May 12, 2020

Neutrino BBSM Course

Lecture VI

LMU

Spring 2020

It's neutrino, stupid

Lecture 6

From Fermi to SM

weak int

$$\boxed{\mathcal{H}_{eff} = \frac{4G_F}{\sqrt{2}} J_\mu^{w} \bar{J}^\mu_w \quad \circledast}$$

$$\boxed{J^\mu_w = \bar{u}_L \gamma^\mu d_L + \bar{\nu}_L \gamma^\mu e_L} \quad 57$$

QED — photon messenger

weak

$$\circledast \quad \frac{g}{\sqrt{2}} W^+_\mu J^\mu_w + h.c.$$

Schwinger

$\mathcal{L}$ $\boxed{U(1) \text{ gauge inv.}}$

$$f \to e^{i\alpha Q} f \qquad Qf = q f$$

$q_e = -1,\ q_u = 2/3 \ldots$

$$\partial_\mu \to \boxed{D_\mu = \partial_\mu - ie A_\mu Q}$$

$W^+_\mu \bar{\nu}_L \gamma^\mu e_L \longleftrightarrow$ spin $\uparrow\downarrow$

$$l_L = \begin{pmatrix} \nu \\ e \end{pmatrix}_L \xrightarrow{W^-}$$ lepton doublet

$\Longleftrightarrow \boxed{SU(2)_L} \leftarrow$ only LH

$\cancel{e_R}$ W

$\boxed{SU(2)_L \text{ is observed in nature}}$

$\boxed{W^+, W^-, Z}$

$\boxed{M_W = 80 \text{ GeV}}$

$M_Z = 90 \text{ GeV}$

A photon

$T_i, \quad i = 1, 2, 3$

$T_i = \frac{\sigma_i}{2}$

$[T_i, T_j] = i \epsilon_{ijk} T_k$

weak spin (iso-spin)

$$Q_{em} \neq T_3 \qquad T_3 = \frac{\sigma_3}{2}$$

$$l: \quad Q_{em} = \begin{pmatrix} 0 & 0 \\ 0 & -1 \end{pmatrix}$$

$$q: \quad Q_{em} = \begin{pmatrix} 2/3 & 0 \\ 0 & -1/3 \end{pmatrix}$$

$$\boxed{Y = 2(Q - T_3)} \quad \text{hyper. charge}$$

$$l: \quad Y_l = 2\left[\begin{pmatrix} 0 & 0 \\ 0 & -1 \end{pmatrix} - \begin{pmatrix} 1/2 & 0 \\ 0 & -1/2 \end{pmatrix}\right]$$

$$= 2\begin{pmatrix} -1/2 & \\ & -1/2 \end{pmatrix} = (-1)\,\mathbb{1}$$

$$[Y_l, T_i] = 0$$

$$q : Y_L = 2\left[\begin{pmatrix} 2/3 & 0 \\ 0 & -1/3 \end{pmatrix} - \begin{pmatrix} 1/2 & 0 \\ 0 & -1/2 \end{pmatrix}\right]$$

$$= 2\begin{pmatrix} 1/6 & 0 \\ 0 & 1/6 \end{pmatrix} = \frac{1}{3}\,\mathbb{1}$$

$$\boxed{[Y, T_i] = 0}$$

$$\boxed{Q = T_3 + \frac{Y}{2}}$$

$$\boxed{SU(2)_L \otimes U(1)_Y}$$

LH int. with W!

$$[T_i, Y] = 0$$

$\boxed{\cancel{B}, \cancel{L}}$ $\boxed{d=6}$

$qqq\,l$ ← Lorentz symmetry

p, n

$\overline{\underline{(u_L^T C\, d_L)\ (u_L^T C\, e_L)}}$ (Lorentz)

$|\uparrow\downarrow - \downarrow\uparrow\rangle$

$\boxed{SU(2)_L}$

$l_L = \begin{pmatrix} \nu \\ e \end{pmatrix}_L$

$\boxed{SU(2)\ \text{inv.}\ ?}$

$$q_L = \begin{pmatrix} u \\ d \end{pmatrix}_L \qquad l_L = \begin{pmatrix} \nu \\ e \end{pmatrix}_L \qquad u_R,\ d_R \ldots \quad e_R$$

