$$
\begin{tikzcd}
 & LR \arrow[dl] \arrow[dr] & \\
C \arrow[d, Rightarrow] & & P \\
\boxed{\text{untangle seesaw}} & &
\end{tikzcd}
$$

$$M_\nu = M_N \frac{v_L}{v_R} - M_D^T \frac{1}{M_N} M_D$$

$$C: \quad M_D = M_D^T \quad \Longrightarrow$$

$$\boxed{M_D = f(\underbrace{M_\nu, M_N}_{\boxed{\text{input}}})}$$

$$\theta_{\nu N} = \frac{1}{M_N} M_D$$

$\hookrightarrow$ dictates decays

$\boxed{\text{· quark sector}}$

$$Y_\phi = Y_\phi^T$$

$$\mathcal{L}_Y^{\Phi} = \overline{f_L}\, Y_\Phi\, \Phi\, f_R + \overline{f_R}\, Y_\Phi^\dagger\, \Phi^\dagger f_L$$

$$C: \quad f_L \longrightarrow C\, \overline{f_R}^T$$

$$\Rightarrow \boxed{Y_\Phi = Y_\Phi^T}$$

$$f_{L,R} = \begin{pmatrix} u \\ d \end{pmatrix}_{L,R} ; \begin{pmatrix} \nu \\ e \end{pmatrix}_{L,R}$$

$$\langle \Phi \rangle = \begin{pmatrix} \phi_1, & \tilde{\phi}_2 \end{pmatrix} = \begin{pmatrix} \phi_1^0 & \phi_2^+ \\ \phi_1^- & -\phi_2^{0*} \end{pmatrix}$$

<u>SM</u> $\langle \phi \rangle = \begin{pmatrix} 0 \\ v \end{pmatrix}$

real by $SU(2)$, $U(1)$

$$\boxed{\langle \Phi \rangle = \begin{pmatrix} v_1 & 0 \\ 0 & -v_2 e^{ia} \end{pmatrix}}$$

$$v_2 < v_1$$

$$\boxed{\varepsilon \equiv \tan 2\beta \sin a \qquad \tan\beta \equiv \frac{v_2}{v_1}}$$

measure of spont. $\not{CP}$ (1)

$$M_q = Y_\Phi \langle \Phi \rangle = M_q^T \quad (2)$$

$$U_{Lq} M_q U_{qR}^\dagger = m_q \quad (*)$$

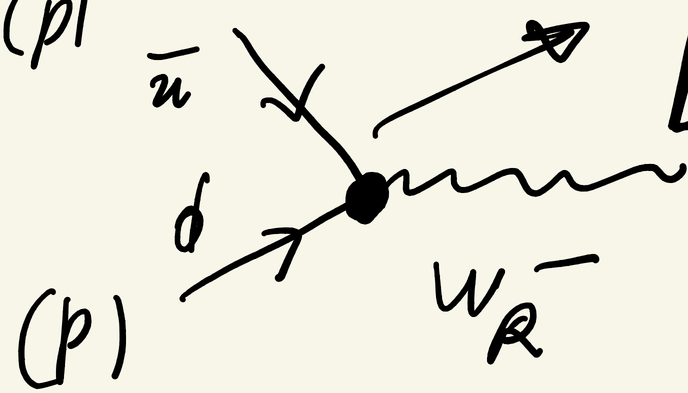
diagonal

$$U_{Lq} M_q U_{Lq}^T = m_q$$

$$\boxed{U_{Rq} = U_{Lq}^*} \quad (3)$$

$$\boxed{\theta_R^q = \theta_L^q} \quad (4)$$

$$\boxed{\cos\theta_{12}^R = \cos\theta_{12}^L}$$



$$\boxed{\circledast \quad U_L \underline{M} V_R^\dagger = m \ (\text{diagonal})}$$

$$\Downarrow$$

$$U_L^\dagger m V_R = M$$

$$H = H^\dagger \Rightarrow \quad U H U^\dagger = \text{diagonal}$$

$$\underline{M} \neq M^\dagger \Rightarrow \quad U_L M V_R^\dagger = \text{—}\,''\,\text{—}$$

