Lecture XXVII

BBSM Neutrino

Course

$$\underbrace{M_D,\ M_u,\ M_d,\ M_e}$$

mass matrices in generation space

see saw

$$\boxed{\begin{gathered} M_\nu = -M_D^T \frac{1}{M_N} M_D \\ i M_D = \sqrt{M_N}\, O \sqrt{M_\nu} \\ O O^T = 1 \\ O \in C \end{gathered}}$$

LR theory

$$S = \frac{v_L}{v_R} - \frac{1}{\sqrt{M_N}} M_\nu^* \frac{1}{\sqrt{M_N}}$$

$$\boxed{H H^T = H H^* = S} \quad (1)$$

$$M_D = \sqrt{M_N}\, H \sqrt{M_N^*}$$

Solving (1)

$$\Rightarrow \boxed{S H = H S^*} \qquad (2)$$

$$\Downarrow \quad 2 \times 2 \ (2\,\text{gen})$$

$$\boxed{H_{2\times 2} = \sqrt{SS^*}\,\frac{1}{\sqrt{S^*}}}$$

We never solved for 3gen

$$\boxed{H_{3\times 3} = ???}$$

- $\underbrace{S = V d V^T}_{\text{did not work}}$

$$HH^T = S$$

$$\boxed{VV^T = 1}$$

$$H = \text{"}\sqrt{S}\text{"}$$

- $$\boxed{\begin{gathered} S = O\, s\, O^T \\ O O^T = 1 \end{gathered}}$$

s = Jorden form

is not always diagonal

Theorem:

$$U_L \underline{M}\, U_R^{\dagger} = d \text{ (diagonal)}$$

$$U_L U_L^{\dagger} = U_R U_R^{\dagger} = 1$$

$$(\underline{M} M^+)^+ = M M^+$$

$$U_L \, \underline{M} \, \underline{M}^+ U_L^+ = d^2$$

$$U_L \, S \, U_R^+ = \text{diagonal} = d$$

$$\underline{M} = S \, \therefore \quad S = S^T$$

$$\Rightarrow \quad U_R = U_L^*$$

$$\Downarrow$$

$$\boxed{V S V^T = d}$$

$$\boxed{M = H \, \therefore \, H^+ = H \Rightarrow V H V^+ = d}$$

Indeed

$$S = O \Lambda O^T$$

$$H = O \sqrt{\Lambda}\, O^{+} \qquad (H = H^{+})$$

$$HH^* = S$$

$$HH^* = O \sqrt{\Lambda}\, \underbrace{O^{+} O^{*}}_{1} \sqrt{\Lambda^*}\, O^T$$

$$= \underbrace{O \sqrt{\Lambda} \sqrt{\Lambda^*}\, O^T}_{\neq\, S}$$

$$\boxed{\text{does not work}}$$

$$H = O \sqrt{\Lambda}\, E\, O^{T}$$

correct

$$H^{*} = O^{*} \sqrt{\Lambda^{*}}\, E^{*} O^{T}$$

$$\downarrow$$

$$HH^{*} = O \sqrt{\Lambda}\, E\, \underbrace{O^{T} O^{*}}_{1} \sqrt{\Lambda^{*}}\, E^{*} O^{T}$$

$$= O \sqrt{\Lambda}\, E \sqrt{\Lambda^{*}}\, E^{*} O^{T}$$

$$\downarrow$$

$$= O \Lambda O^{T}$$

$$E \therefore \quad \sqrt{\Lambda}\, E \sqrt{\Lambda^{*}}\, E^{*} =$$

$$= \sqrt{\Lambda} \sqrt{\Lambda} (E^{*})^{-1} E^{*}$$

$$= \Lambda \qquad \boxed{\text{if } E \text{ exists}}$$

- $HH^* = S$ constraint

$$\left[\begin{array}{ll} \mathrm{Tr}\, HH^* \in \mathbb{R} & \text{(real)} \\ \mathrm{Tr}(HH^*)^2 \in \mathbb{R} & \text{—"—} \\ \mathrm{Tr}(HH^*)^3 \in \mathbb{R} & \text{—"—} \end{array}\right]$$

$$\Downarrow$$

$$\sqrt{S}\, E = E \sqrt{S^*} \qquad E^T = E^* = E^{-1}$$

$$\Downarrow$$

$$\boxed{S E = E S^*} \qquad \boxed{E = E^+}$$

$$\boxed{\text{Take } S = \text{diagonal}}$$

$$\operatorname{Im} \operatorname{Tr} \Delta = \operatorname{Im} \operatorname{Tr} \Delta^2 = \operatorname{Im} \operatorname{Tr} \Delta^3 = 0$$

