

May 12, 2020

Neutrino BSM Course

Lecture VI

L M U

Spring 2020

It's neutrino, stupid

Lecture 6

From Fermi to SD

week 1

$$L_{eff} = \frac{G_F}{\sqrt{2}} J_\mu^W \bar{J}_W^\mu \quad (*)$$
$$J_\mu^W = \bar{u}_L \gamma_\mu d_L + \bar{\nu}_L \gamma_\mu e_L \quad (57)$$

QED — photon messenger

↑

week 2

$$(*) \frac{g}{\sqrt{2}} W_\mu^+ J_\mu^W + h.c.$$

Schnüpp

↓  $(U(1))$  gauge inv.

$$f \rightarrow e^{i\alpha Q} f \quad Qf = qf$$

$$Q_e = -1, Q_u = 2/3 \dots$$

$$\partial_\mu \rightarrow D_\mu = \partial_\mu - ie A_\mu Q$$

$$W_\mu^+ \bar{\nu}_L \gamma^\mu e_L$$



$$l_L = \begin{pmatrix} \nu \\ e \end{pmatrix}_L \quad W^- \quad \text{lepton doublet}$$

$$\Leftrightarrow \boxed{SU(2)_L} \quad \leftarrow \text{only LH}$$

~~$\mathbb{R}$~~   $W$

$SU(2)_L$  is observed  
in nature

$W^+, W^-, Z$

A  
photon

$$M_W = 80 \text{ GeV}$$

$$M_Z = 90 \text{ GeV}$$

$$T_i, \quad i=1,2,3$$

$$T_i = \frac{\sigma_i}{2}$$

$$[T_i, T_j] = i \epsilon_{ijk} T_k$$

weak spin (iso-spin)



$$Q \neq T_3 \quad T_3 = \frac{G}{2\sqrt{3}}$$

$$l: \quad Q_{em} = \begin{pmatrix} 0 & 0 \\ 0 & -1 \end{pmatrix}$$

$$e: \quad Q_{em} = \begin{pmatrix} 2/3 & 0 \\ 0 & -1/3 \end{pmatrix}$$

$$\boxed{Y = 2(Q - T_3)} \quad \text{hypercharge}$$

$$l: \quad Y_e = 2 \left[ \begin{pmatrix} 0 & 0 \\ 0 & -1 \end{pmatrix} - \begin{pmatrix} 1/2 & 0 \\ 0 & -1/2 \end{pmatrix} \right]$$

$$= 2 \begin{pmatrix} -1/2 & \\ & -1/2 \end{pmatrix} = (-1) \mathbb{1}$$

$$[Y_e, T_i] = 0$$

$$Q: Y_e = 2 \left[ \begin{pmatrix} \frac{2}{3} & 0 \\ 0 & -1/3 \end{pmatrix} - \begin{pmatrix} 1/6 & 0 \\ 0 & -1/6 \end{pmatrix} \right]$$

$$= 2 \begin{pmatrix} 1/6 & 0 \\ 0 & 1/6 \end{pmatrix} = \frac{1}{3} \mathbb{1}$$

$$\boxed{[Y, T_i] = 0}$$

$$\boxed{Q = T_3 + \frac{Y}{2}}$$

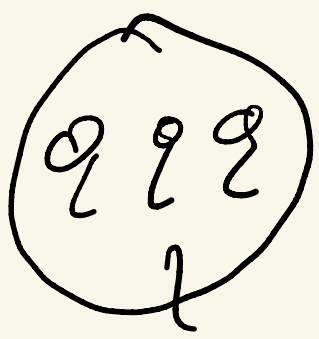
$$\boxed{SU(2)_L \otimes U(1)_Y}$$

LH int. with  $W$ :  $\updownarrow$

$$[T_i, Y] = 0$$

$B, L$

$d=6$



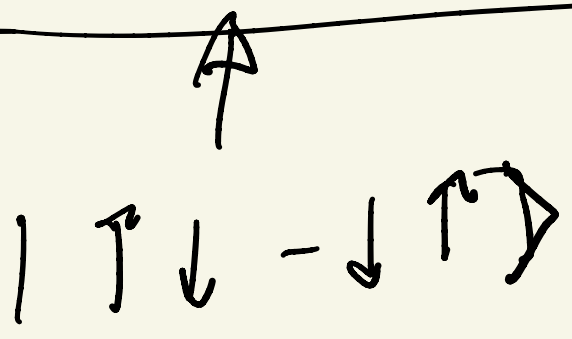
$l \leftarrow$  Lorentz symmetry

$p, n$

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$(u_L^T c d_L) (u_L^T c e_L)$  (Lorentz)