$$U_L \neq U_L^\dagger, \; V_R \neq V_R^\dagger$$

$$U_L \underbrace{M M^\dagger}_{\substack{\| \\ \text{hermitian}}} U_L^\dagger = m^2$$

$$V_R \underbrace{M^\dagger M}_{\text{hermitian}} V_R^\dagger = m^2$$

$$\Rightarrow \quad M = V_L^\dagger m V_R$$

$$M^T = U_R^T\, m\, U_L^* = M$$

$$\Rightarrow \boxed{U_R = U_L^*}$$

$$\underline{M} = U\, m\, U^T$$

$$M^T = U\, m\, U^T \;\checkmark$$

$$\Leftrightarrow \boxed{\begin{array}{l} H = U\, d\, U^\dagger \\ H^\dagger = H \end{array}}$$

$$\boxed{U_R = U_L^*}$$

$\cdot\ V_R = V_L^* \Longrightarrow$ phases connected

$\Big\downarrow$ rotated

CKM form (δ_{KM})

$\Rightarrow$ $\boxed{V_R \text{ phases are arbitrary}}$

Summary

C: $\quad M_D = f(M_\nu, M_N)$

$\theta^e_R = \theta^e_L$, phases <u>not</u> determined (RH)

$\boxed{\text{Parity}}$

$$f_L \longleftrightarrow f_R$$

$$\mathcal{L}_Y^{(\Phi)} = \bar{f}_L Y_\Phi \Phi f_R + \bar{f}_R Y_\Phi^\dagger \Phi^\dagger f_L$$

$$\boxed{Y_\Phi = Y_\Phi^\dagger}$$

$$M = Y_\Phi \langle \Phi \rangle$$

$$\boxed{M - M^\dagger \propto \theta} = \tan 2\beta \sin\alpha$$

$$\tan\beta = v_2/v_1$$

$$\boxed{\varepsilon = 0} \Longrightarrow M = M^\dagger \quad (\langle\Phi\rangle \in R)$$

$$\Downarrow$$

$$\boxed{V_R = V_L}$$

$$\boxed{\varepsilon \approx m_b/m_t} \quad (m_b \ll m_t)$$

Tello

$$(V_R - V_L)_{ij} =$$

$$= -i\epsilon \frac{(V_L)_{ik}\,(V_L^{+} m_u V_L)_{kj}}{m_k^d + m_j^d}$$

- $V_L \in R$ case $(\delta_{KM} = 0)$

$$\Rightarrow \boxed{\theta_R = \theta_L}$$

$$\boxed{\theta_{13} = 0 \Rightarrow \delta_{KM} = 0}$$

$$\boxed{\epsilon_{CP} \simeq \theta_{12}\,\theta_{13}\,\theta_{23}\,\delta_{KM}}$$

$$\Downarrow$$

$$\boxed{\theta_R - \theta_L \ \propto \ \text{small mixing}}$$

$$\Updownarrow$$

$$\boxed{\theta_R^{12} - \theta_L^{12} = \boxed{\epsilon \frac{m_t}{m_s}} \, \theta_{23}^L \, \theta_{13}^L \, \delta_{KM}}$$

$$\theta_{23}^L \uparrow 10^{-2} \qquad \theta_{13}^L \uparrow 10^{-3}$$

· $m_t \simeq 175 \text{ GeV} \longleftrightarrow \theta_{13} \simeq 10^{-3}$

$\theta_{23} \simeq 10^{-2}$

$$\boxed{\theta_R^{12} - \theta_L^{12} \simeq 10^{-2} \cdot \cancel{10^{3}} \cdot 10^{-2} \, \cancel{10^{-3}} \simeq 10^{-3}}$$

$$\Downarrow$$

$$(\delta_R - \delta_L)_{KM} \simeq \epsilon \frac{m_c + m_t s_{23}^2}{m_s}$$

$$V_L = f(\theta^L_{ij=3}, \overset{\equiv \delta_L}{\delta_{KM}})$$

$$V_R = f(\theta^R_{ij=3}, \delta_R; \underbrace{\omega_1, \omega_2 ..., \omega_5}_{\text{phases left from KM "trick"}})$$

