Neutrino BBSM Course

Lecture IV

LMU

Spring 2020

It's neutrino, stupid!

Lecture 4

$\boxed{\text{Baryon} \,\|\, \text{Lepton numbers}}$

- $\underbrace{p, n, \Lambda \ldots}$

the same — except for charge

$\underbrace{}$

Baryon $\longleftrightarrow$ heavy

$$m_p \simeq m_n \simeq 1000\, m_e$$

- $\underbrace{p, n, e, \nu_e}$; A, W^+, W^-, Z

(u, d, e)

- (c, s, μ, ν_μ)
- (t, b, τ, ν_τ)

$u, d \longrightarrow p, n$

$B_q = \frac{1}{3}, \quad B_p = B_n = 1$

$$n \to p + e + \bar{\nu}_e \qquad (m_n \gtrsim m_p)$$

$$\cancel{p \to \bar{e} + \gamma}, \quad \cancel{\bar{e} + \pi^0}, \quad \cancel{\pi^+ + \dots}$$

$$\boxed{\tau_p \gtrsim 10^{34}\,\text{yr}} \qquad \boxed{\text{Super K}}$$

$$\mu \to e + \nu_\mu + \bar{\nu}_e$$
$$\tau_\mu \simeq 10^{-6}\,\text{sec}$$

$\tau_n \simeq 10$ min — anomaly

Lepton # ? e = light

$$n \to p + e + \bar{\nu} \qquad \cancel{L(\nu) = -1}$$

$$L(e) = L(\nu) = 1 \qquad L_q = 0$$

$$B(e) = B(\nu) = 0$$

$$\boxed{B, L\,(A, \ldots) = 0}$$

· <u>global symmetry</u>

$$Q_{em}\,\psi = q\,\psi$$

$$Q_{em} \leftrightarrow U(1)_{em}$$

$$\psi \rightarrow e^{i\alpha Q_{em}}\,\psi$$

$$Q(e) = -1, \quad Q(u) = 2/3,$$

$$Q(\nu) = 0 \qquad Q(d) = -1/3$$

$$\alpha(x) \Longrightarrow \boxed{\text{messengers}}$$

$$\boxed{B} \quad \left\{ \begin{array}{l} D_\mu = \partial_\mu - ie\, Q_{em}\, A_\mu \\ q \to e^{i\beta/3} q, \quad l \to l, \ldots \end{array} \right\}$$

B

$$\cancel{\boxed{\text{gauge } B}} \quad ???$$

$$\partial_\mu \to D_\mu = \partial_\mu - i g_B B X_\mu$$

$$\Rightarrow V_B(r) \simeq \frac{g_B^2}{4\pi} B_1 B_2 \frac{1}{r}$$

$$V_{gr}(r) \simeq G_N \frac{M_1 M_2}{r} =$$

$$= G_N \frac{B_1 B_2\, GeV^2}{r}$$

$$\boxed{G_N \simeq 10^{-38}\, GeV^{-2}}$$

$$\boxed{\frac{g_B^2}{4\pi} \ll 10^{-38} \text{ !!}}$$

$$\boxed{\frac{\alpha_{em}}{4\pi} \simeq 1/100}$$

GUT $\Longleftrightarrow$ grand unification

$\underbrace{\text{strong, em, weak}}$ $\boxed{M_{\text{new gauge}} \simeq 10^{16}\,\text{GeV}}$