$$\Downarrow$$

(i) $\Delta = (\Delta_1, \Delta_2, \Delta_3) \quad \Delta_i \in R$

$$\Rightarrow E = 1$$

(ii) $\Delta = (\Delta, \Delta_0, \Delta^*), \quad \Delta_0 \in R$

$$\Delta^* = (\Delta^*, \Delta_0, \Delta)$$

$$\Rightarrow E = \begin{pmatrix} 0 & 0 & 1 \\ 0 & 1 & 0 \\ 1 & 0 & 0 \end{pmatrix}$$

Summary

measure (vosil)

$$S = \frac{v_L}{v_R} - \frac{1}{\sqrt{M_N}} M_\nu^* \frac{1}{\sqrt{M_N}}$$

(HC) measure

$$HH^* = S$$

$$M_D = \sqrt{M_N}\, H \sqrt{M_N^*}$$

$$S = O \Delta O^T$$

$$\Downarrow$$

$$H = O \sqrt{\Delta}\, E\, O^T$$

<u>no</u> ambiguity

$$\Downarrow$$

$$\Theta = \frac{1}{\sqrt{M_N}} M_D$$

<u>particles</u> $\boxed{E^2 = p^2 + m^2}$

NO

$$\mathcal{M}_D = i\sqrt{M_N}\, O_{amb} \sqrt{m_\nu}$$

$$\Downarrow$$

$$\boxed{O_{amb} = \text{fixed}}$$

<u>QCD</u> $\boxed{m_q = \text{input}}$

$$\mathcal{M}_q = U_L^\dagger\, m_q\, U_R$$

[diagonal, positive]

$$\boxed{\sqrt{\mathbb{1}} = \text{ambiguous}}$$

$$M_\nu = m_\nu \mathbb{1}$$
$$M_N = m_N \mathbb{1}$$
$\Rightarrow$ ambiguity

Neutrino masses

$$\Delta m_A^2 = 10^{-3} eV^2$$

$$\Delta m_\odot^2 = 10^{-4} - 10^{-5} eV^2$$

$$m_\nu \leq 0.1 eV$$
$$(0\nu 2\beta)$$

$$\boxed{\begin{aligned} m_\nu &\gtrsim 1/25\,\text{eV} \\ m_\nu &\leq 1/10\,\text{eV} \end{aligned}}$$

