71

$\boxed{\text{Dvali}}$

<u>Proca</u> $\tilde{A}$ → Proca massive gauge

$$\mathcal{L} = -\frac{1}{4} F_{\mu\nu} F^{\mu\nu} + \frac{1}{2} m_A^2 \tilde{A}_\mu \tilde{A}^\mu$$

$$\tilde{A}_\mu = A_\mu + \frac{1}{m} \partial_\mu G$$

physical $m=0$

longitudinal

$\boxed{U(1)}$

$\boxed{\text{Proca}}$

$$F_{\mu\nu}(\tilde{A}) = F_{\mu\nu}(A)$$

$$(\partial^\mu \partial^\nu G - \partial^\nu \partial^\mu G = 0)$$

$$\mathcal{L}_P = \mathcal{L}_M + \frac{1}{2} m_A^2 (\partial_\mu \, - -)^2$$

$$+ g\left[\left(A_\mu + \frac{1}{m}\partial_\mu G\right)\left(A^\mu + \frac{1}{m}\partial^\mu G\right)\right]^2$$

$\sim A^4$

$$\boxed{\begin{aligned} A_\mu &\to A_\mu + \partial_\mu \alpha \\ G &\to G - \underline{m}\,\alpha \end{aligned}}$$

$$\boxed{\frac{g}{m^4}(\partial_\mu G)^4}$$

$$\boxed{m_A A_\mu \partial^\mu G} \leftarrow \text{bad}$$

- Proca $\overset{\text{NO } G}{}$ = physical = unitary
- Dvali — UV completion (G = long. A)
- $R_\xi \quad (A_\mu, G)$

  $\uparrow$ unphysical

U(1) — more

$$\boxed{\partial^\nu j_\nu = 0}$$

(QED)

$$\mathcal{L}_M = -\frac{1}{4} F_{\mu\nu} F^{\mu\nu} + j^\nu A_\nu$$

$$\Longleftrightarrow \quad \partial^\nu j_\nu = 0$$

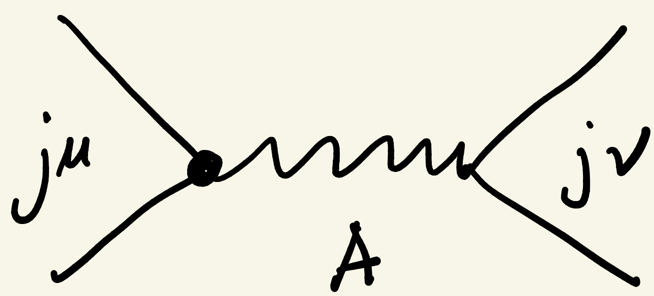
$$j^\mu = \bar{\psi} \gamma^\mu \psi \Rightarrow \partial^\mu \bar{\psi} \gamma_\mu \psi = 0$$

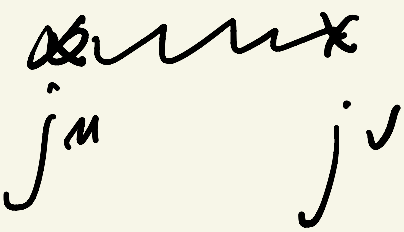
$$\delta \mathcal{L}_M = + \frac{1}{2} m_A^2 A_\mu A^\mu$$

$\hookrightarrow$ Proca

$$\Delta_{\mu\nu} \longrightarrow \boxed{g_{\mu\nu} - \frac{k_\mu k_\nu}{m_A^2}}$$

...





$$k^\mu j_\mu = 0 \Rightarrow \boxed{\text{decouples}}$$

$$\boxed{m_A \lesssim 10^{-34}\,\text{eV}} \qquad \boxed{3 \text{ d.o.f.}}$$

$\rightarrow$ $\boxed{\text{add } G}$ $\left[\left(A_\mu - \frac{1}{m} \partial_\mu G \right)^2 \right]$

$\left[\qquad\qquad \right]^4$

$\rightarrow$ $U(1)$: $D_\mu = \partial_\mu - i g A_\mu Q$

G :

$$\boxed{\begin{aligned} D_\mu &= \partial_\mu - i g\, T_a A_\mu^a \\ [T_a, T_b] &= i f_{abc} T_c \end{aligned}}$$

$$\boxed{\begin{aligned} A_\mu \equiv A_\mu^a T_a &\longrightarrow U A_\mu U^{-1} + \frac{i}{g} (\partial_\mu U) U^{-1} \\ \psi &\longrightarrow U \psi \end{aligned}}$$

$$F_{\mu\nu} \equiv F_{\mu\nu}^a T^a \rightarrow U F_{\mu\nu} U^{-1}$$

$$F_{\mu\nu} = \partial_\mu A_\nu - \partial_\nu A_\mu + i g [A_\mu, A_\nu]$$

global $U = $ const. $A_\mu \to U A_\mu U^{-1}$

$\boxed{YM}$ G gauge bosons carry charges

$U(1)$ $A_\mu \to A_\mu$ photon = neutral

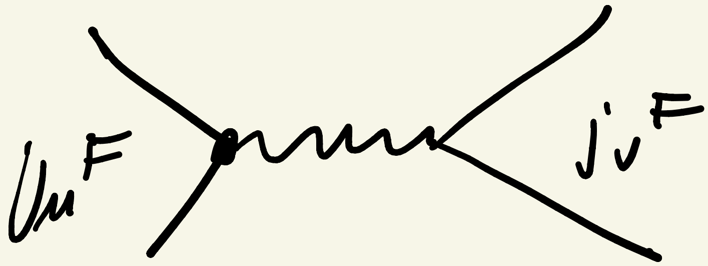
$$F_{\mu\nu}^a = \partial_\mu A_\nu^a - \partial_\nu A_\mu^a + g f_{abc} A_\mu^b A_\nu^c$$