LH = doublet

$$\boxed{q_L^T C\, i\sigma_2\, q_L = SU(2) \text{ inv.}}$$

$$\| \qquad (u_L^T C d_L)$$

$$\underset{\substack{\uparrow \\ \text{doublet}}}{q_L^T} C\, i\sigma_2\, \underset{\substack{\uparrow \\ \text{doublet}}}{l_L} = (u_L^T d_L^T) C \begin{pmatrix} 0 & 1 \\ -1 & 0 \end{pmatrix} \begin{pmatrix} \nu \\ e \end{pmatrix}_L$$

$$\| \qquad \boxed{u_L^T C e_L - d_L^T C \nu_L}$$

$$q_L^T C\, i\sigma_2 q_L\; q_L^T C\, i\sigma_2\, l_L =$$

$$\boxed{= u_L^T C d_L \left(u_L^T C e_L - d_L^T C \nu_L\right)}$$

$\uparrow$ $\boxed{B - L}$ $\boxed{\text{good}}$

$$\boxed{p \to \pi^0 + e^+ \sim \pi^+ + \nu^c}$$

related

$\boxed{B + L\ ??}$

$q_L^T C i\sigma_2 q_L$ | $u_R^T C e_R$

$\left(\frac{1}{3}\right)$ $(u_L^T C d_L)$ | $(2/3 - 1 = -1/3)$

$(2/3 - 1/3)$ | $\cancel{d_R^T C \nu_R}$ $\boxed{\text{no } \nu_R}$

$$\boxed{d_R^T C (\nu^c)_R}$$

Lorentz, Qcu

(q_R, e_R) — interact with A

$\begin{pmatrix} u \\ d \end{pmatrix}_L$ $\quad u_R$, $\cancel{\phi}$ $d_R \xrightarrow{A} d_R$

$u_R \xrightarrow{A} u_R$

$\begin{pmatrix} \nu \\ e \end{pmatrix}_L$ $\quad e_R$, $\cancel{\nu_R}$

$\nu_R \Leftrightarrow \bar{\nu}_L \, \nu_R$

$Q = 0$, <u>no</u> weak int.

phantom

$\boxed{\underline{\text{SM scale}} \Longleftrightarrow E \simeq M_W \simeq 100\ \text{GeV}}$

. $E \gg M_W \Rightarrow \boxed{SU(2) \times U(1)}$

• (D) $\bar{\Psi}_L \Psi_R$ — Lorentz inv.

$\Longleftrightarrow$ electron

• (M) $\nu_L^T C \nu_L$ ||| $\nu_R^T C \nu_R$

||

Majorana

gravity $G_N \frac{m_1 m_2}{r} = V(r)$

$\hookrightarrow E_1 E_2$

$$\boxed{\alpha_{gr} \simeq E^2/M_{pl}^2 \sim 10^{19}\,\text{GeV}}$$

LHC

quarks, gluons

$e^+e^- \to Z \cdots$

h

$\boxed{y_e h \bar{e} e, \quad y_e \simeq 10^{-5}}$

• $\cancel{B}$!

$q_L^T C i\sigma_2 q_L$

$u_L^T C d_L \otimes d_R^T C (\nu^c)_R$

Lorentz, SU(2)

SU(2) singlet

??? why not?

SU(2)

d_R = singlet

$\boxed{(\nu^c)_R \equiv C \bar{\nu}_L^T}$

$\boxed{\nu_L \Leftrightarrow \text{doublet}}$

$\boxed{u_L{}^T C\, d_L\ \cancel{d_R{}^T C (\nu^c)_R}}$ breaks $SU(2)$

$\underbrace{q\,q\,q\,\ell}$ $\Leftarrow$ Lorentz

- $\dfrac{q\,q\,q\,\ell^c (\Phi^* - \text{Higgs})}{\Lambda^3}$ $\boxed{d=7}$

- $q\,q\,q\,\ell\, \dfrac{1}{\Lambda^2} \Rightarrow \boxed{\Lambda > 10^{15}\ \text{GeV}}$