$$\theta = \frac{1}{M_N} M_D \leftarrow \text{crax}$$

$$\Downarrow$$

$$\frac{g}{\sqrt{2}}\,\bar{N}\gamma_\mu \theta\, e\, W_\mu^+$$

$$N \to e + W^-$$

$$\boxed{\Gamma \propto \theta\theta^+}$$

$\cancel{\Theta} \in \mathbb{R}$ $\qquad \Theta = \frac{1}{M_N} M_D$

$$\Theta^T = M_D^T \frac{1}{M_N}$$

$$\boxed{\Theta \Theta^T = \frac{1}{M_N} M_D M_D^T \frac{1}{M_N}}$$

up to signs $\cancel{\Theta}$

<u>no</u> ambiguity

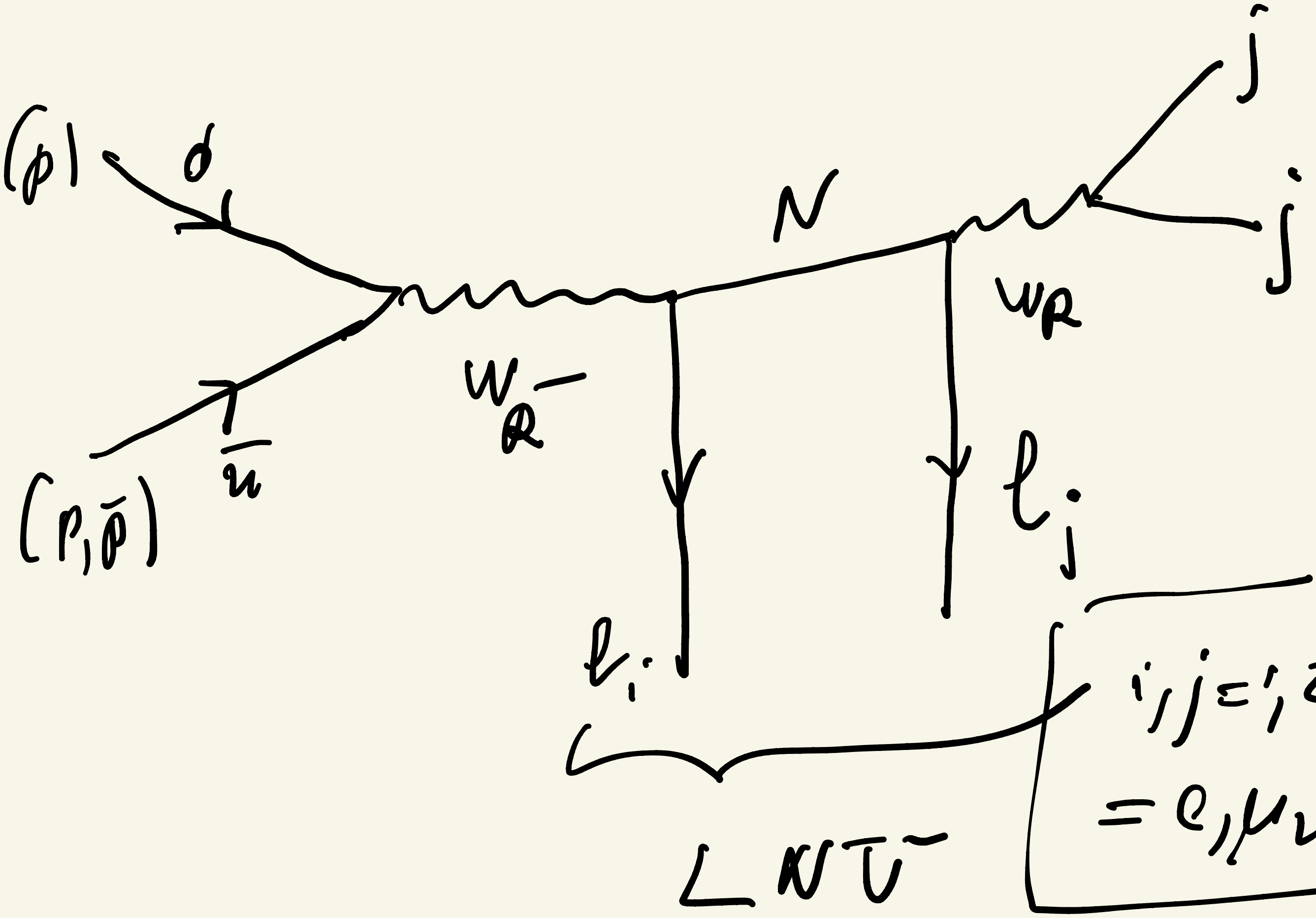
$$\boxed{\underline{M}_D \simeq \text{"} i \sqrt{M_N} \sqrt{M_\nu} \text{"}}$$

$(v_L = 0)$

"seesaw gets solved"

Summary of physic of probe of neutrino mass

(i) find N and probe M_N



$i, j = 1, 2, 3$
$= e, \mu, \tau$

$\Downarrow$

$$N \to l + j_1 + j_2$$

$N \to \bar{\ell} + j_1' + j_2'$

$$\boxed{\begin{aligned} 50\% &= \ell \\ 50\% &= \bar{\ell} \end{aligned}}$$

Probe of Majorana nature

$$\boxed{M_N = V_R \, m \, V_R^T}$$

lepton mixings

$V_R = $ - " - in gen space

$\boxed{LFV}$ Lepton flavour Violation

(a) neutrino oscillation

(ii) $\boxed{\mu \to e\gamma, \ \mu \to e + e + \bar{e}}$

$\mu + N \to e + N$

what happens in SM?
why is it suppressed?

$$B(\mu \to e\gamma) \equiv \frac{\Gamma(\mu \to e\gamma)}{\Gamma(\mu \to e\nu\bar{\nu})} \leq 10^{-13}$$