$$\boxed{\omega_3 \simeq -\epsilon \frac{m_t}{2 m_b}}$$

$$\omega_{1,2,3,4} = f(\omega_3)$$

genesis

$\boxed{\cancel{B+L} \text{ in SM} \\ \Uparrow \\ \text{anomaly}}$

- SM $\Rightarrow$ baryogenesis $\leftarrow$ not enough

$$\frac{n_B}{n_\gamma} \simeq 10^{-10}$$

- LR $\Rightarrow$ $\boxed{\text{baryogenesis failed}}$

$\boxed{\text{leptogenesis} \Rightarrow M_{W_R} \gtrsim 30\,\text{TeV}}$

$$\boxed{\theta_{12}^R - \theta_{12}^L \equiv \theta_C^R - \theta_C^L = - 6\, \theta_{23}^L \theta_{13}^L \delta_{KM} \frac{m_t}{m_s}}$$

- $\varepsilon = 0$ $\quad \Phi \to \Phi^+$

$$\Phi \to U_L \Phi U_R^+ \quad \text{(group)}$$
$$\Phi^+ \to U_R \Phi^+ U_L^+$$

$\langle \phi \rangle \in \mathbb{C} \Rightarrow$ break $\langle \Phi \rangle \to \langle \Phi^+ \rangle$

$\underbrace{}_{\text{such sector}}$

$$\boxed{\varepsilon = \frac{\text{measure of } \cancel{CP}}{\text{measure of } \cancel{P}}}$$

$$\langle \Delta_R \rangle = v_R \qquad \langle \Delta_L \rangle = 0$$

$$\not{\phi}$$

$$\boxed{M_{W_L}^2 = \left(\frac{g}{2}\right)^2 (v_1^2 + v_2^2)}$$

$$\boxed{\varepsilon = \tan 2\beta \sin a}$$

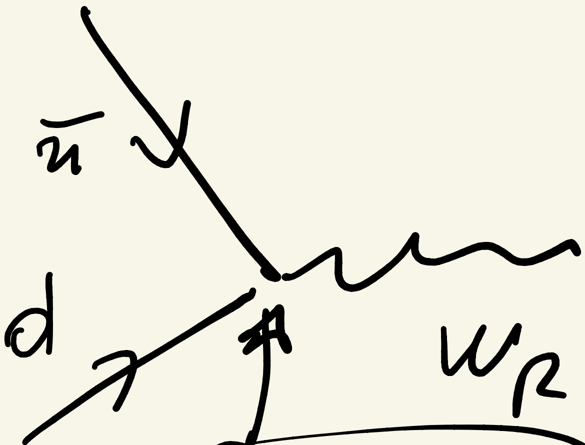
$$\langle \phi \rangle = \begin{pmatrix} v_1 & \\ & v_2 e^{ia} \end{pmatrix}$$