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$SU(2)_L$

$\uparrow$   
 $l_L = \begin{pmatrix} \nu \\ e \end{pmatrix}_L$

$SU(2)$  inv. ?

$$Q_L = \begin{pmatrix} u \\ d \end{pmatrix}_L \quad l_L = \begin{pmatrix} \nu \\ e \end{pmatrix}_L$$

$u_R, d_R, \dots$   
 $e_R$

$LH = \text{doublet}$

$$\rightarrow Q_L^T C i\sigma_2 Q_L = SU(2) \text{ inv.}$$

$$(u_L^T C d_L)$$

$$Q_L^T C i\sigma_2 l_L = \begin{pmatrix} u_L^T C d_L^T \\ -d_L^T C \nu_L \end{pmatrix} \begin{pmatrix} 0 & 1 \\ -1 & 0 \end{pmatrix} \begin{pmatrix} \nu \\ e \end{pmatrix}_L$$

$\uparrow$  doublet                       $\uparrow$  doublet

$$u_L^T C e_L - d_L^T C \nu_L$$

$$Q_L^T C i\sigma_2 Q_L \quad Q_L^T C i\sigma_2 l_L =$$

$$= u_L^T C d_L (u_L^T C e_L - d_L^T C \nu_L)$$



$B-L$  good

$p \rightarrow \pi^0 + e^+ \sim \pi^+ + \nu^c$

related

$B+L$  ??

$q_L^T C i\sigma_2 q_L$   
 $(\frac{1}{3}) (u_L^T c d_L)$   
 $(\frac{2}{3} - \frac{1}{3})$

$u_R^T C e_R$   
 $(\frac{2}{3} - 1 = -\frac{1}{3})$

~~$d_R^T C \nu_R$~~   $\underline{u}_R \nu_R$

$d_R^T C (\nu^c)_R$

Lorentz, QCD

$(q_R, e_R)$  - interact with  $A$

$$\begin{pmatrix} u \\ d \end{pmatrix}_L$$

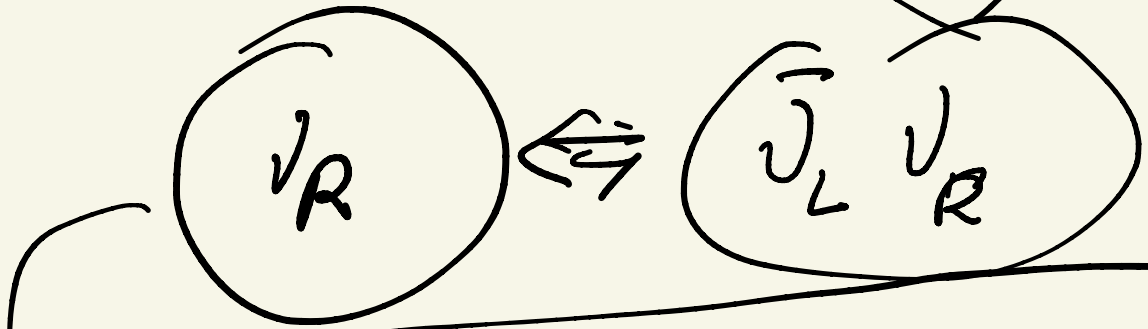
$$u_R, \cancel{d_R} \xrightarrow{A} d_R$$