Fermi: $\boxed{G_F\, J_W\, \bar{J}_W\,,}$ $E \ll \dfrac{1}{\sqrt{G_F}}$

$$\Uparrow\ G_F = \frac{1}{\Lambda_F^2} = \frac{g^2}{8M_W^2}$$

$E \ll \Lambda_F \Rightarrow$ leading $d=6$

4 fermion

Effective $q\,q\,e\,l$

$(u_L d_L)(u_L e_L - d_L \nu_L)$

$(u_R d_L)\, u_R\, e_R$

$u_R\, d_R\, u_R\, e_R$

$- - - -$

$\boxed{B-L}$

Tool towards fundamental theory

$\nu_L \rightarrow$ 2 d.o.f. (Majorana)

$\nu_L, \nu_R \rightarrow 2+2 = 4$ d.o.f. (Dirac)

weak

June 22 – July 2

Neutrino 2020

$\boxed{\Delta L \neq 0}$ $\boxed{\Delta B = 0}$ effective ?

$\nu_L^T C \nu_L$

SU(2) triplet!

Lorentz

NOT singlet

scalar

$T_3 = \frac{1}{2} + \frac{1}{2} = 1 \neq 0$

W — weak int.

$\hookrightarrow \frac{g}{\sqrt{2}}\left(\bar{u}_L \gamma^\mu d_L + \bar{\nu}_L \gamma^\mu e_L\right) W_\mu^+ + h.c.$

$$\boxed{M_W \neq 0}$$

Proca theory $=$ $\boxed{\text{Maxwell} + m_A}$

$\boxed{U(1)}$ $\mathcal{L}_P = \mathcal{L}_M + \frac{1}{2} m^2 A_\mu A^\mu$

$$= \underbrace{-\frac{1}{4} F_{\mu\nu} F^{\mu\nu}} + \frac{1}{2} m^2 A_\mu A^\mu$$

$$F_{\mu\nu} \equiv \partial_\mu A_\nu - \partial_\nu A_\mu$$

$\Downarrow$

$$\mathcal{L}_P = -\frac{1}{2}(\partial_\mu A_\nu - \partial_\nu A_\mu)\,\partial^\mu A^\nu + \frac{1}{2} m^2 A_\mu A^\mu$$

int. by part

$$= +\frac{1}{2}(A_\nu \Box A^\nu - A_\nu \partial^\mu \partial^\nu A_\nu) + \frac{1}{2} m^2 A_\mu A^\mu$$

$$= \frac{1}{2} A_\mu \left[(\Box + m^2) g^{\mu\nu} - \partial^\mu \partial^\nu \right] A_\nu$$

$$\Downarrow$$

check

$$\partial_\mu \quad \boxed{\left[(\Box + m^2) g^{\mu\nu} - \partial^\mu \partial^\nu \right] A_\nu = 0} \quad (*)$$

KG field $\Rightarrow$ $\boxed{(\Box + m^2)\phi = 0}$

$$\Uparrow$$

$$\mathcal{L}_{KG} = \frac{1}{2}(\partial_\mu \phi)(\partial^\mu \phi) - \frac{1}{2} m^2 \phi^2$$

$$\phi = e^{-ipx}\tilde{\phi}(p) \Rightarrow p^2 = m^2$$

$$\Downarrow$$

$$\boxed{E^2 = \vec{p}^{\,2} + m^2}$$

$$\circledast \Rightarrow \cancel{(\Box + m^2)\partial^\mu A_\mu} - \cancel{\Box \partial^\mu A_\mu} = 0$$

$$\Rightarrow \boxed{m^2 \partial^\mu A_\mu = 0}$$

$$\Downarrow$$

$$\boxed{\partial^\mu A_\mu = 0}$$

A : $m \neq 0$ spin 1 $\Rightarrow$ $\underline{3 \text{ d.o.f.}}$

A_μ (4 d.o.f.) $\Rightarrow$ $\boxed{\text{constraint}}$

$\circledast + \mathcal{L}_{\mathrm{Dirac}} = i \bar{\Psi} \gamma^\mu D_\mu \Psi$

$$\boxed{\psi \to e^{i\alpha}\psi \Rightarrow \partial^\mu j_\mu = 0 \quad \alpha = \text{cst.}}$$