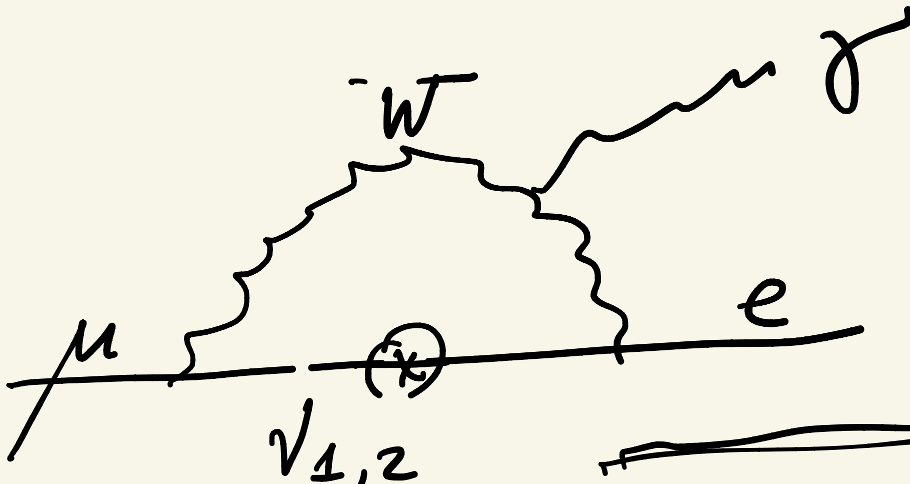
$$B(\mu \to ee\bar{e}) = \frac{\Gamma(\mu \to ee\bar{e})}{\Gamma(\mu \to e\nu\bar{\nu})} \leq 10^{-13}$$

$$\boxed{\mathcal{B}(\mu \to e\gamma)_{SM} = ?}$$

- weak int of ν $\Rightarrow$ <u>ν oscillation</u>

$(m_\nu \neq 0)$

$\Rightarrow$ LFV $\therefore$ $\mu \to e\gamma$



$$\mathcal{B}(\mu \to e\gamma) \simeq \frac{\alpha}{\pi}\boxed{\left(\frac{\Delta m^2}{M_W^2}\right)^2} \text{ mixing}$$

$$\Delta m^2 \simeq 10^{-3}\,eV^2 \simeq 10^{-3}\cdot 10^{-18}\,GeV^2$$

$\Downarrow$

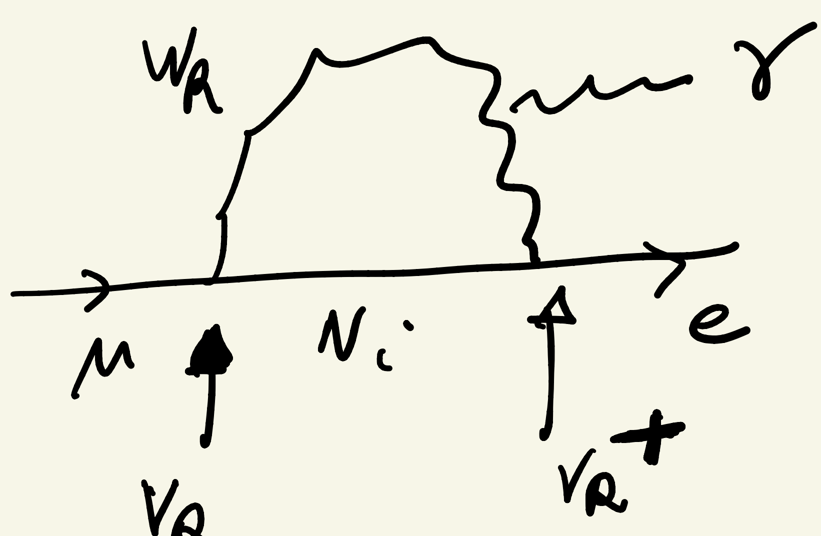
$$\frac{\Delta m^2}{M_W^2} = 10^{-21}/10^4 \simeq 10^{-25}$$

$$\Rightarrow \boxed{B(\mu \to e\gamma) \simeq 10^{-50}}$$

"SM"

"SM" = SM + neutrino mass

$$\boxed{B_{LR}(\mu \to e\gamma) \simeq \frac{\alpha}{\pi}\left(\frac{M_{W_L}}{M_{W_R}}\right)^4 \frac{\Delta m_N^2}{M_{W_R}^2}}$$



$$l_i \to l_j + \gamma$$

$$\boxed{M_N M_N^+}$$

$\boxed{\text{LHC}}$ $M_{W_R} \gtrsim 4\,\text{TeV}$

$M_{W_R} \simeq 8\,\text{TeV}$ (limit)