$$\downarrow A$$

$$u_R$$

$$\begin{pmatrix} \nu \\ e \end{pmatrix}_L$$

$$e_R, \cancel{\nu_R}$$



$Q = 0$ , no weak int.  
 photon

SM scale  $\Leftrightarrow E \approx M_W$   
 $\approx 100 \text{ GeV}$

$E \gg M_W \Rightarrow \boxed{SU(2) \times U(1)}$

• (D)  $\bar{\Psi}_L \Psi_R$  - Lorentz inv.  
 $\Leftrightarrow$  electron

• (M)  $\nu_L^T C \nu_L$  ~~||~~  $\nu_R^T C \nu_R$   
 ||  
 Majorana

gravity  $G_N \frac{m_1 m_2}{\sqrt{\quad}} = \nu(\nu)$

$\hookrightarrow$   $(E_1, E_2)$

$$\lambda_{gr} \approx \frac{E^2}{M_{pl}^2} \sim 10^{19} \text{ GeV}$$

LHC

quarks, leptons

$$e^+ e^- \rightarrow Z \dots$$

$h$

$$g_e h \bar{e} e, \quad g_e \approx 10^{-5}$$

$$q_L^T C i \sigma_2 q_L$$

• ~~B~~

$$u_L^T C d_L$$

$$\otimes \bar{d}_R C (v^e)_R$$

Lorentz,  $SU(2)$

$SU(2)$  singlet

??? why not?

$SU(2)$

$\bar{d}_R = \text{singlet}$

$$(v^e)_R \equiv C \bar{v}_L^T$$



$V_L \Leftrightarrow$  doublet

~~$u_L^T C d_L d_R^T C (V^c)_R$~~  breaks  $SU(2)$

$qqql \Leftrightarrow$  Lorentz

$qqql^c (\Phi - \text{Higgs})$   $d=7$   
 $\uparrow^3$

$qqql \frac{1}{\Lambda^2} \Rightarrow \Lambda > 10^{15} \text{ GeV}$

Fermi  $G_F \approx \frac{1}{\Lambda^2}$   $E \ll \frac{1}{\sqrt{G_F}}$

$$\uparrow\uparrow G_F = \frac{1}{\Lambda_F^2} = \frac{g^2}{8M_W^2}$$

$E \ll \Lambda_F \Rightarrow$  body  $d=6$   
4 fermion

Effective

$q q e l$

$$(n_L d_L) (u_L e_L - d_L \nu_L)$$

$$(u_R d_L) u_R e_R$$

$$u_R d_R u_R e_R$$

$B-L$

→ tool towards fundamental theory

$\nu_L \rightarrow 2 \text{ d.o.f. (Majorana)}$

$\nu_L, \nu_R \rightarrow 2+2=4 \text{ d.o.f. (Dirac)}$

$\nu_{e\mu\tau}$

June 22 - July 2  
Neutrino 2020

$\Delta L \neq 0$

$\Delta B = 0$

effective?

$\nu_L^T C \nu_L$

SU(2) triplet!

Lorentz  $\downarrow$

NOT triplet

$$T_3 = \frac{1}{2} + \frac{1}{2} = 1 \neq 0$$

scalar

$W$  - weak int.

$$\hookrightarrow \frac{g}{\sqrt{2}} (\bar{u}_L \gamma^\mu d_L + \bar{\nu}_L \gamma^\mu e_L) W_\mu^\dagger + \text{h.c.}$$

↗

$M_W \neq 0$

Proca theory = Maxwell +  $m_A$

$\mathcal{L}(1)$

$$\mathcal{L}_P = \mathcal{L}_M + \frac{1}{2} m^2 A_\mu A^\mu$$

$$= \underbrace{-\frac{1}{4} F_{\mu\nu} F^{\mu\nu}} + \frac{1}{2} m^2 A_\mu A^\mu$$

$$F_{\mu\nu} \equiv \partial_\mu A_\nu - \partial_\nu A_\mu$$

↘

$$\mathcal{L}_P = -\frac{1}{2} (\partial_\mu A_\nu - \partial_\nu A_\mu) \partial^\mu A^\nu$$

ind. by part

$$+ \frac{1}{2} m^2 A_\mu A^\mu$$

$$= +\frac{1}{2} (A_\nu \square A^\nu - A_\nu \partial^\mu \partial^\nu A_\mu)$$

$$+ \frac{1}{2} m^2 A_\mu A^\mu$$

$$= \frac{1}{2} A_\mu \left[ (\square + m^2) g^{\mu\nu} - \partial^\mu \partial^\nu \right] A_\nu$$