$$\Downarrow$$

$$\boxed{\theta_R \simeq \theta_L}$$

physics of W_R under control

LHC: $M_{W_R} \gtrsim 4\,\mathrm{TeV}$

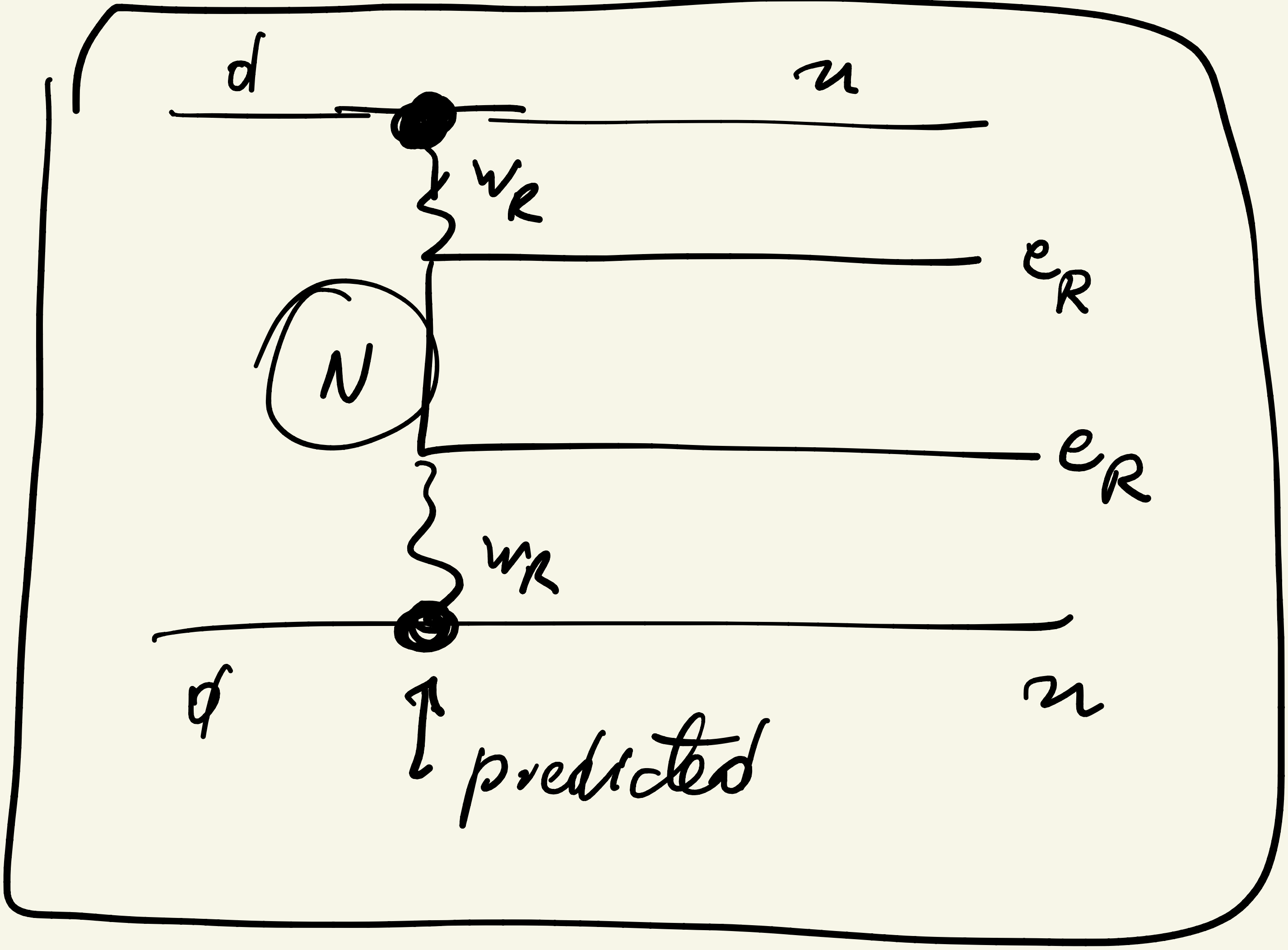


$$g_R (\cos\theta_C)_R = g_L (\cos\theta)_L$$

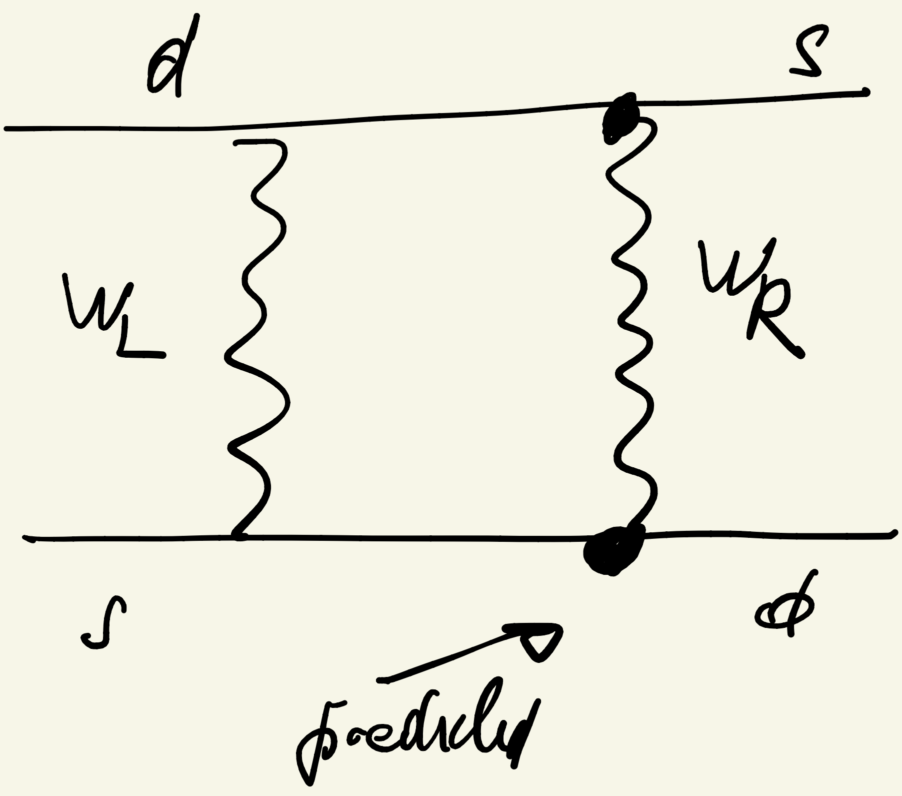
· Low energy manifestation of W_R

· $0\nu 2\beta$

$\Downarrow$

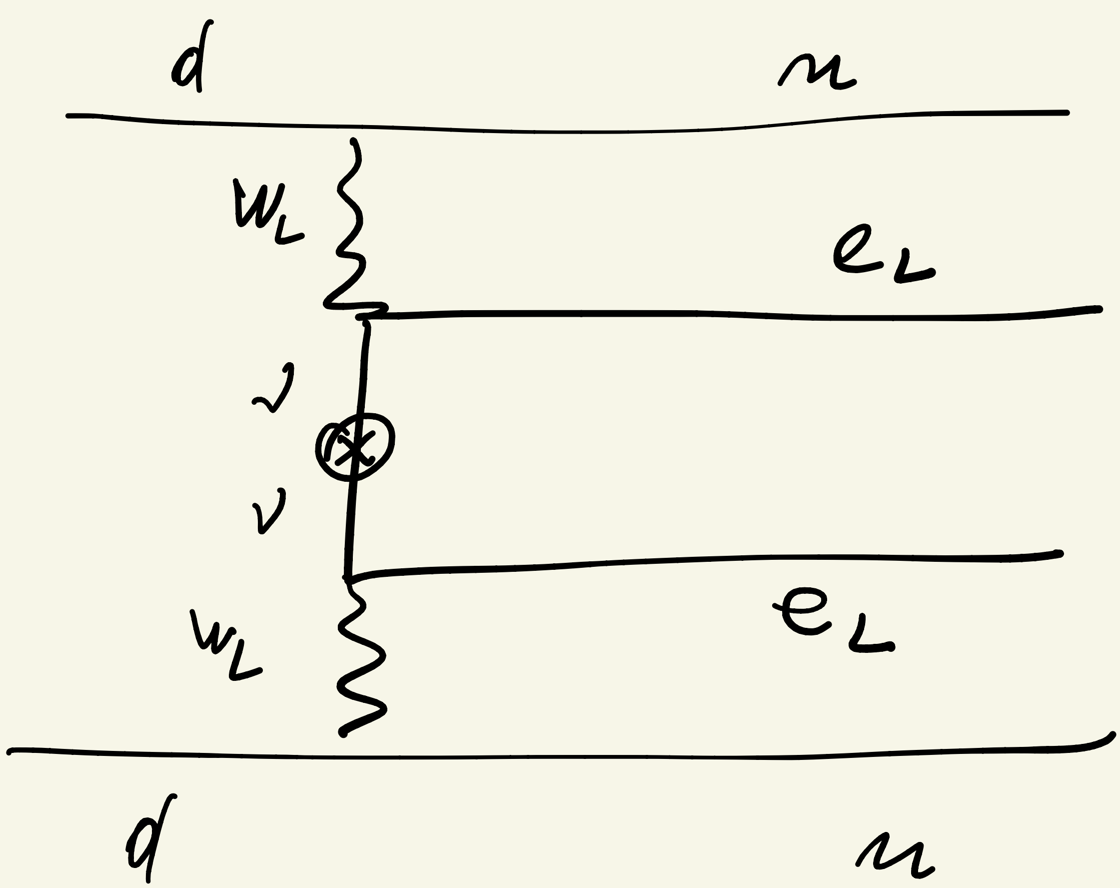


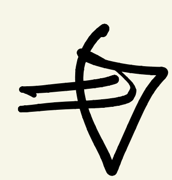
$K - \bar{K}$ mixing



$$\boxed{M_{W_R} = ?}$$

$\boxed{0\nu 2\beta}$



Imagine $\boxed{e = e_R}$ 

<u>not</u> through neutrino

$\Downarrow$

LR

$\Downarrow$

$$\boxed{M_{W_R} \lesssim 10\ \text{TeV}}$$

<u>LFV:</u> $\mu \to e\gamma$

$\mu \to e e \bar{e}$