GUT theory

q, l together $\Rightarrow \cancel{B}$

$$\boxed{B = \text{global symmetry}}$$

B: $q \to e^{i\beta/3} q$, $\ell \to \ell$, ---- invariant

L: $q \to q$, $\ell \to e^{i\gamma} \ell$, ----

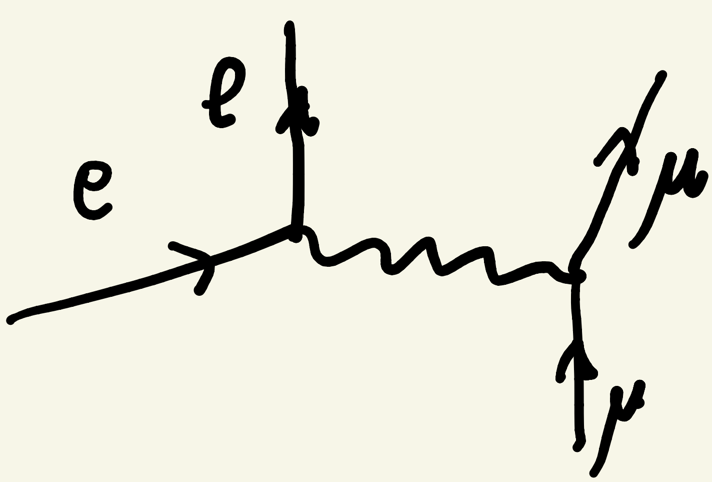
$\ell = (e, \nu)$ – leptons

$$\boxed{\Delta B = \Delta L = 0}$$

$$\boxed{\text{Search for } \Delta B, \Delta L = \\ = \text{Holy Grail}}$$

$\underline{\text{em}}$ + $\underline{\text{strong}}$ $\Longleftrightarrow$ $\Delta L = \Delta B = 0$

$\underbrace{\bar{e} \gamma^\mu e}\, A_\mu$



$\cancel{\bar{e} \gamma^\mu \mu A_\mu}$

Lepton Flavour = conserved

$\underbrace{e, \mu, \tau \text{ - - -}}_{\text{flavour }(F)}$ $m_e \neq m_\mu \neq m_\tau$

$\left.\begin{array}{l} \underbrace{u, c, t} \\ \text{Quark flavour} \\ \underbrace{d, s, b} \end{array}\right\}$ conserved in em + strong

NO
LNV
Lepton Number Violation

LFV

BNV (Baryon ----)
BFV

Weak

$\pi^+ = u\bar{d}$

$K^+ = u\bar{s}$ $\quad K^+ \to \pi^+ \pi^0$

$\Delta S \neq 0, \quad \Delta C \neq 0$

charm

$\boxed{QF\bar{V} \ (BF\bar{V})}$

$\boxed{LF\bar{V}}$ $\quad \mu \to \gamma + e + \bar{\nu}_e$

$\mu \to e + \bar{\nu}_e + \nu_\mu$

$\mu \to e\gamma$ $\quad (LF\bar{V})$

$\mu \to \underline{e + e} + \bar{e}$ $\quad (LF\bar{V})$

$\left[\ B(\mu \to e\gamma) \leq 10^{-13} \ \right]$

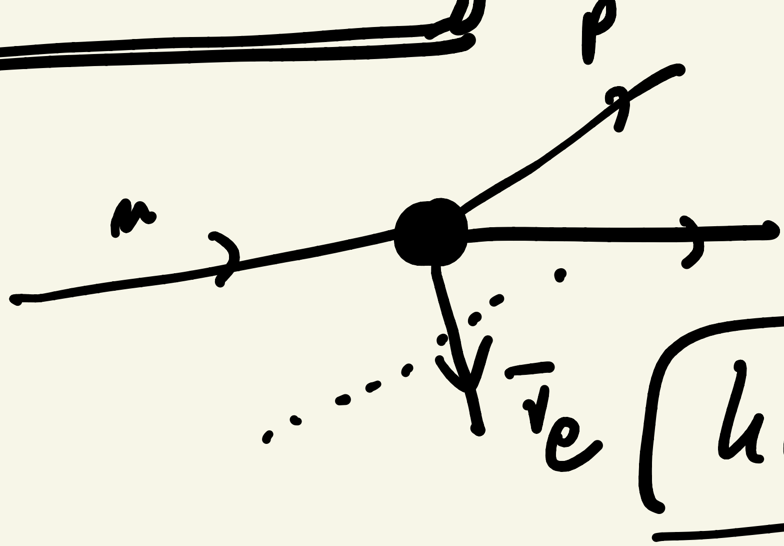
$$\boxed{\mathcal{B}(\mu \to 3e = e, e, \bar{e}) \leq 10^{-13}}$$