(check)

⇓

$\partial_\mu \left[ (\square + m^2) g^{\mu\nu} - \partial^\mu \partial^\nu \right] A_\nu = 0$

KG field  $\Rightarrow (\square + m^2) \phi = 0$

$$\mathcal{L}_{KG} = \frac{1}{2} (\partial_\mu \phi) (\partial^\mu \phi) - \frac{1}{2} m^2 \phi^2$$

$\Rightarrow \phi = e^{-ipx} \tilde{\phi}(p) \Rightarrow p^2 = m^2$

⇓

$$E^2 = \vec{p}^2 + m^2$$

$$\textcircled{*} \Rightarrow \cancel{(\square + m^2) \partial^\mu A_\mu - \square \partial^\mu A} = 0$$

$$\Rightarrow \boxed{m^2 \partial^\mu A_\mu = 0}$$

$$\boxed{\partial^\mu A_\mu = 0}$$

$A$  :  $m \neq 0$  spin 1  $\Rightarrow$  3 d.o.f.

$A_\mu$  (4 d.o.f.)  $\Rightarrow$  constraint

$$\textcircled{*} + \mathcal{L}_{\text{Dirac}} = i \bar{\psi} \gamma^\mu D_\mu \psi$$

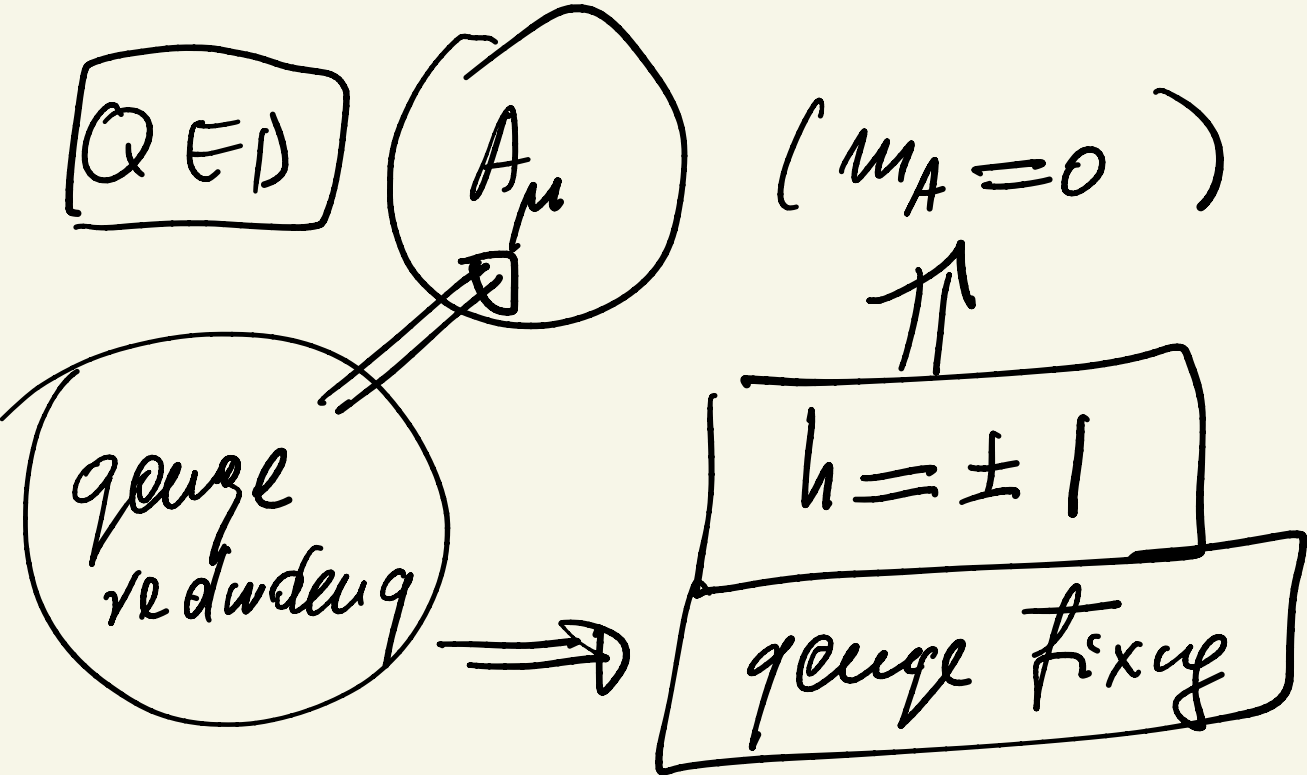
$$\psi \rightarrow e^{i\alpha} \psi \Rightarrow \partial^\mu j_\mu = 0$$