(M_{W_R}) the scale can be ~ 100 TeV

Parity

$\boxed{\text{Leptonic sector}}$

$$M_\nu = Y_{\Delta_L} v_L - M_D^T \frac{1}{M_N} M_D$$

$$\mathcal{L}_Y^{(\Delta)} = l_L^T C\, i\sigma_2\, \Delta_L Y_{\Delta_L} l_L + \boxed{l_R^T C\, i\sigma_2\, \Delta_R Y_{\Delta_R} l_R}$$

$$\Rightarrow \boxed{Y_{\Delta_L} = Y_{\Delta_R}}$$

$$\Downarrow$$

$$M_\nu = \underbrace{\left(Y_{\Delta_R} v_R\right)}_{M_{\nu_R}} \frac{v_L}{v_R} - M_D^T \frac{1}{M_N} M_D$$

$$N = C\,\bar{\nu}_R^{\,T} \alpha\, \nu_R^{*}$$

$* * *$

$$\boxed{M_\nu = M_N^{*}\,\frac{v_L}{v_R} - M_D^{T}\,\frac{1}{M_N}\,M_D}$$

type II

?

type I

$$\boxed{M_D = f(M_\nu, M_N)}$$

P: $Y_\Phi = Y_\Phi^{\dagger}$ / $\boxed{M_D - M_D^{\dagger} \propto \epsilon}$

$$M_D = y_D (Y_\Phi) \langle \Phi \rangle$$

$$\langle \Phi \rangle - \langle \Phi^* \rangle \propto \epsilon$$

$$\boxed{\epsilon = 0} \Rightarrow \boxed{M_D = M_D^*}$$

$$\Downarrow$$

$$\frac{1}{\sqrt{M_N}} \Big/ M_\nu^* = M_N \frac{v_L}{v_R} - \overset{M_D}{\overset{\shortparallel}{M_D^+}} \frac{1}{M_N^*} M_D^* \Big/ \frac{1}{\sqrt{M_N}}$$

$$\Downarrow$$

$$\frac{1}{\sqrt{M_N}} M_\nu^* \frac{1}{\sqrt{M_N}} = \frac{v_L}{v_R} -$$

$$- \frac{1}{\sqrt{M_N}} M_D \frac{1}{\sqrt{M_N^*}} \frac{1}{\sqrt{M_N^*}} M_D^* \frac{1}{\sqrt{M_N}}$$

$$\boxed{\frac{v_L}{v_R} - \frac{1}{\sqrt{M_N}} M_\nu^* \frac{1}{\sqrt{M_N}} = H H^*}$$