$\boxed{\text{LFV}}$

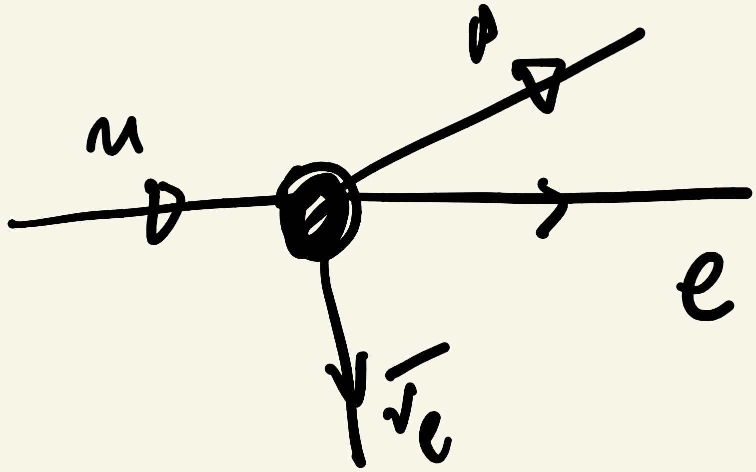
$$\boxed{\begin{array}{l} \nu_e \leftrightarrow \nu_\mu \\ \nu_\mu \leftrightarrow \nu_\tau \end{array}}$$

$$\boxed{\Rightarrow \Delta m_\nu^2 \neq 0}$$

$\boxed{\bullet\ \cancel{\text{LNV}}}$

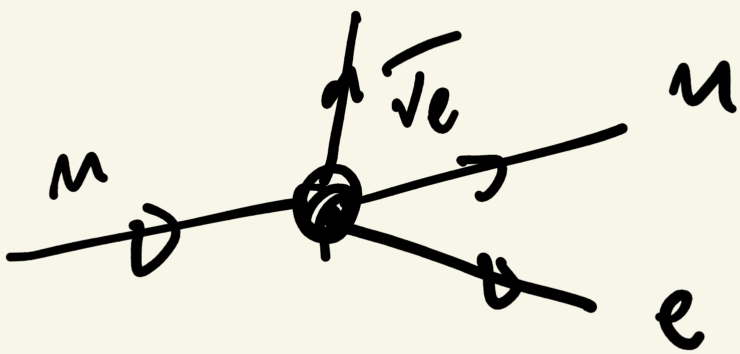


$\boxed{\text{hopeless}}$



2β decay

$$Ge \to Se + e + e + \bar{\nu}_e + \bar{\nu}_e$$



$$\cancel{Ge \to Ar + e + \bar{\nu}_e}$$

$$m_{Ar} > m_{Ge}$$

$$\boxed{\tau_{2\beta} \simeq 10^{21}\ yr}$$

$$n \to p + e + \underbrace{\text{missing}}$$

kinetic $T_e = Q = \cancel{M_i - M_f} - m_e$

$n + n \to p + p + e + e$

? $T_{e_1} + T_{e_2} = Q$ Fermi'38

$\Uparrow$

$\boxed{\text{Majorana}}$

<u>electron</u> $\psi_e = e = \begin{pmatrix} u_L \\ u_R \end{pmatrix}$

$\psi = \underline{\psi_L} + \underline{\psi_R}$

$L \leftrightarrow R$ <u>in QED</u>

- $\boxed{e_L, e_R}$ L, R notion

- e, $\bar{e}$ (positron) $\qquad \boxed{C \equiv i\gamma_2\gamma_0}$

$$\gamma + \gamma \Rightarrow e + \bar{e}$$

$$\bar{e} = \bar{\psi}_e \Leftrightarrow (\psi_e)^c \equiv \psi^c \equiv e^c$$

$$\psi^c = C\bar{\psi}^T = i\gamma_2 \psi^*$$

$$\left[\, i\gamma_2 = \begin{pmatrix} 0 & i\sigma_2 \\ -i\sigma_2 & 0 \end{pmatrix} \right] \Rightarrow$$

$$(\psi_L)^c = C\bar{\psi}_L^T = i\gamma_2 \begin{pmatrix} u_L^* \\ 0 \end{pmatrix}$$

$$= \begin{pmatrix} 0 \\ -i\sigma_2 u_L^* \end{pmatrix} = \begin{pmatrix} 0 \\ u_R \end{pmatrix}$$

$$= (\psi^c)_R$$

$$(\psi_R)^c = (\psi^c)_L$$

$$\underbrace{e_L,\ (e^c)_L}_{e,\ e^c} \equiv C\,\overline{e_R}^{T} = i\sigma_2\, e_R^{*}$$