$\alpha = \text{const.}$

$$A \sim \text{photon} = A_\mu j^\mu \therefore \partial^\mu j_\mu = 0$$

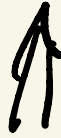
⇒ Lag. Component is decoupled

$A \neq 4 \text{ d.o.f.}$



Proca

$$m^2 A_\mu A^\mu$$



~~$$A_\mu \rightarrow A_\mu + \partial_\mu \alpha(x)$$~~

gauge!?

QED

2 d.o.f. — physical

4  $A_\mu$

$\Rightarrow$  gauge inv.

Proca

limitate Maxwell

3 d.o.f. — physical

5  $(A_\mu, \phi)$  — gauge covariant

$\Downarrow$  Stueckelberg



$$\frac{1}{2} m^2 A_\mu A^\mu = \frac{1}{2} m^2 \left( A_\mu - \frac{1}{m} \partial_\mu \phi \right)^2$$

$$\left( A_\mu - \frac{1}{m} \partial_\mu \phi \right)$$

gauge

massive

$$A_\mu \rightarrow A_\mu + \partial_\mu \alpha(x)$$

$$\phi \rightarrow \phi + m \alpha(x)$$

loops

Proca

$$\partial_\mu A^\mu = 0$$

$$A_\mu = e^{-ipx} \epsilon_\mu(p) \Rightarrow$$

↑  
pol. Tensor

$$p^\mu \epsilon_\mu = 0$$

at rest

$$p^0 \epsilon_0 = 0 \Rightarrow \epsilon_0 = 0$$

$$\epsilon_{\mu} = (0; \underbrace{a, b, c}_{\text{spin 1}})$$

$S = 1$  particle  $\iff$  adjoint repr.  
of  $SU(2)$   
(triplet, vector)

$$(T_i)_{jk} = -i \epsilon_{ijk}$$

$$[T_i, T_j] = i \epsilon_{ijk} T_k$$

$$Q \in \mathbb{D}$$

$$SU(2)_L$$

$$D_{\mu} = \partial_{\mu} - ie Q A_{\mu}$$
$$D_{\mu} = \partial_{\mu} - ig T_a A_{\mu}^a$$

$$T_a = \sigma_a / 2 \quad \text{FUNO}$$

$$l = \begin{pmatrix} \nu \\ e \end{pmatrix} \Rightarrow \bar{l} \gamma^\mu g \underbrace{T_a A_\mu^a}_{\text{adjoint = adjoint to generators}} l$$

$$A_\mu = T_a A_\mu^a$$

$$l \rightarrow U l \quad U \equiv e^{i \theta_a T_a}$$

$$T_a = \tau_a / 2$$

$$\Rightarrow \theta = \text{const.}$$

$$A_\mu \rightarrow U A_\mu U^\dagger$$

gauge

$$\bar{l} \dots T_a A_\mu^a l \rightarrow \bar{l} \underbrace{U^\dagger U}_{=} T_a \underbrace{A_\mu^a}_{=} \underbrace{U^\dagger U}_= l$$