$$\boxed{H \equiv \frac{1}{\sqrt{M_N}} M_D \frac{1}{\sqrt{M_N^*}}}$$

$$\boxed{***}$$

$$\boxed{M_N = M_N^T}$$

$$H^+ = \frac{1}{\sqrt{M_N}} M_D^+ \frac{1}{\sqrt{M_N^*}} = H$$

$$\left(M_D^+ = M_D\right)$$

$$S \equiv \frac{v_L}{v_R} - \frac{1}{\sqrt{M_N}} M_\nu^* \frac{1}{\sqrt{M_N}}$$

$$S^T = \frac{v_L}{v_R} - \frac{1}{\sqrt{M_N}} M_\nu^* \frac{1}{\sqrt{M_N}} = S$$

$$\Downarrow$$

$$\boxed{HH^* = S} \leftarrow \text{symmetric}$$

$$H = H^+$$

$$\Downarrow$$

$$\boxed{M_D = \sqrt{M_N}\, H \sqrt{M_N}^*}$$

if $H = f(S) = f(M_N, M_N)$

$$\Downarrow \quad M_D = f(M_N, M_N)$$

$$\boxed{S = HH^T}$$

$$\Rightarrow \quad SH = HH^T H$$

$$S^* = H^* H^+ = H^T H$$

$$H S^* = H H^T H$$

$$\Downarrow$$

$$\boxed{SH = HS^*} \quad \boxed{S = HH^T}$$

2x2

S

$$H_{2\times 2} = \sqrt{S S^*} \frac{1}{\sqrt{S^*}}$$

$$\parallel$$

$$H^+_{2\times 2}$$

Prove

$\sqrt{M}$ = given (Wikipedia)

$$S = \frac{v_L}{v_R} - \frac{1}{\sqrt{M_N}} M_\nu^* \frac{1}{\sqrt{M_N}}$$

LHC

neutrino oscillation

LHC

$$S \xrightarrow[\text{compute}]{} H\,(M_D)$$

$$\Theta = \frac{1}{M_N} M_D$$

$\boxed{\text{physics}}$

$$N \to e W^-$$

$$\boxed{S = V d V^T} \qquad (S^T = S)$$

$$HH^T = S$$

$$H = V \sqrt{d}\, V^\dagger$$

$$H^T = V^* \sqrt{d^*}\, V^T$$

$$\cancel{HH^T = V \sqrt{d}\, \underbrace{V^\dagger V^*}_{????} \sqrt{d^*}\, V^T}$$

$$\boxed{VV^\dagger = 1}$$

$$\boxed{\begin{matrix} V_L V_L^\dagger = 1 \\ V_R V_R^\dagger = 1 \end{matrix}}$$

$$\boxed{M = V_L\, m\, V_R^\dagger}$$

$$M = S \Rightarrow V_R^* = V_L \Rightarrow$$

$$\left[\ \underline{M}\, M^{\dagger} = V_L\, m\, V_R^{\dagger} V_R\, m\, V_L^{\dagger}\right.$$

Hermiten $= V_L\, m^2\, V_L^{\dagger}$

$(M M^{\dagger})^{\dagger} = M M^{\dagger}$ $\boxed{V_L V_L^{\dagger} = 1}$

$M^{\dagger} M = V_R\, m^2\, V_R^{\dagger}$ $(M^{\dagger} M)^{\dagger} = M^{\dagger} M$

$\boxed{\begin{array}{l} M M^{\dagger} \text{ — diag. by } V_L \\ M^{\dagger} M \text{ — diag by } V_R \end{array}}$ $\boxed{V_R^{\dagger} V_R = 1}$

$\boxed{S = V d V^{T} \qquad V V^{T} = 1}$

$\boxed{\Leftrightarrow \quad H = V h V^{\dagger} \qquad V V^{\dagger} = 1}$

Jordan decomposition

$$S = O\, s\, O^T$$

$OO^T = 1$ $\quad\hookrightarrow$ Jordan form

Jordan form $\neq$ diagonal in general

$$H = O \sqrt{s}\, O^+$$

almost works

$$H^* = O^* \sqrt{s^*}\, O^T$$

$$HH^* = O \sqrt{s}\, \underbrace{O^+ O^*}_{1} \; ---$$

$\sqrt{d} = \pm$ ambiguity

vs

$$\boxed{M_D = \sqrt{M_N}\, O \sqrt{M_V}}$$

$OO^T = 1$ $\quad$ $O \to$ [illegible]