(Left handed)

$$\mathcal{L}_{M}^{(D)} = i\,\bar{\Psi}\gamma^\mu D_\mu \Psi - m_D \bar{\Psi}\Psi - \frac{1}{4}F_{\mu\nu}F^{\mu\nu}$$

$$\gamma^0 = \begin{pmatrix} 0 & 1 \\ 1 & 0 \end{pmatrix}$$

$$m_D:\quad \bar{\Psi}\Psi = \psi^\dagger \gamma^0 \psi$$

$$= \bar{\Psi}_L \psi_R + \bar{\Psi}_R \psi_L$$

$$= \left(u_L^\dagger\ u_R^\dagger\right)\gamma^0 \begin{pmatrix} u_L \\ u_R \end{pmatrix} =$$

$$= \left(u_L^\dagger\ u_R^\dagger\right)\begin{pmatrix} 0 & 1 \\ 1 & 0 \end{pmatrix}\begin{pmatrix} u_L \\ u_R \end{pmatrix} =$$

$$= \underbrace{u_L^\dagger u_R + u_R^\dagger u_L}_{\text{mass } (Q_{em},\, L,\, B)}$$

Ψ = Baym = ξ

$$\Rightarrow \bar{\Psi}\Psi = \underline{u_L^+ u_R} + h.c.$$

$$\left(\begin{array}{l} \psi \to e^{i\alpha B}\psi \quad \bar{\Psi} \to \bar{\Psi} e^{-i\alpha B} \\ u_{L,R} \to e^{i\alpha B} u_L, u_R \end{array} \right)$$

- ψ_L, ψ_R
- $(\psi, \psi^C)_L$

$$(\psi^c)_L = C\,\overline{\psi_R}^{\,T}$$

$$m_D \underline{\overline{\psi_R}\psi_L} + h.c. = m_D (\psi^c)_L^T C \psi_L + h.c.$$

$$(\psi^c)_L^T = \overline{\psi_R}\, C^T \quad \because \boxed{C^T C = 1}$$

$$\boxed{\begin{array}{c} Q, B, L, \cdots \\ \text{cunserved} \end{array}}$$

$$m_D (\psi^c)_L^T \, C \, \psi_L = \text{Lorentz} +$$

Majorana

$$m_M \, \psi_L^T C \psi_L = \text{Lorentz inv.}$$

$$\boxed{\begin{aligned} \psi^c &\to \Lambda \psi^c \\ \psi &\to \Lambda \psi \end{aligned}}$$

→ breaks any "charge" that ψ_L carries

- $\cancel{e_L^T C e_L}$ violate Q_{em}

$$\Downarrow$$

$\boxed{\text{Only neutral} = \text{Majorana}}$

$$\psi_L^T C \psi_L = (u_L \;\; 0)^T\, i\sigma_2\gamma_0 \begin{pmatrix} u_L \\ 0 \end{pmatrix}$$

$$= (u_L^T \;\; 0)\, i \begin{pmatrix} 0 & \sigma_2 \\ -\sigma_2 & 0 \end{pmatrix} \begin{pmatrix} 0 & 1 \\ 1 & 0 \end{pmatrix} \begin{pmatrix} u_L \\ 0 \end{pmatrix}$$

$$= u_L^T i\sigma_2 u_L = \text{Inv.}$$

$$u_L \to e^{i\vec{\sigma}/2\,(\underset{\text{ROT}}{\vec{\theta}} + i\underset{\text{BOOST}}{\vec{\varphi}})} u_L$$

$$\vec{\theta} + i\vec{\varphi} = \vec{x}$$

$$u_L^T i\sigma_2 u_L \to u_L^T e^{i\vec{\sigma}^T/2\,\vec{x}}\, i\sigma_2\, e^{i\vec{\sigma}/2\,\vec{x}} u_L$$

$$\sigma_2^T = -\sigma_2, \quad \sigma_{1,3}^T = \sigma_{1,3}$$

$$[\sigma_2, \sigma_2] = 0, \quad \{\sigma_{1,3}, \sigma_2\} = 0$$

$$= u_L^T i\sigma_2 \underbrace{e^{-i\vec{\sigma}/2\,\vec{x}}\, e^{i\vec{\sigma}/2\,\vec{x}}}_{1} u_L$$

$$\boxed{\begin{array}{c} u_L^T\, i\sigma_2 u_L \longrightarrow u_L^T\, i\sigma_2 u_L \\ \text{Lorentz} \end{array}} \quad \boxed{\text{Why not?}}$$