$\boxed{A \sim \text{photon}}$ $\Rightarrow$ $A_\mu j^\mu$ $\therefore \partial^\mu j_\mu = 0$

$\Rightarrow$ Lag. component is decoupled

$$\boxed{A \neq 4 \text{ d.o.f.}}$$

QED A_μ $(m_A = 0)$

$\Uparrow$

$h = \pm 1$

gauge redundancy $\Rightarrow$ gauge fixing

$\boxed{\text{Proca}}$ $m^2 A_\mu A^\mu$

$\Uparrow$ $\cancel{A_\mu \to A_\mu + \partial_\mu \alpha(x)}$

$\boxed{\text{gauge!?}}$

<u>QED</u> 2 d.o.f. — physical
4 A_μ $\not\Rightarrow$ gauge inv.

<u>Proca</u> Imitate Maxwell

3 d.o.f. — physical

5 (A_μ, σ) — gauge covariance

$\Downarrow$ Stueckelberg

$$\frac{1}{2} m^2 A_\mu A^\mu = \frac{1}{2} m^2 \left(A_\mu - \frac{1}{m}\partial_\mu G\right) \times$$

$$\left(A^\mu - \frac{1}{m}\partial^\mu G\right)$$

gauge

massive

$$A_\mu \to A_\mu + \partial_\mu \alpha(x)$$

$$G \to G + m\,\alpha(x)$$

loops

Proca

$$\partial_\mu A^\mu = 0$$

$$A_\mu = e^{-ipx}\,\epsilon_\mu(p) \Rightarrow \boxed{p^\mu \epsilon_\mu = 0}$$

pol. Tensor

at rest $\Rightarrow$ $\boxed{p^0 \epsilon_0 = 0 \Rightarrow \epsilon_0 = 0}$

$$\boxed{\epsilon_\mu = (0;\ \underbrace{a, b, c}_{\text{spin } 1})}$$

$\boxed{S = 1 \text{ particle}}$ $\Longleftrightarrow$ adjoint repr. of $SU(2)$ (triplet, vector)

$$\boxed{(T_i)_{jk} = -i\,\epsilon_{ijk}}$$

$$\Updownarrow$$

$$\boxed{[T_i, T_j] = i\,\epsilon_{ijk} T_k}$$

$\boxed{QED}$ $\quad D_\mu = \partial_\mu - ieQA_\mu$

$\boxed{SU(2)_L}$ $\quad D_\mu = \partial_\mu - ig\,T_a A_\mu^a$

$$\boxed{T_a = \sigma_a/2 \qquad \text{FUND}}$$

$$\boxed{l = \begin{pmatrix} \nu \\ e \end{pmatrix} \Rightarrow \bar{l}\, \gamma^\mu g\, \underbrace{T_a A_\mu{}^a}\, l}$$

adjoint = adjoint to generators

$$A_\mu = T_a A_\mu{}^a$$

$$l \to U l \qquad U \equiv e^{i\theta_a T_a}$$

$$T_a = \sigma_a/2$$

$$\Rightarrow \boxed{\theta = \text{const.}}$$

$$\boxed{A_\mu \to U A_\mu U^\dagger +} \quad \boxed{\text{gauge}}$$

$$\bar{l} \ldots T_a A_\mu{}^a l \to \bar{l}\, \underbrace{U^\dagger U}_{= 1}\, T_a A_\mu{}^a \underbrace{U^\dagger U}_{1} l$$

$$= \text{inv.}$$

$\epsilon_\mu = (0; \epsilon_i)$

$$\epsilon = \begin{pmatrix} \epsilon_1 \\ \epsilon_2 \\ \epsilon_3 \end{pmatrix} \qquad \epsilon \to O\,\epsilon$$

vector of $SO(3)$

$$O^T O = O O^T = 1$$

$$\det O = 1$$

$$\Rightarrow O = e^{i\theta_a T_a} \Longrightarrow \quad \det O = 1 \Downarrow \operatorname{Tr} T_a = 0$$