$M_{W_L} \simeq 80\,\text{GeV}$

$$\Rightarrow \boxed{M_{W_R} \simeq 100\, M_{W_L}}$$

$$\boxed{B(\mu \to e\gamma) \simeq \underset{\left(\frac{\alpha}{\pi}\right)}{10^{-2}} \cdot 10^{-8}\, \frac{\Delta m_N^2}{M_{W_R}^2}}$$

$$\boxed{\text{GIM in SM} \Rightarrow \frac{m_c^2}{M_W^2} \leq 10^{-3}}$$

$$\boxed{\mu \to e e \bar{e}}$$

$$\boxed{\delta^{++}}$$

$$\boxed{\text{also enters}}$$

$$\mathcal{L}_Y = l_R^T \Delta_R Y_R l_R + h.c.$$

$$M_N^* = y_R \langle \Delta_R \rangle$$

$$l_i (M_N^*)_{ij} l_j \delta^{++}$$

$$\boxed{\delta^{--} \to l_i l_j \; (\propto M_N^{ij})}$$

Tello, G. S. 2020

Correlations

$\boxed{\text{LHC}}$ $0\nu 2\beta$, LFV ($\mu \to e\gamma$ ...)

$\boxed{\nu \text{ oscillations}}$, m_ν (KATRIN)

LFV, $0\nu 2\beta$ $\Rightarrow$ new physics

- LFV

LR $\quad B(\mu \to e) \simeq \frac{\alpha}{\pi}\left(\frac{M_L}{M_R}\right)^4 \frac{\Delta m_N^2}{M_R^2}$

$$10^{-14} = \frac{\alpha}{\pi} 10^{-1} \left(\frac{M_L}{M_R}\right)^4 \qquad \frac{\Delta m_N^2}{M_R^2} = O(1/10)$$

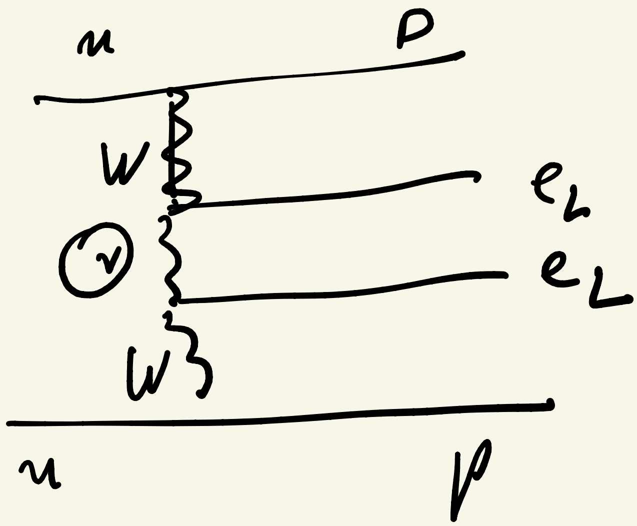
$$\simeq 10^{-3} \left(M_L/M_R\right)^4$$

$\Downarrow$

$$\boxed{\begin{gathered}(M_L/M_R)^4 \simeq 10^{-11}\\ M_R \simeq 10^3 M_L \ !!!\end{gathered}}$$

0ν2β

"SM"



$$\propto G_F^2 \left(\frac{1}{M_L^4}\right) \frac{m_\nu^{M}}{p^2}$$

$\underline{W_R}$:

$$\propto G_F^2 \left(\frac{M_L}{M_R}\right)^4 \frac{m_N}{\cancel{p^2} - m_N^2}$$

$$\simeq G_F^2 \left(\frac{M_L}{M_R}\right)^4 \frac{1}{m_N}$$

"SM" $\quad \dfrac{G_F^2 \, 10^{-10}\,\text{GeV}}{(p = 100\,\text{MeV})^2} \simeq G_F^2 \, 10^{-8} \dfrac{1}{\text{GeV}}$

LR $\quad G_F^2 \left(\dfrac{M_L}{M_R}\right)^4 \dfrac{1}{m_N} \simeq G_F^2 \, 10^{-8} \dfrac{1}{\text{GeV}}$

$$\boxed{\left(\frac{M_L}{M_R}\right)^4 \frac{1}{m_N} \simeq 10^{-8} \frac{1}{\text{GeV}}}$$

tight constraint

$\rightarrow$ e_R !!!

M_N @ LHC (N = Majorana)

$\Downarrow$

$$\begin{cases} M_D = i\,\text{"}\sqrt{\mu_N}\,\sqrt{M_V}\,\text{"} \\ \Updownarrow \\ \Theta = \dfrac{1}{\mu_N} M_D \end{cases}$$