<u>weak</u>

$$H_{eff} = \frac{4 G_F}{\sqrt{2}} J^W_\mu \bar{J}^\mu_W \qquad G_F = 10^{-5}\,\text{GeV}^{-2}$$

$$J^W_\mu = \bar{\nu}_L \gamma^\mu e_L + \bar{u}_L \gamma^\mu d_L$$

$$\Leftarrow \frac{e}{\sqrt{2}} W^+_\mu J^\mu_W$$

$A(\gamma)$: $e \to e$

W : $e \to \nu$

<u>SU(2)</u> $\begin{pmatrix} \nu \\ e \end{pmatrix} \circlearrowleft W$

<u>weak</u> : $\boxed{\nu = e \text{ (leptons)}}$

$$\boxed{m_\nu \le 10^{-6}\, m_e}$$

$\begin{pmatrix} u \\ \phi \end{pmatrix} 2W \Longleftrightarrow m_u \simeq m_d$ (cup $\simeq m_u$)

$$\boxed{Q_\nu = 0 \;, Q_e = -1}$$

- $m_D\, \bar{\nu}\, \nu \longleftrightarrow m_e\, \bar{e}\, e$

$\underbrace{\boxed{\bullet\ \frac{1}{2} m_M\ \nu_L{}^T C\ \nu_L}}_{\text{violates } L \text{ (lepton \#)}}$ Majorana

$$\boxed{\Delta L = 2}$$

$\boxed{\psi :}\ \boxed{S_F(p) = \frac{i}{\not{p} - m}}$

$$\psi_M = \psi_L + \underbrace{C\bar{\psi}_L^T}_{(\psi^c)_R} \leftrightarrow \psi_D$$

$$= \begin{pmatrix} u_L \\ 0 \end{pmatrix} + \begin{pmatrix} 0 \\ -i\sigma_2 u_L^* \end{pmatrix}$$

$$\boxed{\underline{\text{Majorana}} = \tfrac{1}{2}\ \text{particle} + \tfrac{1}{2}\ \text{anti particle}}$$

$$\boxed{\psi_M = \begin{pmatrix} u_L \\ -i\sigma_2 u_L^* \end{pmatrix}}$$

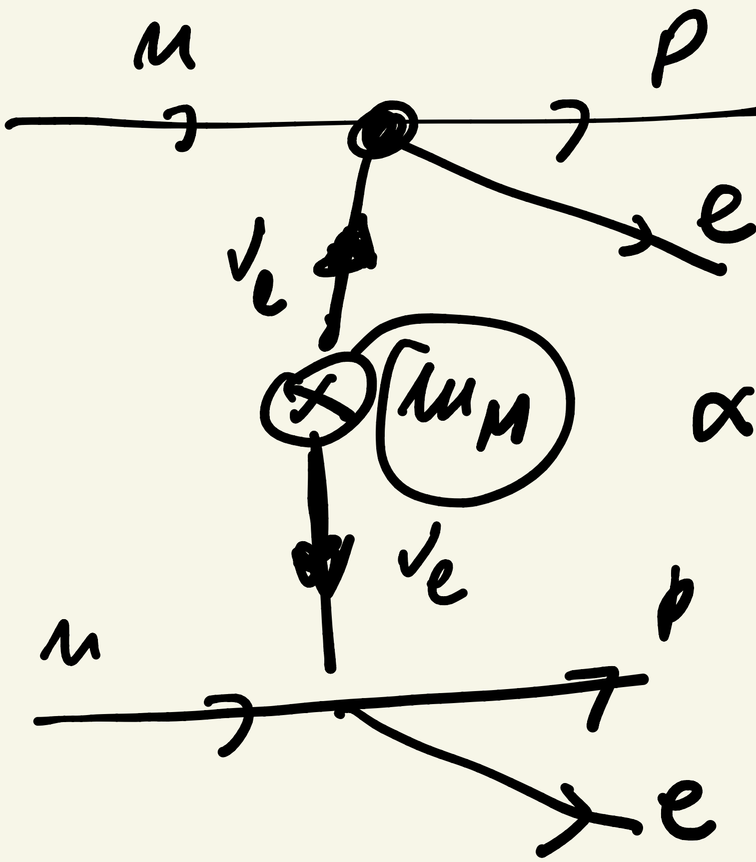
$$\bar{\psi}_M \psi_M = \bar{\psi}_{ML}\psi_R + h.c. = 2\left(\psi_L^T C \psi_L + h.c.\right)$$