$$O^T O = 1 \Rightarrow T_a^T = -T_a$$

$$O \in R \Longrightarrow T_a = \text{imaginary}$$

$$\Downarrow$$

$$\boxed{(T_a)_{ij} = -i\epsilon_{ijk}}$$

$$\boxed{\epsilon_a T_a \longrightarrow U\, \epsilon_a T_a U^+}$$

$$\theta_a = \theta_a(x) \Longleftrightarrow \partial_\mu \to D_\mu = \partial_\mu - i g T_a A^a_\mu$$

$$T_a A^a_\mu \equiv \mathbf{A}_\mu$$

$$\boxed{A_\mu \to U A_\mu U^\dagger} \cancel{+ \frac{i}{g}(\partial_\mu U) U^\dagger}$$

What happens when $\theta = \text{cst}$?

$\nexists$ photon!!!

photon

U(1)

$$A_\mu \to A_\mu + \partial_\mu \alpha(x)$$

$$\alpha = \text{cst.} \quad A_\mu \to A_\mu$$

$$\Longleftrightarrow A_\mu = \text{neutral}$$

$$\boxed{\varepsilon_\mu \longrightarrow \text{ROTATIONS}}$$

$\varepsilon_\mu = (0; \varepsilon_i)$ at rest

$$\begin{pmatrix}\varepsilon_1\\ \varepsilon_2\\ \varepsilon_3\end{pmatrix} \longrightarrow O \begin{pmatrix}\varepsilon_1\\ \varepsilon_2\\ \varepsilon_3\end{pmatrix}$$

$$O = e^{i\hat{T}_a\theta_a}$$

$$(\hat{T}_a)_{ij} = -i\epsilon_{ija}$$

$$\boxed{\varepsilon_a T_a \longrightarrow U\, \varepsilon_a T_a\, U^\dagger}$$

$$U \equiv e^{i\theta_a T_a} \qquad T_a = \sigma_a/2$$

- $\vec{\varepsilon} = \text{vector}$; $\varepsilon = \begin{pmatrix}\varepsilon_1\\ \varepsilon_2\\ \varepsilon_3\end{pmatrix}$;

$$\hat{\varepsilon} = T_a\varepsilon_a \qquad \varepsilon_a = 1,2,3$$

small U

$$\epsilon \to U \epsilon U^\dagger \qquad \sigma_a/2$$

$$\varepsilon_a T_a \to U \varepsilon_a T_a U^\dagger$$

$$= \left(1 - i\theta_b \frac{\sigma_b}{2}\right) \varepsilon_a \frac{\sigma_a}{2} \left(1 + i\theta_b \frac{\sigma_b}{2}\right)$$

$$= \varepsilon_a \frac{\sigma_a}{2} - i\theta_b \left[\frac{\sigma_b}{2}, \frac{\sigma_a}{2}\varepsilon_a\right]$$

$$= \varepsilon_a \frac{\sigma_a}{2} - i\theta_b \, i\varepsilon_{bac} \frac{\sigma_c}{2} \epsilon_a$$

$$= \varepsilon_a \frac{\sigma_a}{2} \; \varepsilon_{abc} \theta_b \frac{\sigma_c}{2} \varepsilon_a$$

$$= \frac{\sigma_c}{2} \left[\varepsilon_c - \underline{\varepsilon_{abc}} \theta_b \varepsilon_a \right]$$

$$\Updownarrow$$

$$\boxed{(T_a)_{bc} = -i\varepsilon_{abc}}$$

$\vec{V}$, $\quad V = \begin{pmatrix} V_1 \\ V_2 \\ V_3 \end{pmatrix}$, $\quad V = V_i T_i$

$T_i = \frac{\sigma_i}{2}$

$$\mathbb{V} \equiv V_i T_i \longrightarrow U \, V_i T_i U^\dagger$$

$\parallel$

fundamental

$$\boxed{\begin{gathered} V = \begin{pmatrix} V_1 \\ V_2 \\ V_3 \end{pmatrix} \longrightarrow O \begin{pmatrix} V_1 \\ V_2 \\ V_3 \end{pmatrix} \\ O = 1 + i\theta_a T_a, \quad (T_a)_{ij} = -i\varepsilon_{aij} \end{gathered}}$$