$$= \text{inv.}$$

$$\epsilon_\mu = (0; \epsilon_i)$$

$$\xi = \begin{pmatrix} \xi_1 \\ \xi_2 \\ \xi_3 \end{pmatrix}$$

vector of  $SO(3)$

$$\xi \rightarrow \underline{O} \xi$$

$$O^T O = O O^T = 1$$

$$\det O = 1$$

$$\Rightarrow O = e^{i \theta_a T_a} \quad \det O = 1$$

$$\Downarrow$$

$$T_a T_a = 0$$

$$O^T O = 1 \Rightarrow T_a^T = -T_a$$

$$O \in R \Rightarrow T_a = \text{imaginary}$$

$\Downarrow$

$$(T_a)_{ij} = -i \epsilon_{ij\mu}$$

$$\xi_a T_a \rightarrow U \xi_a T_a U^\dagger$$

$$\Theta_a = \Theta_a(x) \Leftrightarrow \partial_\mu \rightarrow D_\mu = \partial_\mu - i g T_a A_\mu^a$$

$$T_a A_\mu^a \equiv A_\mu$$

$$A_\mu \rightarrow U A_\mu U^\dagger + \frac{i}{g} (\partial_\mu U) U^\dagger$$

What happens when  $\Theta = \text{const}$ ?

!! photon !!!

photon

$$A_\mu \rightarrow A_\mu + \partial_\mu \alpha(x)$$

$G(U)$

$$\alpha = \text{const.} \quad A_\mu \rightarrow A_\mu$$

$$\Leftrightarrow A_\mu = \text{neutral}$$

$\Sigma_\mu \rightarrow$  ROTATIONS

$\epsilon_\mu = (0; \epsilon_i)$   
at rest

$\begin{pmatrix} \epsilon_1 \\ \epsilon_2 \\ \epsilon_3 \end{pmatrix} \rightarrow 0 \begin{pmatrix} \epsilon_1 \\ \epsilon_2 \\ \epsilon_3 \end{pmatrix}$

$\dot{0} = e^{i T_a \dot{\theta}_a}$

$(T_a)_{ij} = -i \epsilon_{ij}^k \dot{\theta}_k$

$\Sigma_a T_a \rightarrow U \Sigma_a T_a U^\dagger$

$U = e^{i \theta_a T_a}$

$T_a = \sigma_a / 2$

$\vec{\Sigma} = \text{vector}; \quad \mathcal{E} = \begin{pmatrix} \epsilon_1 \\ \epsilon_2 \\ \epsilon_3 \end{pmatrix};$

$\vec{\Sigma} = T_a \mathcal{E}_a \quad \Sigma_a = 1, 2, 3$

$$\underline{\underline{\epsilon}} \rightarrow U \underline{\underline{\epsilon}} U^\dagger \quad \xrightarrow{\text{small } U} \quad \sigma_a/2$$

$$\epsilon_a T_a \rightarrow U \epsilon_a T_a U^\dagger$$

$$= \left( 1 - i \theta_b \frac{\sigma_b}{2} \right) \epsilon_a \frac{\sigma_a}{2} \left( 1 + i \theta_b \frac{\sigma_b}{2} \right)$$

$$= \epsilon_a \frac{\sigma_a}{2} - i \theta_b \left[ \frac{\sigma_b}{2}, \frac{\sigma_a \epsilon_a}{2} \right]$$

$$= \epsilon_a \frac{\sigma_a}{2} - i \theta_b i \epsilon_{bac} \frac{\sigma_c}{2} \epsilon_a$$

$$= \epsilon_a \frac{\sigma_a}{2} + \sum_{abc} \theta_b \frac{\sigma_c}{2} \epsilon_a$$

$$\equiv \frac{\sigma_c}{2} \left[ \epsilon_c - \underline{\underline{\epsilon_{abc} \theta_b \epsilon_a}} \right]$$

$$\boxed{(T_a)_{bc} = -i \epsilon_{abc}}$$

$$\vec{V}, \quad V = \begin{pmatrix} v_1 \\ v_2 \\ v_3 \end{pmatrix}, \quad V = v_i T_i$$

$$T_i = \frac{\sigma_i}{2}$$

$$V \equiv v_i T_i \rightarrow U v_i T_i U^\dagger$$

"fundamental"

$$V = \begin{pmatrix} v_1 \\ v_2 \\ v_3 \end{pmatrix} \rightarrow 0 \begin{pmatrix} v_1 \\ v_2 \\ v_3 \end{pmatrix}$$

$$0 = 1 + i\theta_a T_a, \quad (T_a)_{ij} = -i\varepsilon_{2ij}$$