$$F_{\mu\nu}^a F^{\mu\nu a} \to \cdots + \boxed{g^2 f f AAAA}$$

global $\psi \to U\psi$, $A_\mu \to U A_\mu U^{-1}$

$$\partial^\mu j_\mu^a = 0$$

$$j_\mu^a = \bar\psi \gamma^\mu T^a \psi + g F_{\mu\nu}^b A^{\nu c} f_{abc}$$



$$\partial^\mu j_\mu^F \neq 0$$

$$\partial^\mu \bar{\psi} \gamma_\mu T_a \psi \neq 0$$

$\boxed{U(1)}$ PROCA

$g A^\mu j_\mu \qquad j_\mu = \bar{\psi} \gamma_\mu (a + b\gamma_5) \psi$

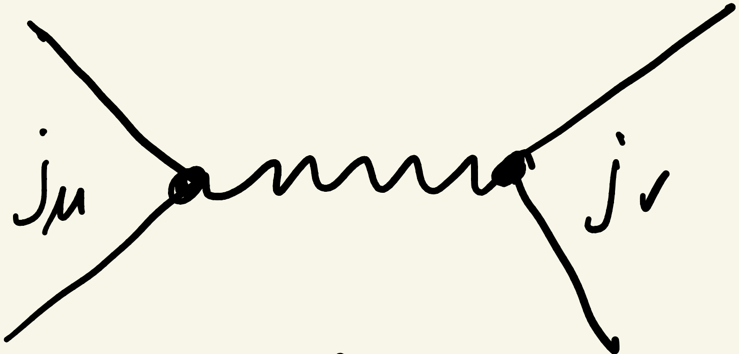
<u>weak</u> $a = b = 1$ (W) $\boxed{m_A \neq 0}$

$a_Z, b_Z = ?$ (Z)

$$\boxed{\partial^\mu j_\mu \neq 0}$$

$\partial^\mu \bar{\psi} \gamma_\mu \psi = 0$
(Dirac)

$$\boxed{\partial^\mu \bar{\psi} \gamma_\mu \gamma_5 \psi = m_f \bar{\psi} \gamma_5 \psi \, b} \quad ?$$



$$k^\mu j_\mu \neq 0$$

$$j^\mu j^\nu \, \frac{g_{\mu\nu} - \frac{k_\mu k_\nu}{m_A^2}}{k^2 - m_A^2}$$

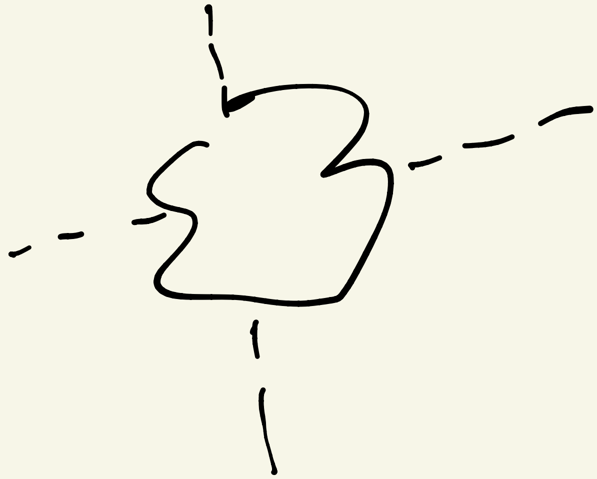
Wboam!

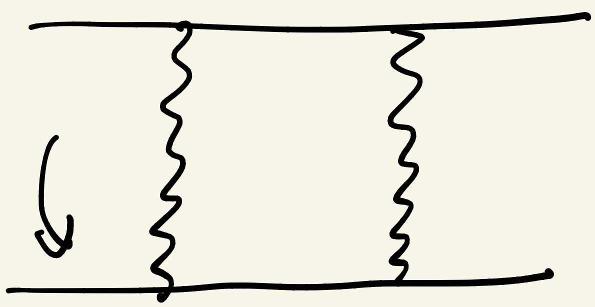
$$= \frac{j_\mu j^\mu}{k^2 - m_A^2} + \boxed{\left(\frac{m_\psi}{m_A}\right)^2 b^2 \frac{1}{k^2 - m_A^2}}$$

$m_A \to 0$ <u>not</u> smooth

· <u>Andrei:</u> can I work with Proca

$$\Delta \sim \frac{g - kk/m_A^2}{k^2 - m_A^2}$$





$$S_F = \frac{i}{\not k - m_f} \xrightarrow[k\to\infty]{} \frac{1}{\not k}$$

$$g - g_{\mu\nu}$$

$$\int^{\Lambda} d^4k \, \frac{1}{\not k}\, \frac{1}{\not k}\, \frac{1}{(k^2)^2}\left(g - \frac{kk}{m^2}\right)^2$$

① ②

$$① = \int^{\Lambda} d^4k \, \frac{1}{k^6} = \int^{\Lambda} k^2 dk^2 \, \frac{1}{k^6} \sim \frac{1}{\Lambda^2} \xrightarrow[\Lambda\to\infty]{} 0$$

$$② \int^{\Lambda} \cancel{k^2} dk^2 \, \frac{1}{\cancel{k^6}} \, \frac{\cancel{k^4}}{m_A^4} = \frac{\Lambda^2}{m_A^4} \xrightarrow[\Lambda\to\infty]{} \infty$$

$$\boxed{\text{Proca} \equiv \xi \to \infty \text{ limit of } R_\xi \text{ theory}}$$

- $\xi = 1$ 't Hooft – Feynman

  $\xi = 0$ Landau

keep ξ and check that it cancels