$$\mathcal{L}_M = i\,\bar{\psi}_L \gamma^\mu \partial_\mu \psi_L - \frac{m_M}{2}\left(\psi_L^T C \psi_L + h.c.\right)$$

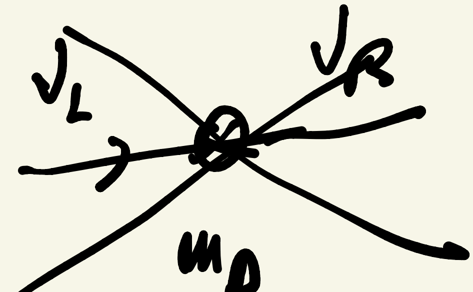
$$= \cancel{\tfrac{1}{2}}\underbrace{\left[i\,\bar{\psi}_M \gamma^\mu \partial_\mu \psi_M - m_M \bar{\psi}_M \psi_M\right]}_{\text{the same "Dirac"}}$$

$$\boxed{\Rightarrow S_M = \frac{i}{\not p - m_M} = \frac{i(\not p + m_M)}{p^2 - m_M^2}}$$

$\boxed{0\nu 2\beta}$ $\quad \frac{m_M}{2}\left(\underline{\nu_L^T C \nu_L} + \nu_L^\dagger C^\dagger \nu_L^*\right)$



$\propto \boxed{m_M}$



$\boxed{\nu_R \text{ has never been seen}}$

$$\boxed{m_M^{\nu} \leq 1\,eV}$$

$$\boxed{\tau_{\text{overp}} \gtrsim 10^{25}\,yr}$$

$$\boxed{m_{\nu}^{\text{katrin}} \leq 1\,eV}$$

$$n \to p + e + \bar{\nu}_e$$

$$\boxed{m_{\nu}^{\text{cosmo}} \leq 1\,eV}$$

(<u>digression</u>) $m_A \neq 0$

• <u>direct</u>

potential

$$\frac{1}{r} \longleftrightarrow e^{-mr}/r$$

$$m_A = 0 \qquad\qquad m_A \neq 0$$

$$\boxed{m_A \leq 10^{-14}\,eV}$$

"galactic" limit

$$m_A \leq 10^{-34}\,\text{eV} \quad (?)$$

NOT a true limit

- Adelberger, Dvali, Gruzinov '20...

$\Rightarrow$ valid only if m_A by hand

$\Leftrightarrow$ $m_A \neq 0 \leftarrow$ due to Higgs

$\Downarrow$

$$\boxed{m_A \leq 10^{-14}\,\text{eV}}$$

$\boxed{m_A \leftarrow \text{Higgs}}$ $\boxed{\text{BSM}}$

(a) <u>give by hand</u> $\Longleftrightarrow$ consistent

$$m_A \leq 10^{-34}\,eV \simeq 10^{-40}\,m_e$$

$$\boxed{\begin{array}{l} \ast\ \tau_p \gtrsim 10^{34}\,yr \gtrsim 10^{42}\,sec \gtrsim 10^{48}\,\tau_\mu \\ \tau_\mu \simeq 10^{-6}\,sec \\ \text{GUT} \longrightarrow p \text{ decays} \end{array}}$$

(b) Higgs $m_A \to$ <u>hard</u>

$$\langle \phi^+ \rangle \neq 0$$

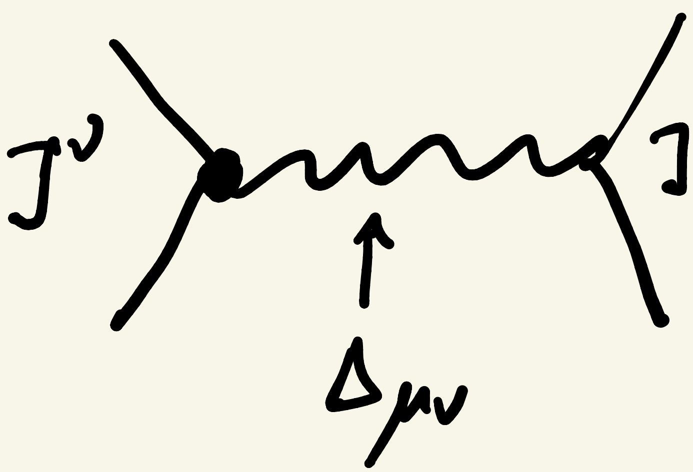
[charge $\leq 1/10^{20}$]

$$\boxed{\begin{array}{ll} m_A = e\langle\phi\rangle & \langle\phi\rangle \leq 10^{-14}\,eV \\ T,\ B \text{ large} \Rightarrow & \langle\phi\rangle = 0 \end{array}}$$

$\boxed{m_A \neq 0:}$ $$\Delta_{\mu\nu} = \frac{g_{\mu\nu} - k_\mu k_\nu / m_A^2}{k^2 - m_A^2}$$

(i) $k \to \infty \Rightarrow \Delta_{\mu\nu} \to \frac{1}{m_A^2}$

(ii) $m_A \to 0 \Rightarrow \Delta_{\mu\nu} \to \frac{1}{m_A^2} \to \infty$



$J_\mu^{em} A^\mu$

$$\partial_\mu J^\mu_{em} = 0$$
$$\Downarrow$$
$$k_\mu \tilde{J}^\mu_{em}(k) = 0$$

$$J^\mu_{em} \left(\frac{g_{\mu\nu}}{k^2 - m_A} - \cancel{\frac{k_\mu k_\nu / m_A^2}{k^2 - m_A^2}} \right) J^\nu_{em}$$

longitudinal photon decouples

$m_A \neq 0$ 3 d.o.f. (degrees of freedom)

$m_A = 0$ 2 d.o.f.

QED $\longleftrightarrow$ $U(1)$: $\partial^\mu J_\mu = 0$

2 d.o.f.

$$\boxed{m_A \to 0 \text{ limit is smooth}}$$

$$\nu_L \longleftrightarrow (\nu_L)^c = C\bar{\nu}_L^{\,T} = (\nu^c)_R$$

$$\psi_M = \begin{pmatrix} u_L \\ -i\sigma_2 u_L^* \end{pmatrix}$$

$$\nu_R = \text{new} \longleftrightarrow \boxed{N_L = C\bar{\nu}_R^{\,T}}$$

$\nu_R = N_R$

$N_L \neq (\nu^c)$

sterile $\Longleftrightarrow$ <u>no</u> weak int.

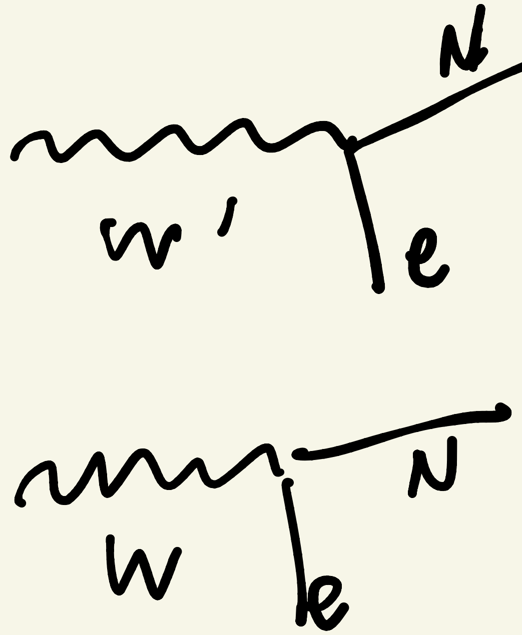
<u>no</u> int. with W

$\nu_R \to N_L$ = sterile

new gauge boson W'?

• $N_L W' e_{(R)}$ LHC

decay



Majorana

sterile ν_R is light $\simeq$ eV

oscillations